Lecture 11  From coulomb field of a point charge to Gauss Law

We first define Electric Flux:

Consider E-lines hitting an area $\Delta A$. The electric flux through $\Delta A$ is defined by:

$$\Delta \phi = E \Delta A_\perp$$

For a point charge $Q$, the total electric flux hitting the spheres surface with radius $r$ is given by:

$$\sum_{\text{sphere}} \Delta \phi = \sum_{\text{sphere}} E \Delta A_\perp$$

$$= E \sum_{\text{sph}} \Delta A_\perp = E 4\pi r^2$$

$$= \frac{kq}{r^2} \cdot 4\pi r^2 = 4\pi kQ = \frac{Q}{\epsilon_0}$$

In other words: **Total Flux** emitted by the point charge $= \frac{Q}{\epsilon_0}$
**General Statement of Gauss Law**

Define Gaussian surface $S$ to be the surface which enclosed the charge then flux emitted through $S$ equates to: \[
\frac{Q}{\epsilon_0}.
\]

Turns out, $S$ can be any arbitrary shape, $Q$ can be any charge distribution within $S$.

We have \[
\Phi_S = \frac{Q}{\epsilon_0}.
\]

Porcupine-needle analogy: by counting the needles, one can determine the size of the porcupine.
Field due to charges which have a spherical symmetry.

Find \( \mathbf{E}_P \)

\[
\Phi_S = \int E_P dA_{\perp} = E_P 4\pi r^2 = \frac{Q_1 + Q_2 + Q_3}{\varepsilon_0}
\]

\[
\therefore \mathbf{E}_P = \frac{(Q_1 + Q_2 + Q_3)}{4\pi \varepsilon_0 r^2}
\]

So far as it is spherically symmetric:

\[
\mathbf{E}_P = \frac{k(Q_1 + Q_2 + Q_3)}{r^2}
\]

\( r < R \)     Proof: \( \mathbf{E}_P = 0 \)

through \( P \) draw Gaussian surface \( S \)

\[
E_P 4\pi r^2 = \frac{Q_S}{\varepsilon_0} = 0 \Rightarrow E_P = 0
\]
Consider a charge $Q$ enclosed within a thick spherical, conducting shell carrying total charge $Q_1$. See Fig11-2

A Gaussian surface $S$ of arbitrary shape is drawn entirely within the conducting medium.
Clicker 11-1

a. What is the total flux through $S$?

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<tr>
<th>Choice</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_s$</td>
<td>$Q/\epsilon_0$</td>
<td>0</td>
<td>$-Q/\epsilon_0$</td>
</tr>
</tbody>
</table>

b. Determine $Q'$, the charge at the inner surface of the shell, and $Q''$, the charge on the outer surface of the shell.

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<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>$Q'$</td>
<td>$-Q$</td>
<td>$Q$</td>
<td>0</td>
</tr>
<tr>
<td>$Q''$</td>
<td>$Q + Q_1$</td>
<td>$-Q + Q_1$</td>
<td>$Q_1$</td>
</tr>
</tbody>
</table>
Cylindrical Symmetry

Given: Long uniformly charged rod

Find: Field at P

Gaussian surface – cylinder through P

$E_r$ radially outwards

$$\Phi_S = \oint E_r dA_\perp = \frac{Q}{\varepsilon_0}$$

$$E_r \oint dA_\perp = h2\pi r E_r = \frac{\lambda h}{\varepsilon_0}$$

$$\therefore E_r = \frac{\lambda}{2\pi r \varepsilon_0}$$
Plane symmetry: one plane

From analytic integration, results we have

$$E_x = \frac{\sigma}{2\epsilon_0} = \frac{\Delta Q / \Delta A}{2\epsilon_0}$$

$\Phi_S = \frac{Q_S}{\epsilon_0}$

$\Phi = EdA_\perp \Rightarrow \Phi_S = \Phi_1 + \Phi_2 + \Phi_3 = 2E?\Delta A = \frac{\Delta Q}{\epsilon_0}$

$\therefore E? = \frac{\Delta Q / 2\Delta A}{\epsilon_0} = \frac{\sigma}{2\epsilon_0}$
A thin neutral foil of area $A_1$ is placed inside a parallel plate capacitor whose plates have surface charge density $\sigma$. The foil is parallel to the plates, and $A_1 \ll A_{plates}$. Find the total charge $q_1$ on the right-side surface of the foil.

<table>
<thead>
<tr>
<th>Choice</th>
<th>$q$</th>
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<tbody>
<tr>
<td>1</td>
<td>$\sigma A_1$</td>
</tr>
<tr>
<td>2</td>
<td>$2\sigma A_1$</td>
</tr>
<tr>
<td>3</td>
<td>$(\sigma A_1)/2$</td>
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</tbody>
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