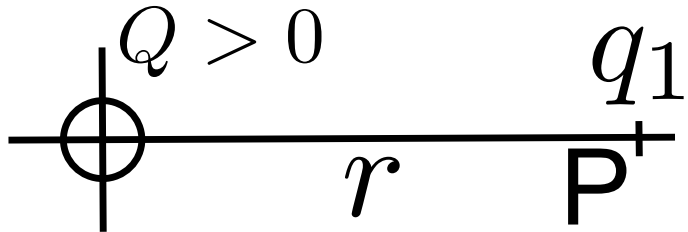


Lecture 7

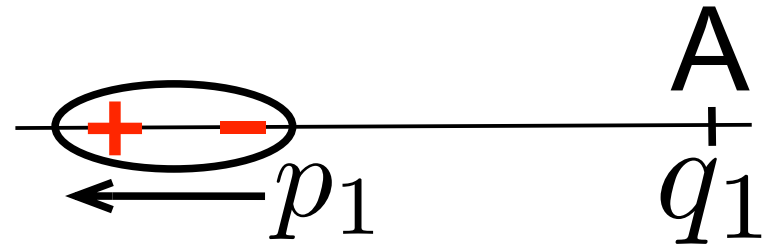
Define: $E = \lim_{q \rightarrow 0} \frac{F}{q}$

What happens when q is finite? Look at two examples: $q_2 > q_1 > 0$



$$E_P^Q = \frac{kQ}{r^2}$$

Test charge q_1 at P : induces p_1



$$E_A = \underbrace{E_A^Q}_{\rightarrow} + \underbrace{E_A^{p1}}_{\leftarrow}$$



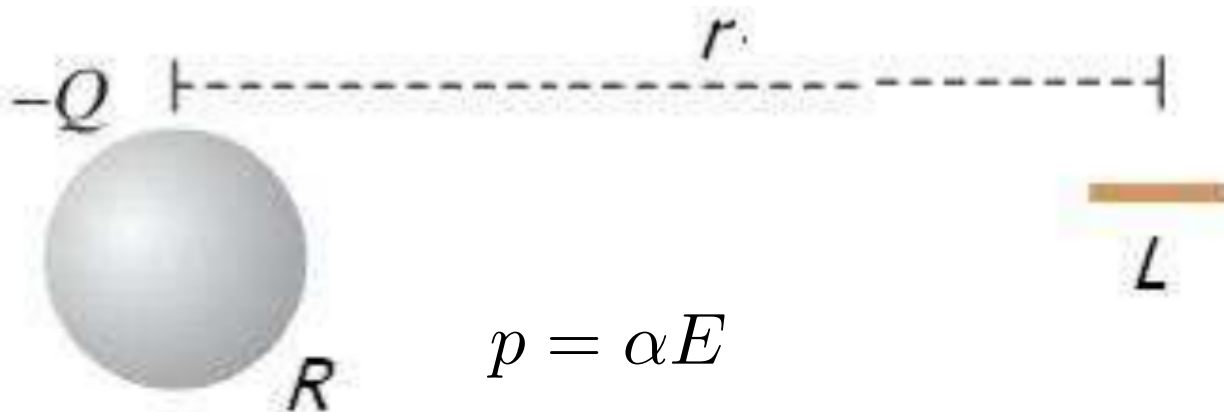
$$q_2 > q_1$$

$$E'_A = \underbrace{E_A^Q}_{\rightarrow} + \underbrace{E_A^{p2}}_{\leftarrow}$$



$$E'_A < E_A$$

Example Polarizability



$$p = \alpha E$$

$$qL = \alpha \left(\frac{kQ}{r^2} \right)$$

Find q, α

Assume origin at the pipe: $E_0 = E_0^Q + E_0^{dipole}$

$$qL = \alpha \frac{2kq}{\left(\frac{L}{2}\right)^2}$$

$$\alpha = \left(\frac{L}{2}\right)^2 \frac{L}{2k} = \frac{\left(\frac{L}{2}\right)^2}{k}$$

$$\begin{array}{c} \uparrow \\ \frac{kQ}{r^2} \end{array} \quad \begin{array}{c} \uparrow \\ \frac{2kq}{\left(\frac{L}{2}\right)^2} \end{array}$$

“Neutralize” the metal ball

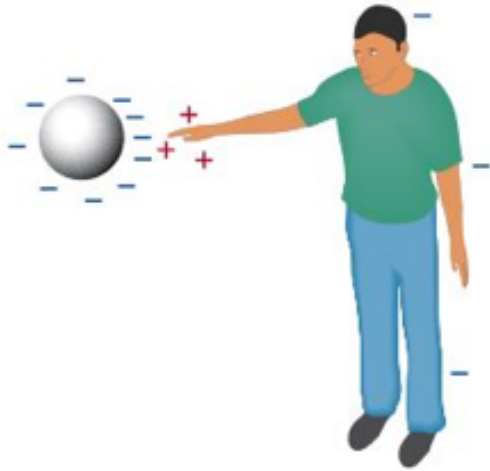


Figure 15.43 The metal is charged, and the person is uncharged but slightly polarized.

