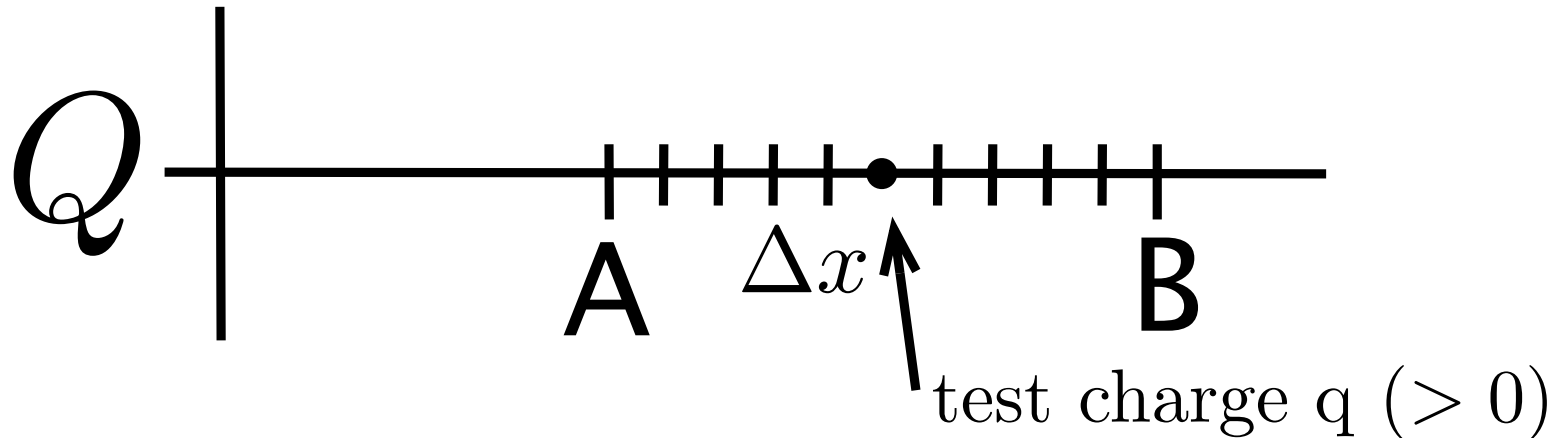


Lecture 15

ΔU and ΔV due to a point charge

let $Q > 0$, placed at 0. $\vec{E} = \frac{kQ}{r^2} \hat{r}$ Along x-axis



Consider going from A to B

Iterative Reasoning

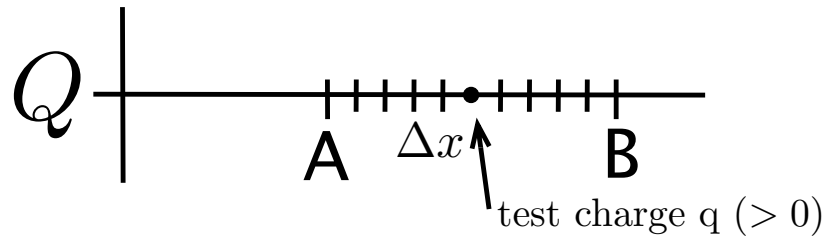
E is to the right.

Δx is along E

so $\Delta U = U(B) - U(A) < 0$ Going downhill.

This agrees with $\Delta U = -E \cdot \Delta x < 0$

ΔU and ΔV due to a point charge continued...



Math: $\Delta U = U(\infty) - U(r)$

$$= - \int_r^{\infty} E \cdot dr = - \int_r^{\infty} \frac{kQ}{r^2} dr$$

$$= - \left[\frac{-kQ}{r} \right] \Big|_r^{\infty}$$

$$= - \frac{kQ}{r}$$

Sol. $U(\infty) = 0$, $U(r) = \frac{kQ}{r}$

$$\Delta V = \frac{\Delta U}{q} = \frac{kQ}{x} = V(x) - V(\infty) \quad \text{here} \quad V(\infty) = \frac{U(\infty)}{q} = 0$$

