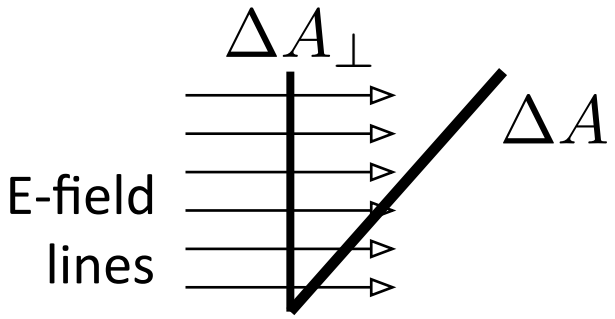
Lecture 11 From coulomb field of a point charge to Gauss Law

We first define Electric Flux:

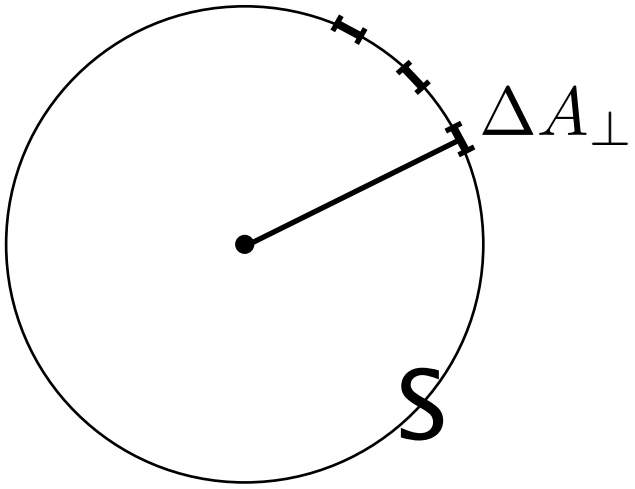


Consider E-lines hitting an area ΔA.

The electric flux through ΔA is defined by:

$$\Delta\phi = E\Delta A_\perp$$

For a point charge Q, the total electric flux hitting the spheres surface with radius r is given by:



$$\sum_{sphere} \Delta\phi = \sum_{sphere} E\Delta A_\perp$$

$$= E\sum_{sph} \Delta A_\perp = E4\pi r^2$$

$$= \frac{kq}{r^2}\cdot 4\pi r^2 = 4\pi kQ = \frac{Q}{\epsilon_0}$$

In other words: **Total Flux** emitted by he point charge $= \dfrac{Q}{\epsilon_0}$

General Statement of Gauss Law

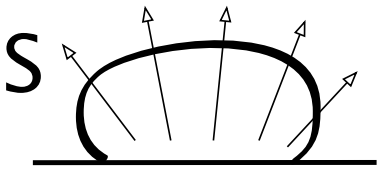
Define Gaussian surface S to be the surface which enclosed the charge then flux emitted through S equates to: $\frac{Q}{\epsilon_0}$.

Turns out, S can be any arbitrary shape, Q can be any charge distribution within S.

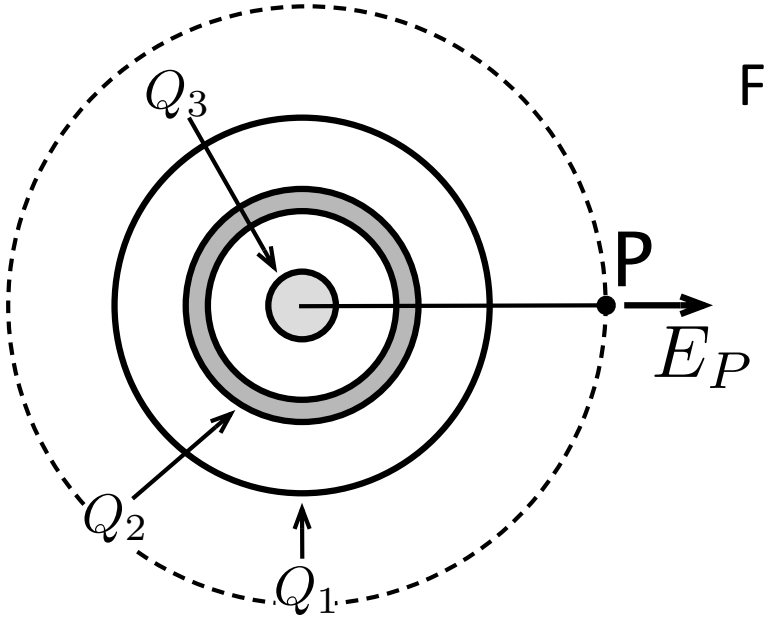
We have $\Phi_S = \frac{Q}{\epsilon_0}$.

Porcupine-needle analogy:

by counting the needles, one can determine the size of the porcupine.