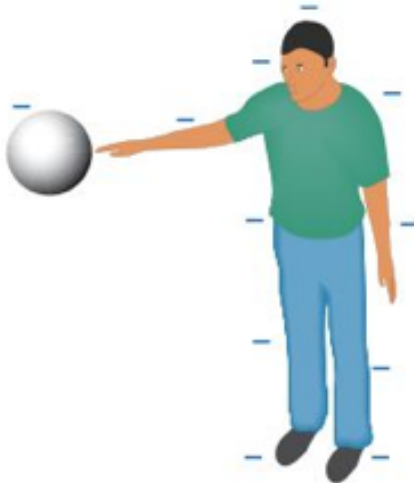


Figure 15.44 The net negative charge is distributed over a much larger area, nearly neutralizing the metal.

“Neutralize” the tape

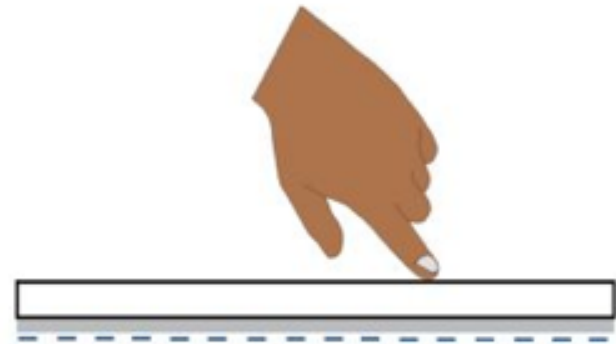


Figure 15.45 You run your finger along the slick side of the tape, and the tape seems to become neutralized.

■ Related experiment: 15.EXP.24

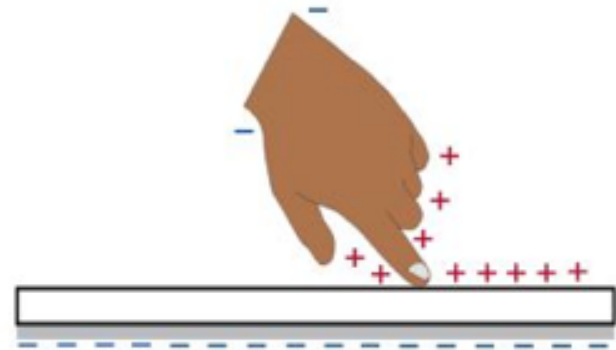
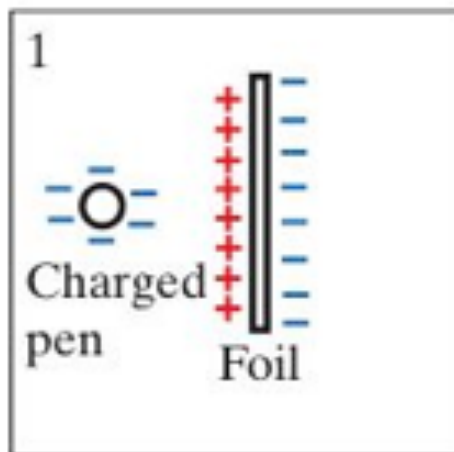
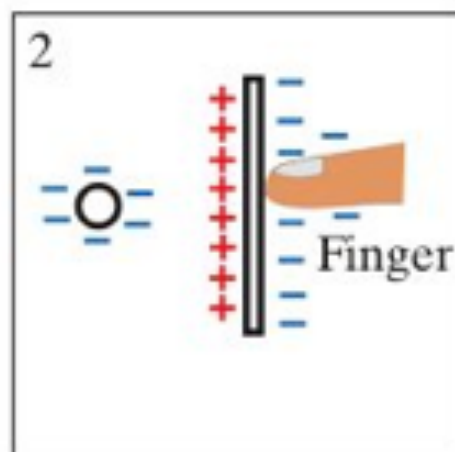


Figure 15.46 Positive ions from the salt solution on your skin are attracted to the negatively charged tape.