Point Charge Q

$$\text{Field: } \vec{E} = \frac{kQ}{r^2} \hat{r}$$

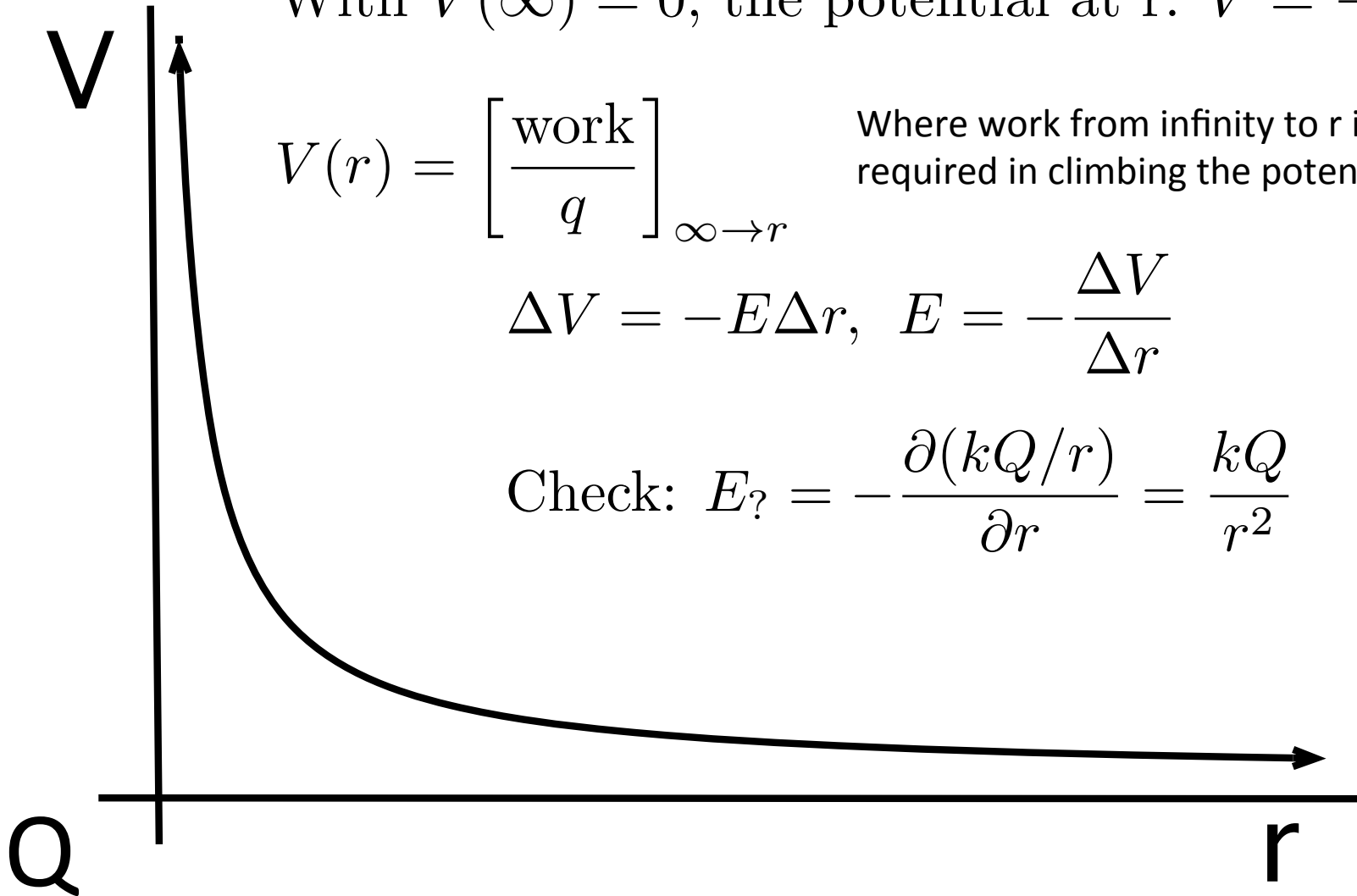
$$\text{With } V(\infty) = 0, \text{ the potential at } r: V = \frac{kQ}{r}$$

$$V(r) = \left[\frac{\text{work}}{q} \right]_{\infty \rightarrow r}$$

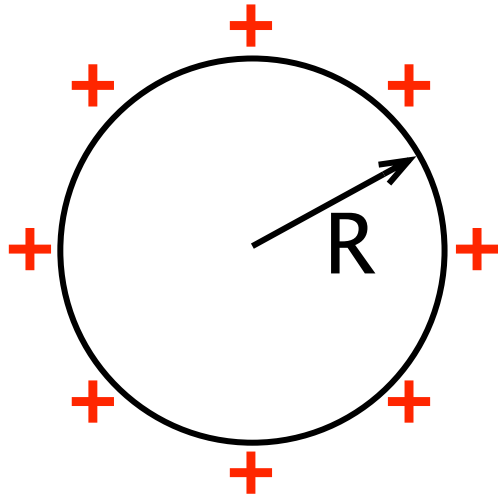
Where work from infinity to r is energy required in climbing the potential hill

