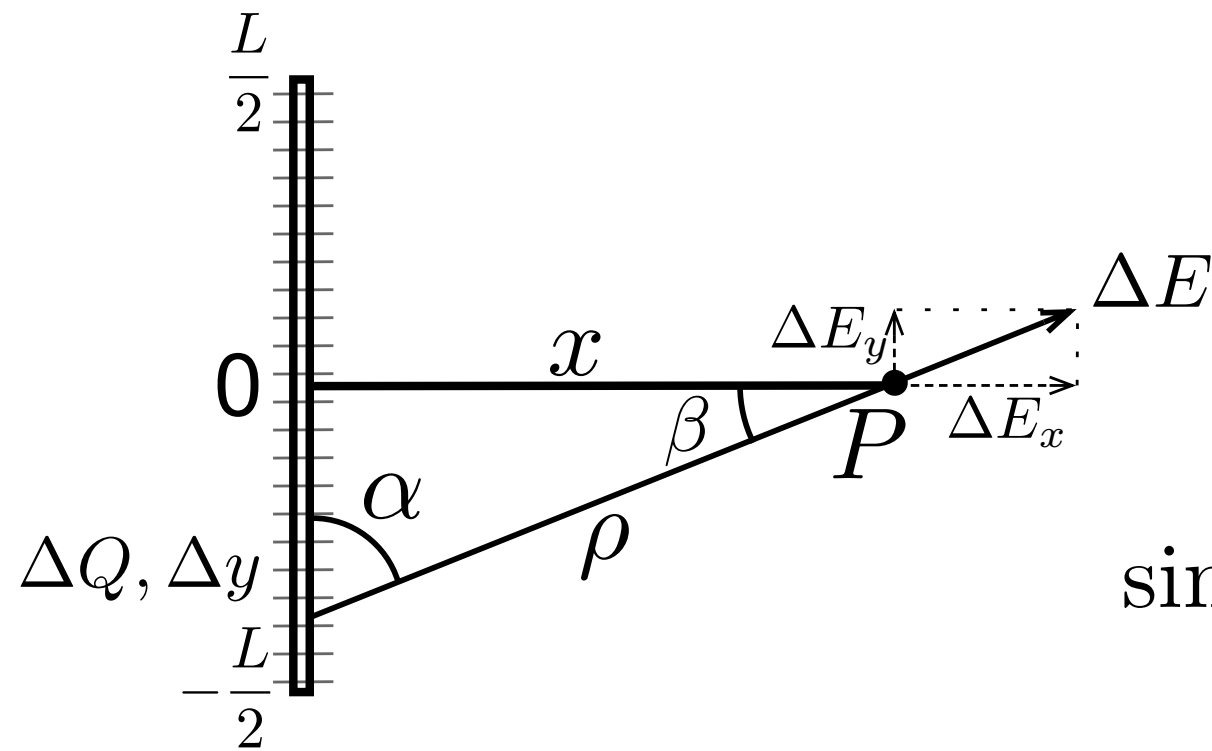


Lecture 8

Uniformly charged rod, Q , L , with charge density $\frac{\Delta Q}{\Delta y} = \lambda = \frac{Q}{L}$

Find E at P :

