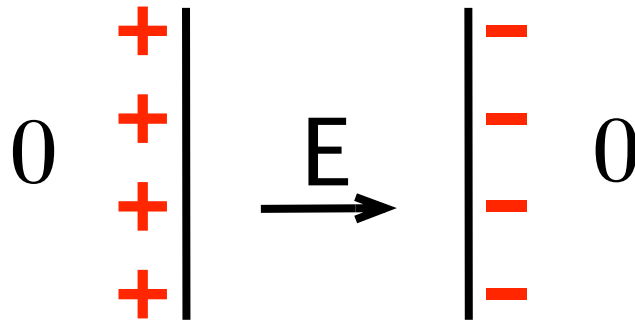


Lecture 10

A capacitor is a device that stores electric energy

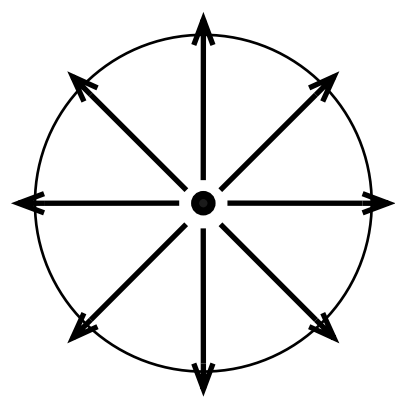
$$\text{Energy Density} = \frac{U}{\Delta V} = \frac{1}{2} \epsilon_0 E^2$$



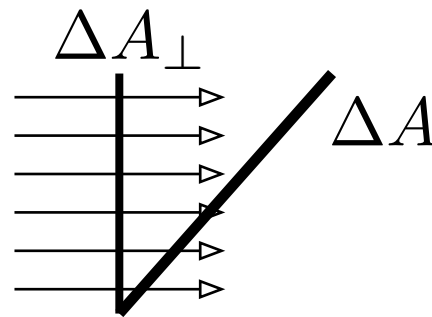
$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$$U_{stored} = \left(\frac{1}{2} \epsilon_0 E^2 \right) Ad$$

Point Charge Q, E-lines, E-flux, Gauss Law



$$\Delta\phi = E\Delta A_{\perp}$$



$$\Phi = \frac{kQ}{r^2} \cdot A\pi r^2$$

$$= \frac{Q}{\epsilon_0}$$

Simple Example of Gauss Law:

$$\Phi_S = \oint_S \vec{E} \cdot \Delta\vec{A}$$

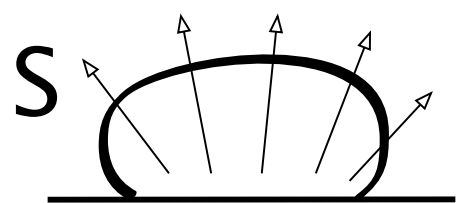
$$= \oint_S \frac{kQ}{r^2} \times 4\pi r^2$$

$$= \oint_S \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \times 4\pi r^2$$