$$\Delta V = -E \Delta r, \quad E = -\frac{\Delta V}{\Delta r}$$

$$\text{Check: } E? = -\frac{\partial(kQ/r)}{\partial r} = \frac{kQ}{r^2}$$



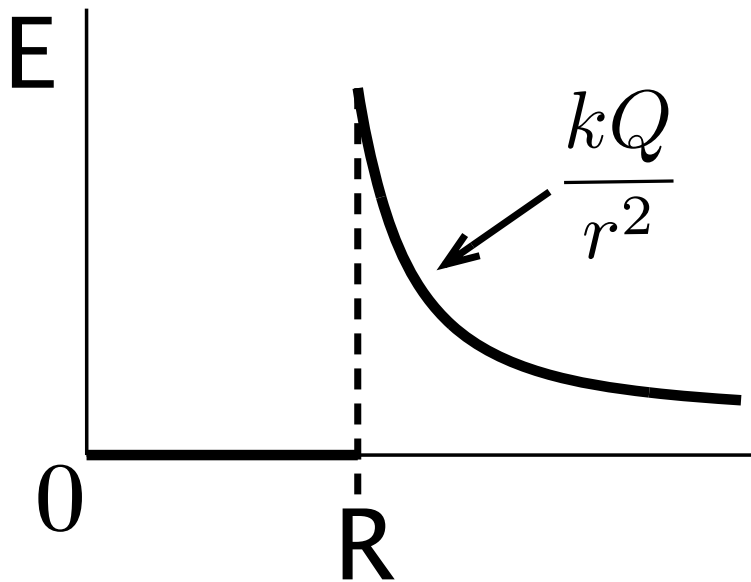
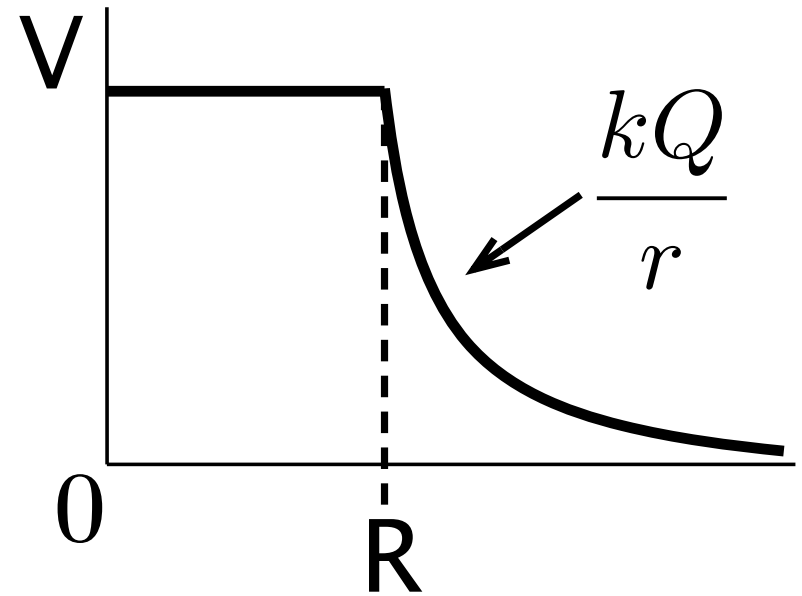
$V(r)$ for a uniformly charged plastic shell



Uniformly charged shell, Q , R .

Find: $V(r)$ by inspection

$$\Delta V = -E\Delta r$$



Fig(clicker) 15.1

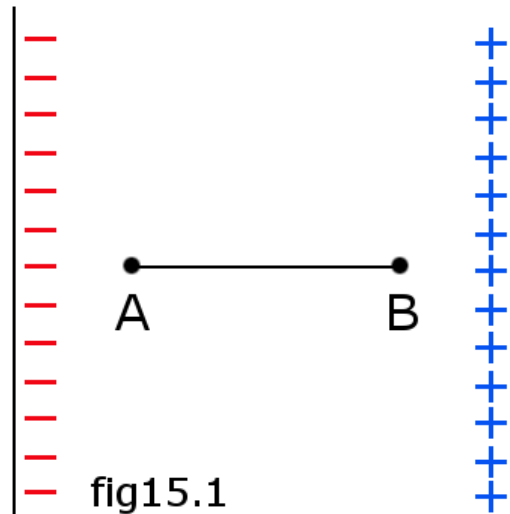
Potential difference across regions of constant E 's.