Field due to charges which have a spherical symmetry.



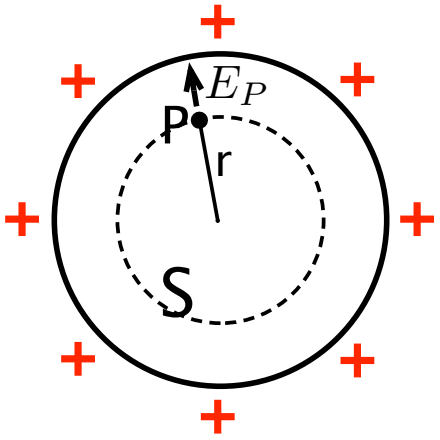
Find E_P

$$\Phi_S = \oint E_P dA_\perp$$

$$= E_P 4\pi r^2 = \frac{Q_1 + Q_2 + Q_3}{\epsilon_0}$$

$$\therefore E_P = \frac{(Q_1 + Q_2 + Q_3)}{4\pi\epsilon_0 r^2}$$

So far as it is spherically symmetric: $E_P = \dfrac{k(Q_1 + Q_2 + Q_3)}{r^2}$

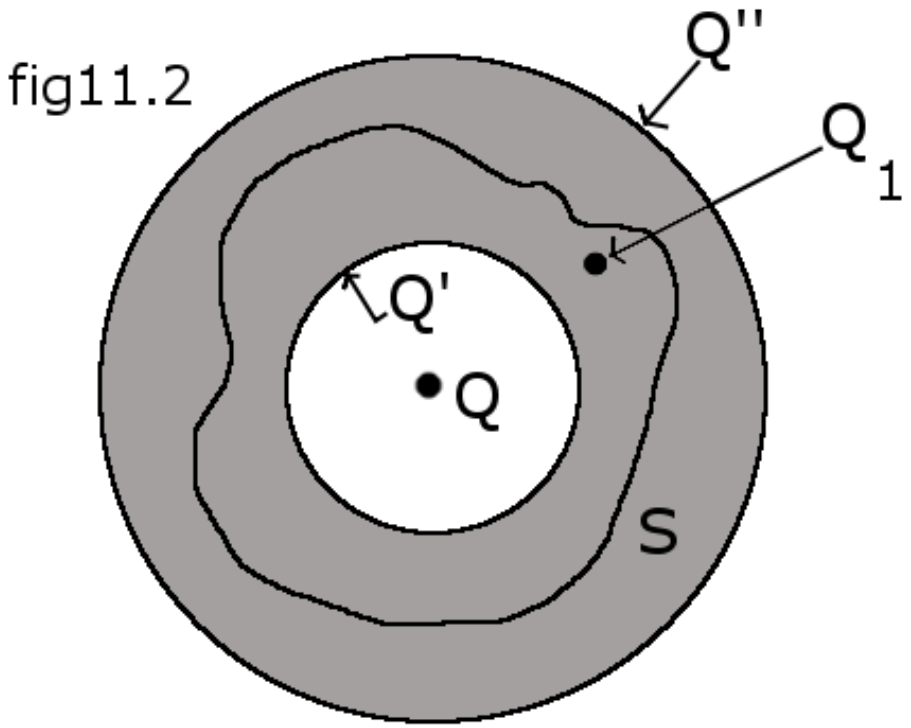


$r < R$ Proof: $E_P = 0$

through P draw Gaussian surface S

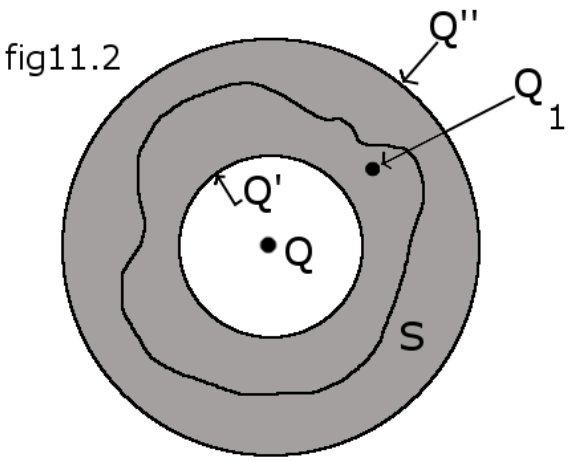
$$E_P 4\pi r^2 = \frac{Q_S}{\epsilon_0} = 0 \Rightarrow E_P = 0$$

Consider a charge Q enclosed within a thick spherical, conducting shell carrying total charge Q_1. See Fig11-2



A Gaussian surface S of arbitrary shape is drawn entirely within the conducting medium.

Clicker 11-1



a. What is the total flux through S?