$$= \frac{Q_S}{\epsilon_0}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

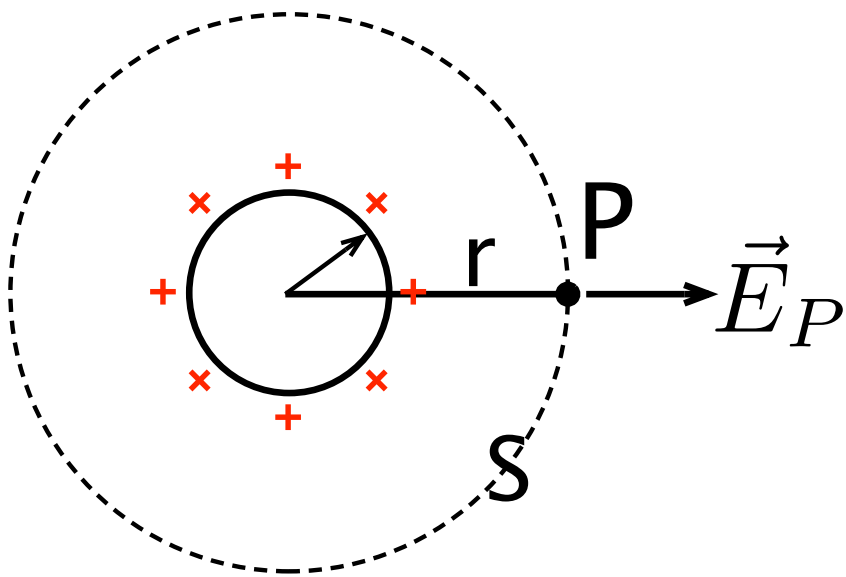
Porcupine-Needle Analogy



Gauss Law:

Electric flux $\longrightarrow \Phi_S = \frac{Q_S}{\epsilon_0}$ $\longleftarrow$ Charge enclosed by surface

Given: A Sphere conductivity shell with Q on the surface



$$\Phi_S = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$\uparrow$
 $E \Delta A$

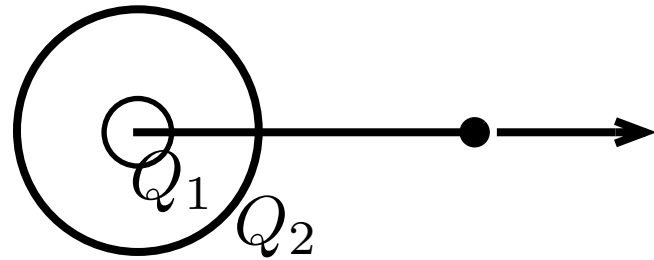
$$E \oint_S dA_{\perp} = E_{?} 4\pi r^2 = \frac{Q_S}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$\therefore E_{?} = \frac{Q}{4\pi\epsilon_0 r^2}$$

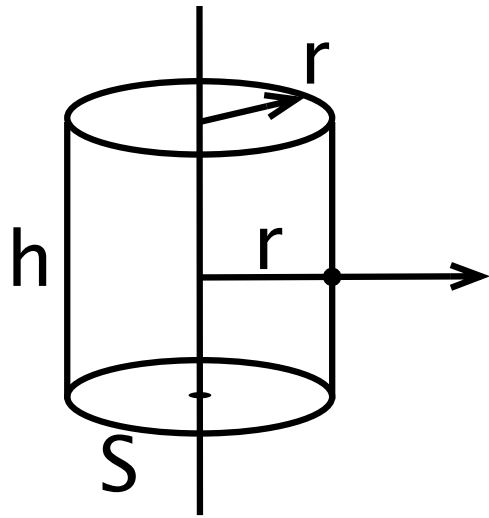
$$E_{?} 4\pi r^2 = \frac{(Q_1 + Q_2 + \dots)}{4\pi\epsilon_0 r^2}$$