Lec15-1 Review $dV = -E \cdot d\ell$ in constant E .

Electric potential difference is defined by $\Delta V = \frac{\Delta U}{q} = -E \cdot \Delta \ell$.

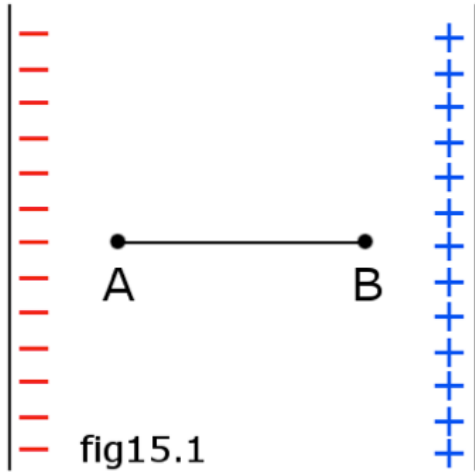
Consider two points within the gap of a parallel plate capacitor, the initial point A and the final point B (see fig15.1).

Determine the sign of $\Delta V = V_B - V_A$.



Choice	Sign of $V_B - V_A$
1	> 0
2	$= 0$
3	< 0

Fig(clicker) 15.1



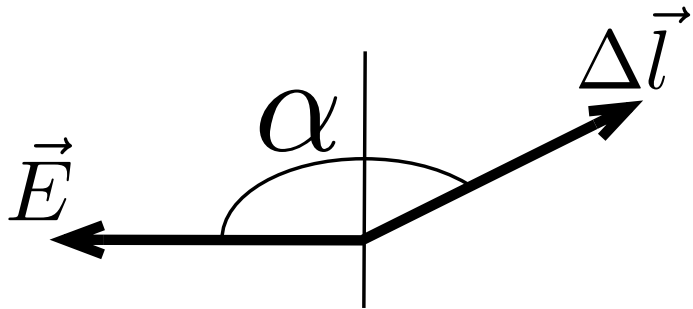
$$\Delta V = -\vec{E} \cdot \Delta \vec{l} = -E \Delta l \cos \alpha = E \Delta l > 0$$

180° ↙

Notice $\Delta V = V_{final} - V_{initial}$

E is the slope of the hill. Consider moving from A to B. Climbing Case $V_B - V_A > 0$

General Case:



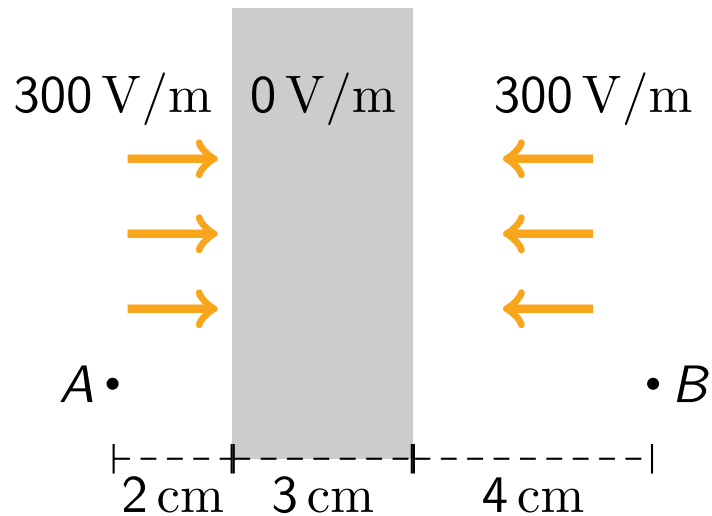
$$\alpha > 90^\circ, \cos \alpha < 0, \Delta V > 0$$

