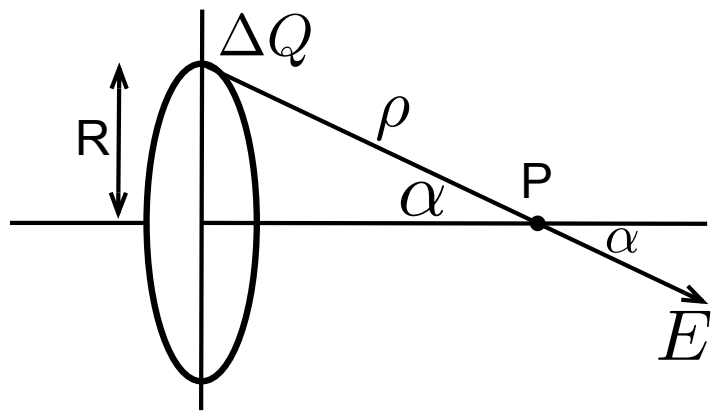


Lecture 9 Given: Uniformly Charged Ring, Q, R (Find Electric Field at P)



$$\Delta E = \frac{k\Delta Q}{\rho^2} \qquad \Delta E_z = \Delta E \cos \alpha = \Delta E \cdot \frac{z}{\rho}$$

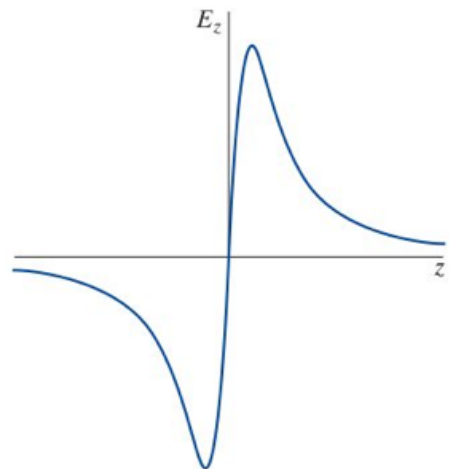
$$E_z = \int dE_z = \int \frac{k\Delta Q}{\rho^2} \cdot \frac{z}{\rho} = \frac{kQz}{\rho^3} \qquad \rho = \sqrt{R^2 + z^2}$$

Far field : $z \gg R$ $E_z = \frac{kQz}{z^3} = \frac{kQ}{z^2}$ (as expected)

Near field : $z \ll R$ $E_z = \frac{kQz}{R^3}$

Oscillation effect:

$$F_z = (-e)E_z$$
$$\ddot{z} = -\omega z$$

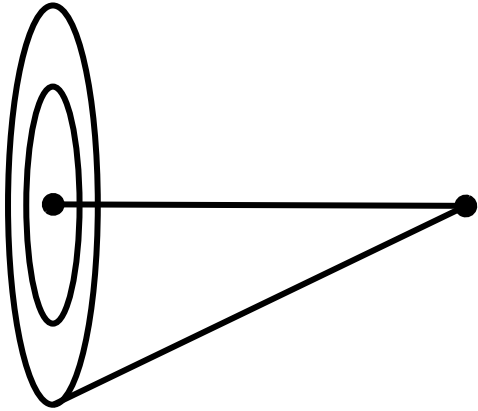


Clicker Question 9-1

For a positively charged ring, determine the sign of the test charge placed near $z = 0$, which will lead to small z oscillations about $z=0$.

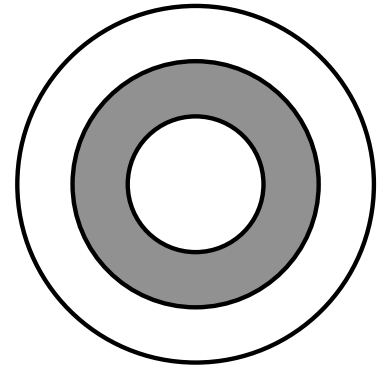
	Sign of Test Charge
1	Positive charge
2	Negative charge

Uniformly Charged Disk



$$Q^{disk} = \sum \Delta Q^{disk}$$

concentric rings



$$\Delta Q^{disk} = \sigma 2\pi r \Delta r, \quad E^{ring} = \frac{k Q^{ring} z}{\rho^3}$$

$$Q^{disk} = \sum \Delta Q^{disk} = \sum \sigma (2\pi r \Delta r)$$

$$\begin{aligned} E^{disk} &= \sum \frac{k \Delta Q^{disk} z}{\rho^3} = \sum \frac{k \sigma 2\pi r \Delta r z}{\rho^3} \\ &= \sum (k \sigma 2\pi z) \frac{r \Delta r}{\rho^3} \\ &= (k \sigma 2\pi z) \int \frac{r dr}{\rho^3} \end{aligned}$$

Uniformly charged disk continued...

$$I = \int_0^r \frac{r dr}{\rho^3} \xrightarrow[\rho^2 = z^2 + R^2 \quad 2\rho d\rho = 2r dr]{\text{change of int var}} \int \frac{\rho d\rho}{\rho^3} = \int \frac{d\rho}{\rho^2} = -\frac{1}{\rho} \Big|_z^{\sqrt{z^2 + R^2}}$$

$$\begin{aligned} E^{disk} &= (k\sigma 2\pi z) \left[\frac{1}{z} - \frac{1}{\sqrt{z^2 + R^2}} \right] \\ &= (k\sigma 2\pi) \left[1 - \frac{1}{(1 + \epsilon)^{1/2}} \right] \end{aligned} \quad \text{where } \epsilon = \frac{R^2}{z^2}$$