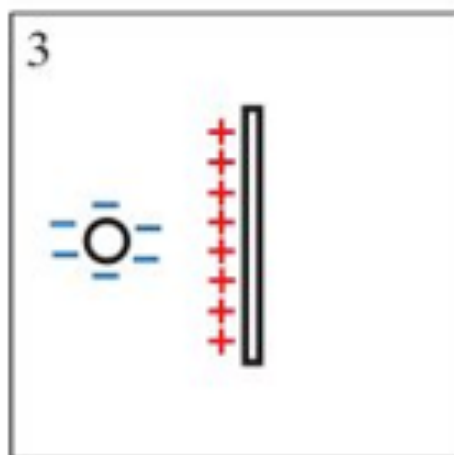
Charging by induction:



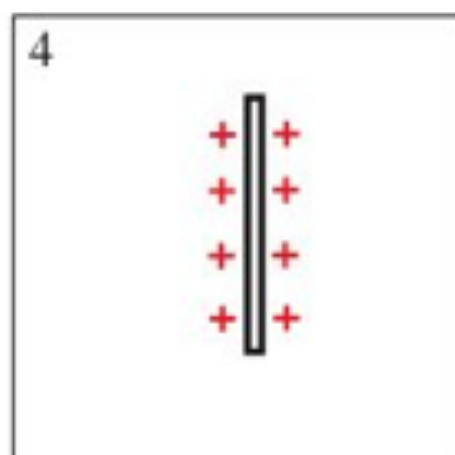
A charged pen is brought near a piece of aluminum foil, which polarizes.



You touch the opposite side of the foil. Negative charge spreads out onto your body.



Leaving the pen in place, you remove your negatively charged finger. The foil is now positive.

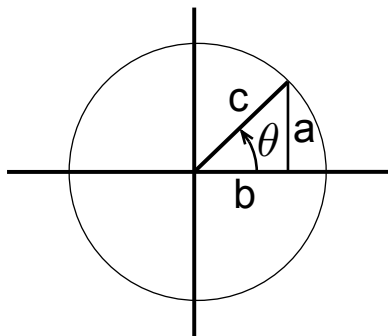


Remove the pen. The positively charged foil is no longer polarized.

Figure 15.47

Review Math

1. Trig Functions:



$$\sin \theta = \frac{a}{c} \quad \cos \theta = \frac{b}{c} \quad \tan \theta = \frac{a}{b} = \frac{\sin \theta}{\cos \theta}$$

$$\sin(\pi - \theta) = \sin \theta, \quad \cos(\pi - \theta) = -\cos \theta$$

2. Derivatives:

$$\frac{d \sin \theta}{d\theta} = \cos \theta$$

$$\frac{d \tan \theta}{d\theta} = \frac{d \frac{\sin \theta}{\cos \theta}}{d\theta} = \frac{\cos \theta \frac{d \sin \theta}{d\theta} - \sin \theta \frac{d \cos \theta}{d\theta}}{\cos^2 \theta}$$

$$\frac{d \cos \theta}{d\theta} = -\sin \theta$$

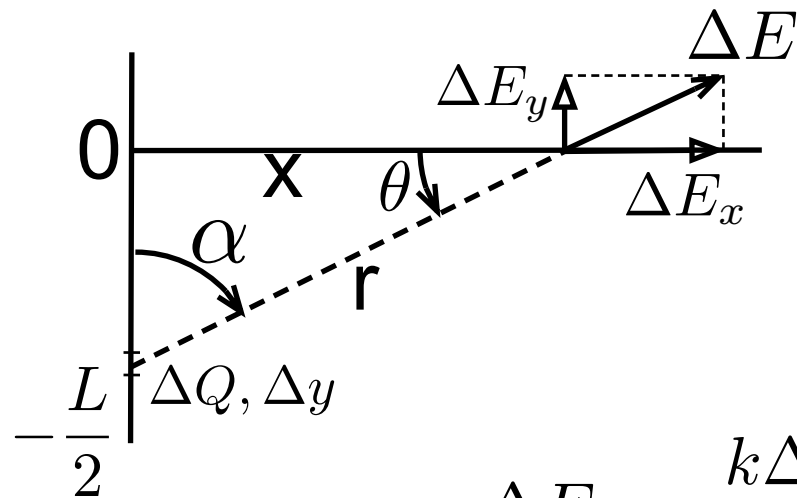
$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