$$\alpha < 90^\circ, \cos \alpha > 0, \Delta V < 0$$

Fig(clicker) 15.2

Lec15-2 $dV = -E \cdot d\ell$ in regions of constant E 's. Exercise-1

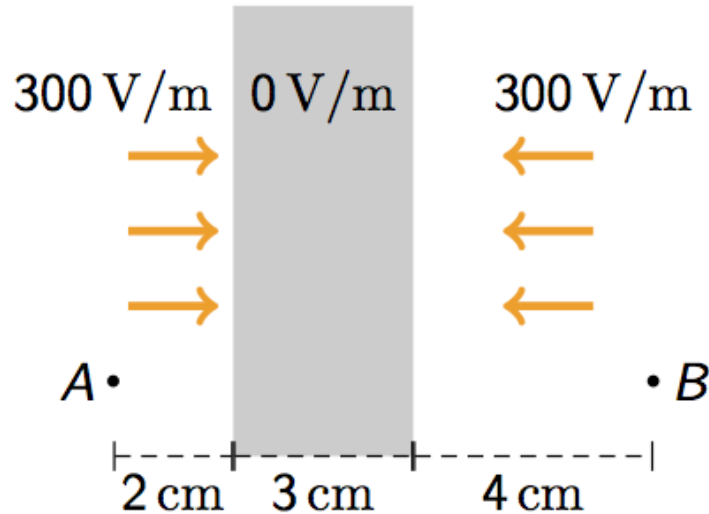
Q16.5B



What is $\Delta V = V_B - V_A$?

1. 270 V
2. 18 V
3. 6 V
4. -6 V
5. -18 V
6. -270 V

Fig(clicker) 15.2



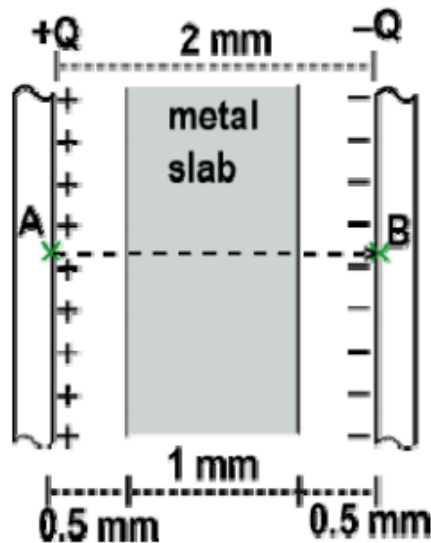
Region 1	$E = 300 \text{ V/m}$	$\longrightarrow$	$\Delta l = 0.02 \text{ m}$	-6
Region 2	$E = 0$			0
Region 3	$E = 300$	$\longleftarrow$	$\Delta l = 0.04 \text{ m}$	$\frac{+12}{+6} \checkmark$

$$\Delta V = \Delta V^I + \Delta V^{II} + \Delta V^{III}$$

Fig(clicker) 15.3

Lec15-3 $dV = -E \cdot d\ell$ in regions of constant E 's. Exercise-2

Q16.5i



In a capacitor $E = (Q/A)/\epsilon_0$

Without the metal slab, $V_B - V_A$ was -1000 volts. Then a metal slab was inserted into the capacitor.

Now $\Delta V = V_B - V_A =$

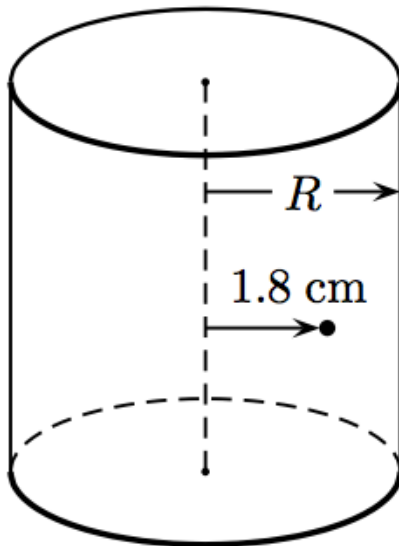
- 1) + 1000 volts
- 2) +500 volts
- 3) 0 volts
- 4) -500 volts
- 5) -1000 volts

Ch17. Hw2:001-003

Long Cylindrical Insulator

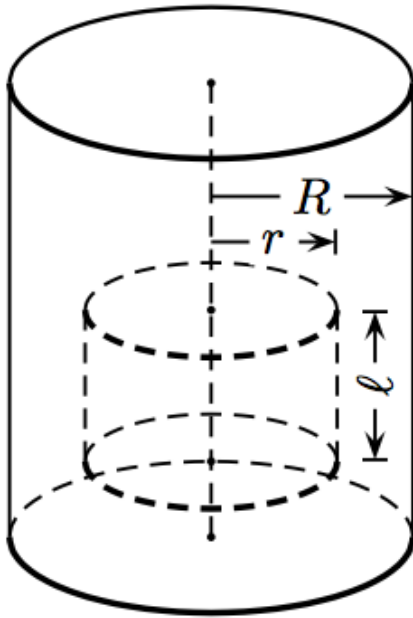
001 (part 1 of 3) 3.0 points

Consider a long, uniformly charged, cylindrical insulator of radius R and charge density $1.1 \mu\text{C}/\text{m}^3$.