$$\therefore \Delta Q = \lambda \Delta y$$



$$\sin \alpha = \frac{\Delta E_x}{\Delta E} = \frac{x}{\rho}$$

Cylindrically symmetric E-field:
At the end replace x with r

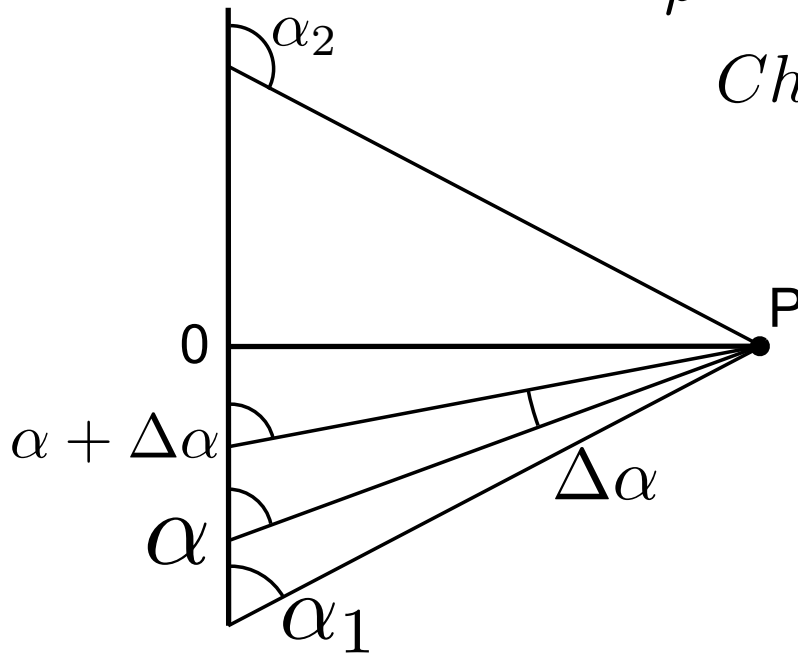
The diagram shows a cylindrical Gaussian surface of radius r and length $2x$. The electric field vector E_x is shown pointing to the right.

Uniformly charged rod continued...

$$\Delta E = \frac{k(\lambda \Delta y)}{\rho^2}, \quad \Delta E_x = \Delta E \sin \alpha$$

$$E_y = \sum \frac{k(\lambda \Delta y)}{\rho^2} \sin \alpha \longrightarrow k\lambda \int \frac{dy}{\rho^2} \sin \alpha$$

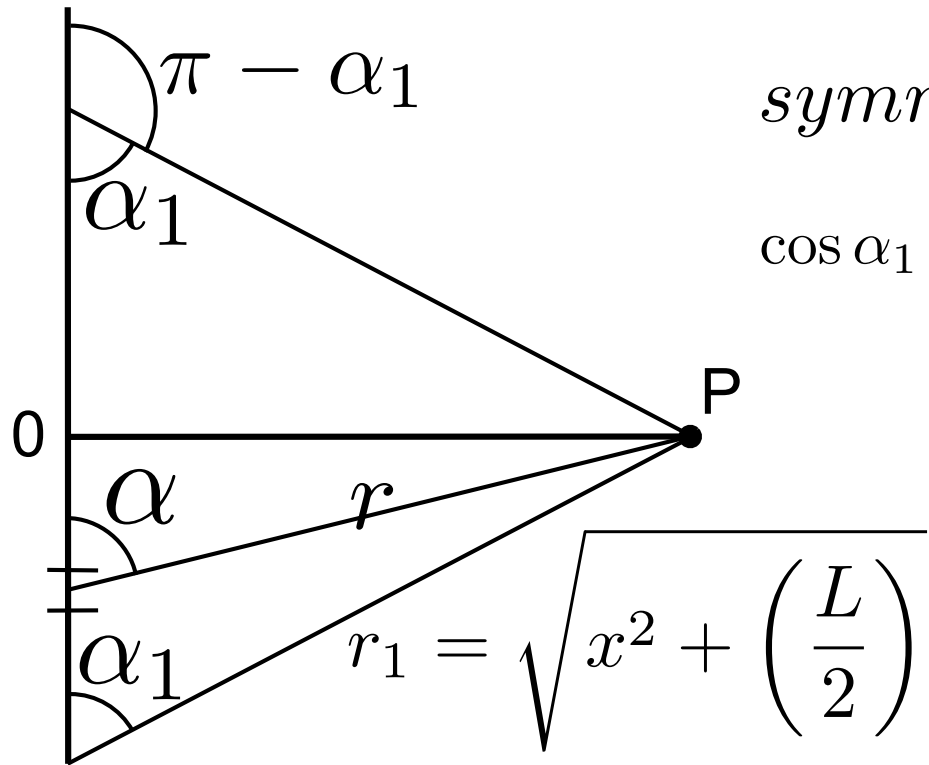
Change variable $\Delta y \rightarrow \Delta \alpha$



I.D. $\frac{dy}{\rho^2} = \frac{d\alpha}{x}$

$$E_x = k\lambda \int_{\alpha_1}^{\alpha_2} \frac{d\alpha}{x} \sin \alpha = \frac{k\lambda}{x} (-\cos \alpha) \Big|_{\alpha_1}^{\alpha_2} = \frac{k\lambda}{x} (\cos \alpha_1 - \cos \alpha_2)$$

Uniformly charged rod continued...