$$= \frac{Q}{4\pi\epsilon_0 r^2}$$

Spherical Cavity: $E_{in} = 0,$



Gauss Law: Long Rod



$$\lambda = \frac{\Delta Q}{\Delta y}$$

$$\oint_S dA_{\perp} = h2\pi r$$

$$Q_S = \lambda h$$

$$\oint_S \vec{E} \cdot d\vec{A} = \oint_S E dA_{\perp}$$

$$= E_{\text{?}} \oint_S dA_{\perp}$$

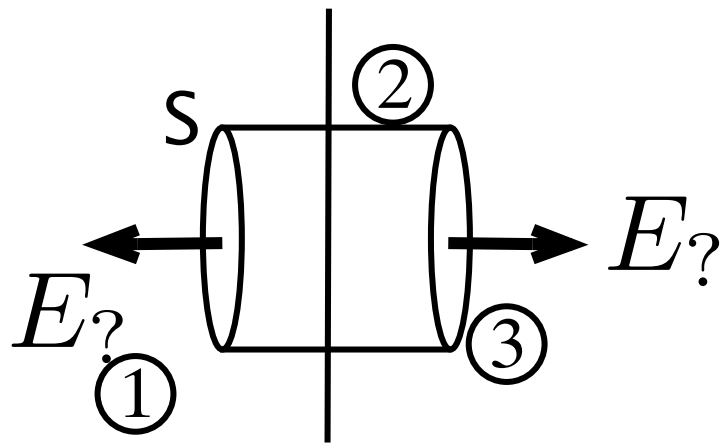
$$= \frac{Q_S}{\epsilon_0}$$

$$= \frac{\lambda h}{\epsilon_0}$$

$$E_{\text{?}} h2\pi r = \frac{\lambda h}{\epsilon_0}$$

$$E_{\text{?}} = \frac{\lambda}{2\pi r \epsilon_0} \quad (\text{agrees with near field expression})$$

Infinite Sheet



$$\text{Given: } \sigma = \frac{Q}{A}$$

Find: $E?$ near the surface

(cylindrical Gaussian surface)

$$\begin{aligned}\Phi_S &= \oint E \Delta A_{\perp} = \Phi_1 + \Phi_2 + \Phi_3 \\ &= E? \Delta A + 0 + E? \Delta A = 2E? \Delta A \\ &= \frac{Q_S}{\epsilon_0} = \frac{\sigma \Delta A}{\epsilon_0}\end{aligned}$$

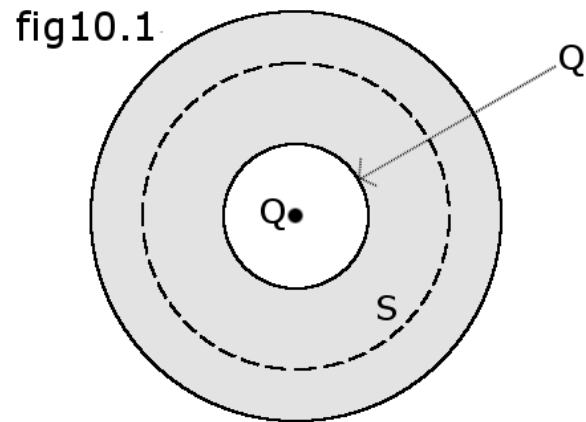
Charge Enclosed by Gaussian Surface: $Q_S = \Delta Q = \sigma \Delta A$

$$\therefore 2E? \Delta A = \frac{\sigma \Delta A}{\epsilon_0} \quad \therefore E? = \frac{\sigma}{2\epsilon_0}$$

Clicker Question: Clicker 10.1

A point charge Q surrounded by a neutral conducting shell

Given: a point charge Q at the center of a neutral spherical conducting shell. Find the total charge Q' on the inner surface of the conducting shell.

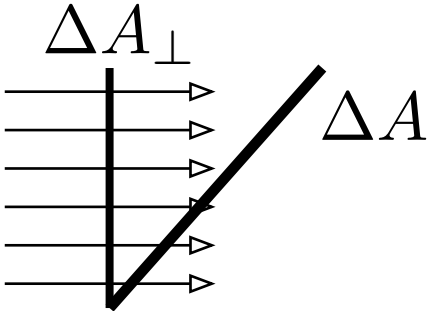


Choice	Q'
1	$-Q$
2	$-Q/2$
3	Q

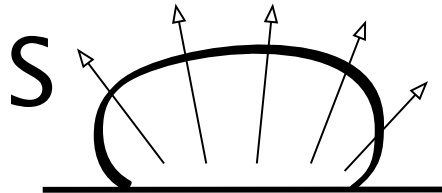
Recap:

Gauss Law: $\Phi_S = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$

$\Delta\phi = E\Delta A_{\perp}$



Porcupine-Needle Analogy



$$\Phi_S = \frac{Q_S}{\epsilon_0}$$