Lecture 9 Field due to uniformly charged disk

Lec9-1 Evaluate E of uniformly charged disk of radius R
when $z \gg R$

For a uniformly charged circular disk in the xy -plane with total charge Q , area $A = \pi R^2$, and centered on the origin,

$$E(z) = \frac{Q/A}{2\epsilon_0} \left[1 - \frac{z}{(R^2 + z^2)^{1/2}} \right] .$$

When $z \gg R$, $[...] = 1 - \frac{1}{(1+\epsilon)^{1/2}}$, where $\epsilon = \frac{R^2}{z^2}$. What is the term in $[\cdot]$ equal to in the small ϵ approximation?

Choice	1	2	3	4
$\left[1 - \frac{1}{(1+\epsilon)^{1/2}} \right]$ in small ϵ approximation	$-\epsilon/2$	$-\epsilon$	$\epsilon/2$	ϵ

Uniformly charged disk continued...

$$E^{disk} = (k\sigma 2\pi) \left[1 - \frac{1}{(1 + \epsilon)^{1/2}} \right]$$

Far Field:

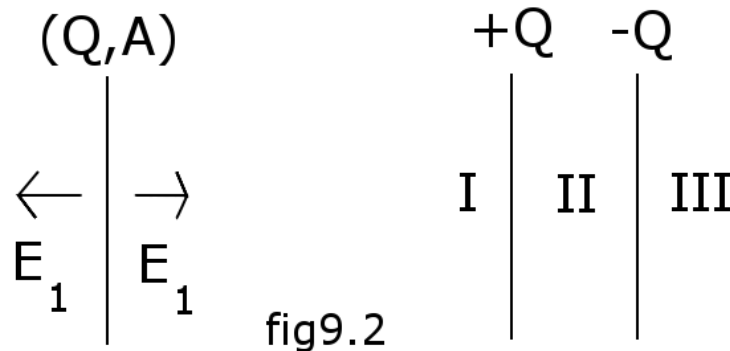
$$\begin{aligned} E^{disk} &= (k\sigma 2\pi) \left[1 - \left(1 - \frac{\epsilon}{2} \right) \right] \\ &= (k\sigma 2\pi) \left[\frac{\epsilon}{2} \right] \\ &= k\sigma 2\pi \left(\frac{R^2}{2z^2} \right) \\ &= \frac{k\sigma \pi R^2}{z^2} \\ &= \frac{kQ}{z^2} \end{aligned}$$

Near Field:

$$\begin{aligned} z &\ll R \\ \frac{1}{z} - \frac{1}{R} &\approx \frac{1}{z} \\ E^{disk} &= (k\sigma 2\pi z) \left[\frac{1}{z} - \frac{1}{\sqrt{z^2 + R^2}} \right] \\ &= k\sigma 2\pi \\ &= \frac{\sigma}{2\epsilon_0} \end{aligned}$$

The field of parallel plate capacitor

For $z \ll R$, $E(z)$ for a charged disk of radius R in the xy -plane reduces to $E(z) = E_1 = (Q/A)/(2\epsilon_0)$; close to the disk and far from its edges, the field is uniform on either side of the disk, as shown on the left in fig. 9.2. Use this fact to analyze the parallel-plate capacitor shown on the right in the figure.



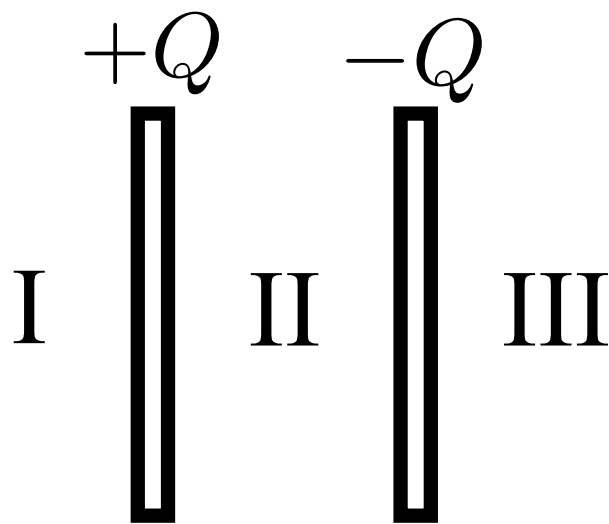
A. Field within the gap (region II)

Choice	1	2	3
E in region II	$(Q/A)/(\epsilon_0)$	$(Q/A)/(2\epsilon_0)$	0

B. Field outside the gap (region I)

Choice	1	2	3
E in region II	$(Q/A)/(\epsilon_0)$	$(Q/A)/(2\epsilon_0)$	0

Capacitor field within the gap



	I	II	III
E^{left}	$-\frac{\sigma}{2\epsilon_0}$	$+\frac{\sigma}{2\epsilon_0}$	$+\frac{\sigma}{2\epsilon_0}$
E^{right}	$+\frac{\sigma}{2\epsilon_0}$	$+\frac{\sigma}{2\epsilon_0}$	$-\frac{\sigma}{2\epsilon_0}$
$E^{resultant}$	0	$\frac{\sigma}{\epsilon_0}$	0