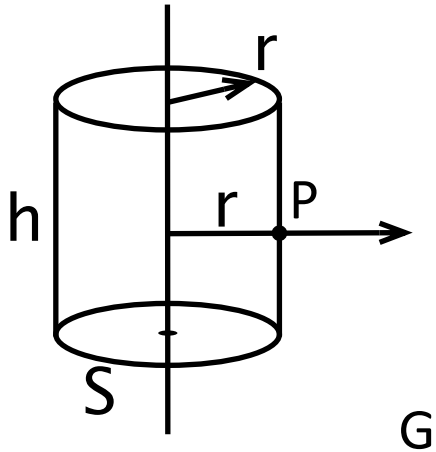
Choice	1	2	3
ϕ_s	Q/ϵ_0	0	$-Q/\epsilon_0$

b. Determine Q', the charge at the inner surface of the shell, and Q'', the charge on the outer surface of the shell.

Choice	1	2	3
Q'	$-Q$	Q	0
Q''	$Q + Q_1$	$-Q + Q_1$	Q_1

Cylindrical Symmetry

Given: Long uniformly charged rod



$$\lambda = \frac{\Delta Q}{\Delta y}$$

Find: Field at P

Gaussian surface – cylinder through P

$E_?$ radially outwards

$$\Phi_S = \oint E_? dA_\perp = \frac{Q}{\epsilon_0}$$

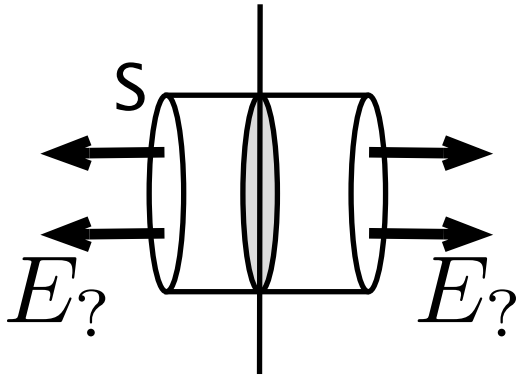
$$E_? \oint dA_\perp = h 2\pi r E_? = \frac{\lambda h}{\epsilon_0}$$

$$\therefore E_? = \frac{\lambda}{2\pi r \epsilon_0}$$

Plane symmetry: one plane

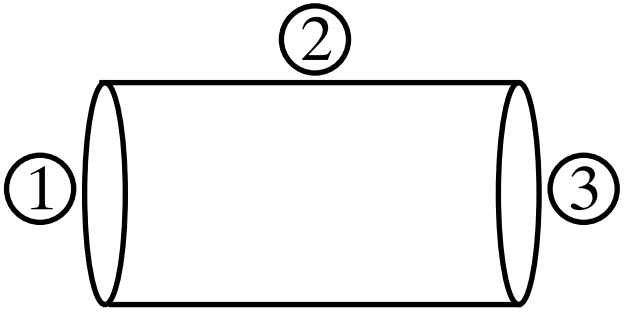
From analytic integration, results we have

$$E_x = \frac{\sigma}{2\epsilon_0} = \frac{\Delta Q/\Delta A}{2\epsilon_0}$$



ΔQ: in the shaded area

ΔA: Ends of the cylinder



$$\Phi_S = \frac{Q_S}{\epsilon_0}$$

$$\Phi = EdA_\perp \Rightarrow \Phi_S = \underset{\|}{\Phi_1} + \underset{\|}{\Phi_2} + \underset{\|}{\Phi_3} = 2E_?\Delta A = \frac{\Delta Q}{\epsilon_0}$$

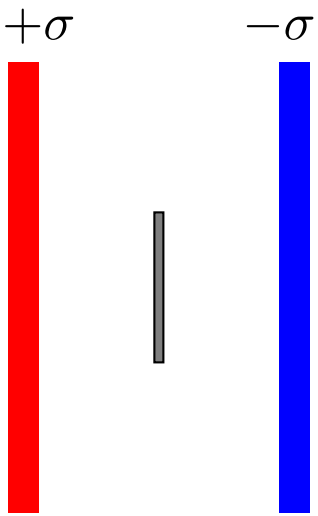
$$E_?\Delta A \quad 0 \quad E_?\Delta A$$

$$\therefore E_? = \frac{\Delta Q/2\Delta A}{\epsilon_0} = \frac{\sigma}{2\epsilon_0}$$

Clicker 10-3

Lec10-3 A metal foil inside of a parallel plate capacitor

A thin neutral foil of area A_1 is placed inside a parallel plate capacitor whose plates have surface charge density σ. The foil is parallel to the plates, and $A_1 \ll A_{plates}$. Find the total charge q_1 on the right-side surface of the foil.



Choice	q
1	σA_1
2	$2\sigma A_1$
3	$(\sigma A_1)/2$