3. Integration:

$$\int_{x_1}^{x_2} \frac{1}{x^2} dx = -\frac{1}{x} \Big|_{x_1}^{x_2} = \frac{1}{x_1} - \frac{1}{x_2}$$

$$\int_{\theta_1}^{\theta_2} \sin \theta d\theta = \int_{\theta_1}^{\theta_2} d(-\cos \theta) = -(\cos \theta_2 - \cos \theta_1) = \cos \theta_1 - \cos \theta_2$$

Review Math continued...



$$E_x = \Sigma \Delta E_x$$

$$E_y = \Sigma \Delta E_y = 0, \text{ by symmetry}$$

$$\Delta E_x = \frac{k\Delta Q}{r^2} \cdot \cos \theta, \quad \cos \theta = \frac{x}{r}$$

$$\text{Put together: } \Delta E_x = \frac{k\Delta Q x}{r^3}$$

Uniform Charge --

$$\frac{\Delta Q}{\Delta y} = \frac{Q}{L}, \quad \Delta Q = \frac{Q}{L} \Delta y \quad \text{or} \quad \Delta E_x = \left(\frac{kQ}{L} x \right) \frac{\Delta y}{r^3}$$

$$E_x = \frac{kQ}{L} x \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dy}{r^3}$$

$$\therefore E_x = \frac{kQx}{L} \Sigma \frac{\Delta y}{r^3}$$

Preview Analytic Result

$$E_x = \frac{kQ}{L} \int \frac{dy}{r^2} \sin \alpha$$

$$E_x = \frac{kQ}{L} \int \frac{d\alpha \sin \alpha}{x}$$

$$E_x = \frac{kQ}{L} \cdot \frac{1}{x} (-\cos \alpha) \Big|_{\alpha_1}^{\pi - \alpha_1}$$

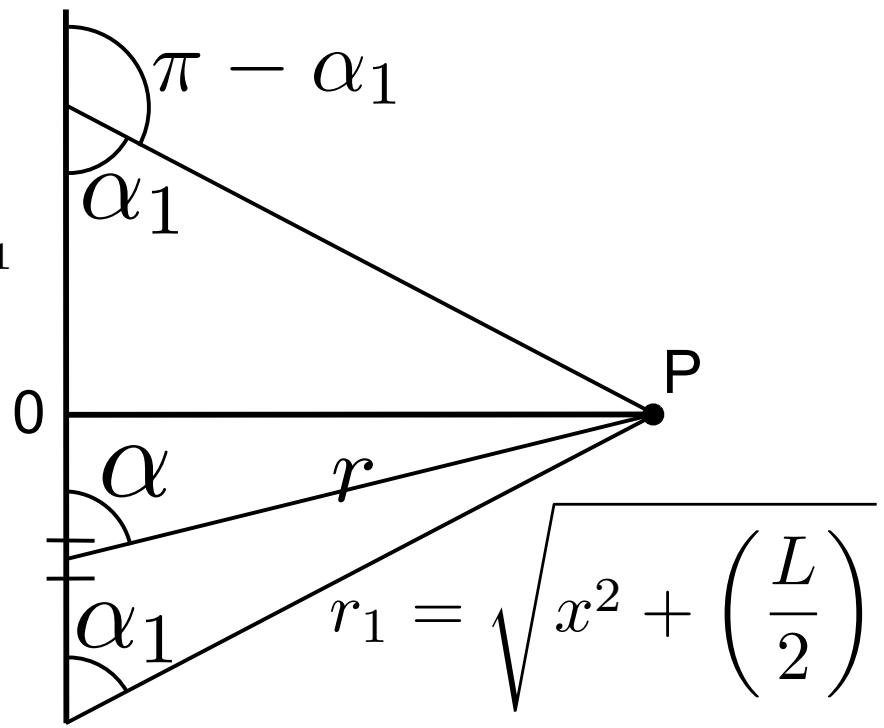
$$E_x = \frac{kQ}{Lx} 2 \cos \alpha_1$$

$$E_x = \frac{kQ}{Lx} \frac{L}{r_1}$$

$$E_x = \frac{kQ}{xr_1} \quad (\text{pg 633})$$

Change of Variable in Integration:

Math ID: $\frac{dy}{r^2} = \frac{d\alpha}{x}$



$$2 \cos \alpha_1 = \frac{2 \frac{L}{2}}{r_1} = \frac{L}{r_1}$$