What is the electric field inside the insulator at a distance $1.8 \text{ cm} < R$ from the axis? The volume of a cylinder with radius r and length ℓ is $V = \pi r^2 \ell$. The value of the permittivity of free space is $8.85419 \times 10^{-12} \text{ C}^2/\text{N}/\text{m}^2$.

Ch17. Hw2:001-003 continued...



Given: $\rho = \frac{\Delta Q}{\Delta vol} = \text{const.}$

Gauss Law: $E_{\text{?}}(2\pi r h) = \frac{\rho \pi r^2 h}{\epsilon_0}$

$$E_{\text{?}} = \frac{\rho \pi r^2 h}{\epsilon_0} / 2\pi r h = \text{const.} \times r$$

Clicker 17-h2:1-3

What is the sign of $V(R) - v(r)$?

1	+
2	0
3	-

N-dependence of ΔV

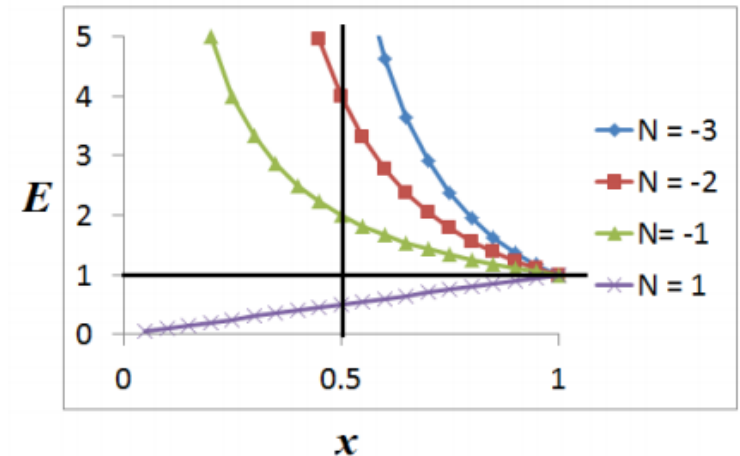
We write the r-dependence of the field for various cases of N as $E = Kr^N$

For dipole: $E = \frac{K}{r^3}$ $N = -3$

point charge: $N = -2$

long rod: $N = -1$

solid rod: $r < R$ $N = +1$



The curves of E vs. x for different N 's are shown in the figure. Notice at $r = 1$, $E = k$, independent of N . In going from $N = -3$ to $N = -1$, the peak at $x = 0$ becomes less and less pronounced. At $N = +1$, the peak turns into a valley.

Lesson: The shape of E vs. x curve is sensitive to the power of N .

EFeildAxisMI17x059 v2
004 (part 1 of 5) 2.0 points

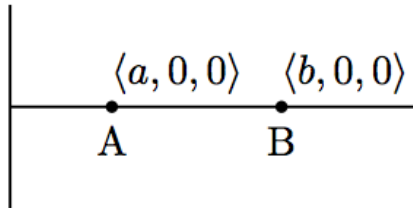
Locations

$$A = \langle a, 0, 0 \rangle$$

and

$$B = \langle b, 0, 0 \rangle$$

are on the $+x$ axis, as shown in the figure below.



Four possible expressions for the electric field along the x axis are given below. For each expression for the electric field, select the correct expression for the potential difference $V_A - V_B$. In each case, K is a numerical constant with appropriate units.

The first possible expression is:

$$\vec{E} = \left\langle \frac{K}{x^2}, 0, 0 \right\rangle$$

Consider the case $a = 0.5$ and $b = 1$. $\Delta V = V(0.5) - V(1)$

How does ΔV vary with N ?

Choice: N increases

1. ΔV increases monotonically
2. $\Delta V = \text{constant}$
3. ΔV decreases monotonically
4. ΔV fluctuates as N increases