symmetry $\alpha_2 = \pi - \alpha_1$

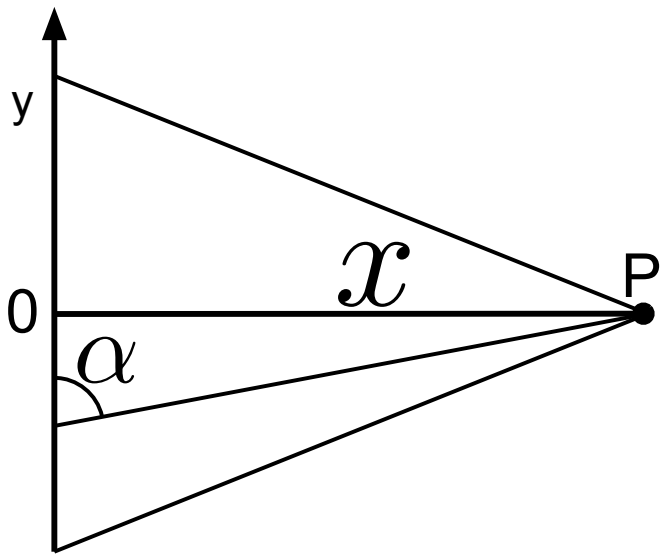
$$\cos \alpha_1 - \cos \alpha_2 = 2 \cos \alpha_1 = 2 \frac{L/2}{\rho_1} = \frac{L}{\rho_1}$$

$$\begin{aligned} \therefore E_x &= \frac{k\lambda}{x_1} \cdot \frac{L}{\rho_1} && \textbf{(see pg 634)} \\ &= \frac{kQ}{x_1 \rho_1} \end{aligned}$$

Derive Math ID:

Change of Variable in Integration:

Math ID: $\frac{dy}{\rho^2} = \frac{d\alpha}{x}$



$$\tan \alpha = \frac{x}{(-y)}$$

$$\frac{d}{d\alpha} LHS = \frac{d}{d\alpha} \tan \alpha = \frac{1}{\cos^2 \alpha} = \frac{1}{\left(\frac{y}{\rho}\right)^2} = \frac{\rho^2}{y^2}$$

$$\frac{d}{d\alpha} RHS = \frac{d}{d\alpha} \frac{x}{(-y)} = (-x) \frac{d}{d\alpha} \frac{1}{y} = (-x) \left(-\frac{1}{y^2}\right) \frac{dy}{d\alpha} = \frac{x}{y^2} \frac{dy}{d\alpha}$$

$$\begin{aligned} \frac{d}{d\alpha} LHS &= \frac{d}{d\alpha} RHS \\ \frac{\rho^2}{y^2} &= \frac{x}{y^2} \frac{dy}{d\alpha} \end{aligned} \quad \longrightarrow \quad \therefore \frac{dy}{\rho^2} = \frac{d\alpha}{x}$$

Back to Electric Field at P:

$$E_r = \frac{kQ}{r \sqrt{r^2 + \left(\frac{L}{2}\right)^2}}$$

$$\text{Far field : } r \gg \frac{L}{2} \quad E_r = \frac{kQ}{r^2} \quad (\text{as expected})$$

$$\text{Near field : } r \ll \frac{L}{2} \quad E_r = \frac{kQ}{r \left(\frac{L}{2}\right)}$$

Later we will see for Gauss Law one finds the near field is given by:

$$\frac{\lambda}{2\pi\epsilon_0 r}$$

Check:

$$\begin{aligned} RHS &= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{\frac{rL}{2}} \\ &= \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{r} \end{aligned}$$

$$F_{drop}^{rod} = (-q)E_{-}^{rod} + qE_{+}^{rod} < 0$$

$$E_r = \frac{2k\lambda}{r}$$

$$|F_{drop}^{rod}| = q \frac{2k\lambda}{r} \left| -\frac{1}{1-\epsilon} + \frac{1}{1+\epsilon} \right|$$

$$\downarrow$$

$$-(1+\epsilon) + (1-\epsilon) = -2\epsilon$$

$$= -2 \frac{s/2}{r}$$

$$= -\frac{s}{r}$$

$$|F_{drop}^{rod}| \frac{2kq\lambda}{r} \cdot \frac{s}{r} \propto \frac{1}{r^2}$$

