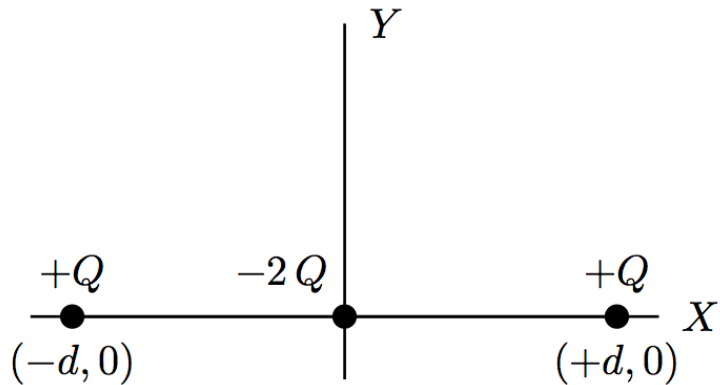


Lecture 16

Second order small ϵ expansion for a quadrupole charge system



Given: Quadrupole charge system is defined (see figure)

Find: $V(x)$, where $x \gg d$

Explanation:

$$\begin{aligned} V(x) &= kQ \left[\frac{1}{x+d} - \frac{2}{x} + \frac{1}{x-d} \right] \\ &= \frac{kQ}{x} \left[\frac{1}{1+\epsilon} - 2 + \frac{1}{1-\epsilon} \right] \\ &= \frac{kQ}{x} f(\epsilon) \end{aligned}$$

$$\epsilon = \frac{d}{x}$$

Notice to the first order in epsilon expansion:

$$f(\epsilon) = (1 - \epsilon) - 2 + (1 + \epsilon) = 0$$

so we need to work to the order of ϵ^2

Second order epsilon expansion

For this, we work with the approximation: $\frac{1}{1-\epsilon} = 1 + \epsilon + \epsilon^2$ (1)

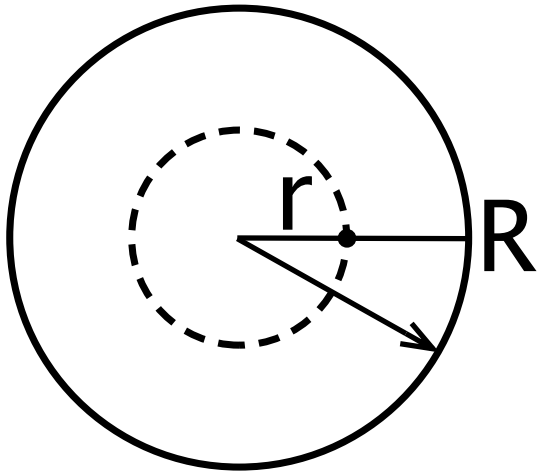
Proof:

$$\begin{aligned} 1 &\stackrel{?}{=} (1-\epsilon)(1+\epsilon+\epsilon^2) = (1+\epsilon+\epsilon^2) - (\epsilon+\epsilon^2+\epsilon^3) \\ &= 1 - \epsilon^3 \quad \text{so (1) is satisfied up to } \epsilon^2 - \text{term} \end{aligned}$$

Using (1)

$$\begin{aligned} f(\epsilon) &= \left[\frac{1}{1+\epsilon} - 2 + \frac{1}{1-\epsilon} \right] \\ &= [1 + (-\epsilon) + (\epsilon)^2] - 2 + [1 + \epsilon + \epsilon^2] \\ &= 2\epsilon^2 = 2 \left(\frac{d}{x} \right)^2 \end{aligned}$$

$$V(x) = \frac{kQ}{x} \cdot \frac{2d^2}{x^2}$$



Given: Uniformly charged sphere, Q, R

Find: $\Delta V = V(0) - V(R)$

Assume $Q > 0$

at any r , E is pointing radially outward.

Climbing upward, when going from R to 0.

$\therefore \Delta V$ is positive.

$$\Delta V = V(0) - V(R) = - \int_R^0 E_r dr$$

Gauss Law:

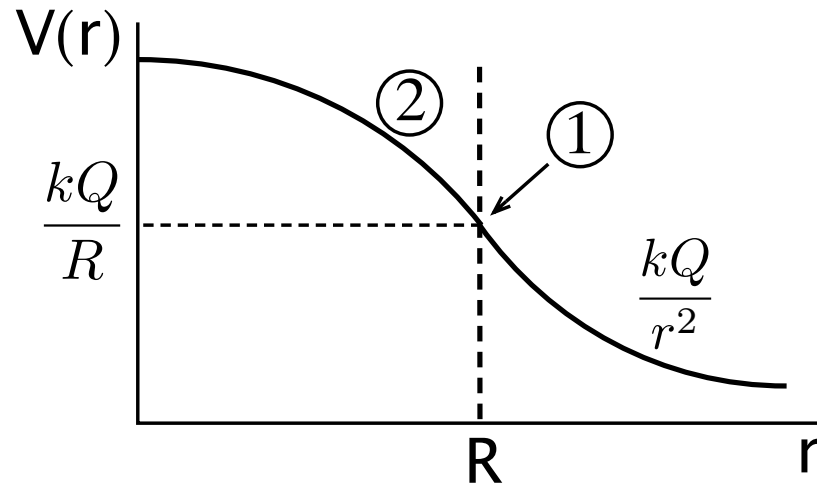
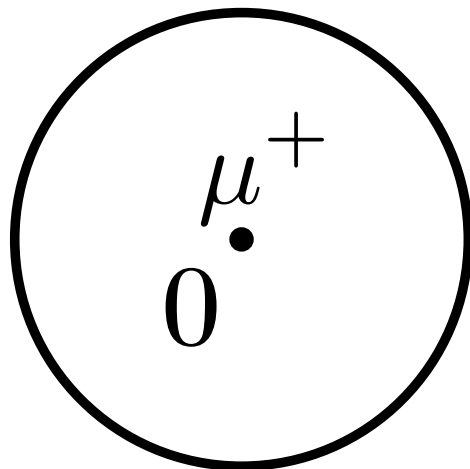
$$4\pi r^2 E_{\text{?}} = \frac{Q \left(\frac{r}{R}\right)^3}{\epsilon_0}, E_{\text{?}} = \frac{kQ}{R^3} r \xrightarrow{r=R} \frac{kQ}{R^2} \checkmark$$

$$\Delta V = V(0) - V(R) = -\frac{kQ}{R^3} \int_R^0 r dr = \frac{kQ}{R^3} \cdot \frac{R^2}{2} = \frac{kQ}{2R}$$

$$\therefore V(0) = \frac{kQ}{2R} + \frac{kQ}{R} = \frac{3}{2} \cdot \frac{kQ}{R}$$

016 (part 1 of 2) 5.0 points

A nucleus contains Z protons of charge e that are assumed to be uniformly distributed throughout a tiny sphere of radius R . What is the potential difference $\Delta V = V_{r=R} - V_{r=0}$?



At $t = 0$, μ^+ created at $r = 0$ with K_i , initial KE

Find: K_f .

Hint, use cons. of energy

$$K_i + U(0) = K_f + U(\infty)$$

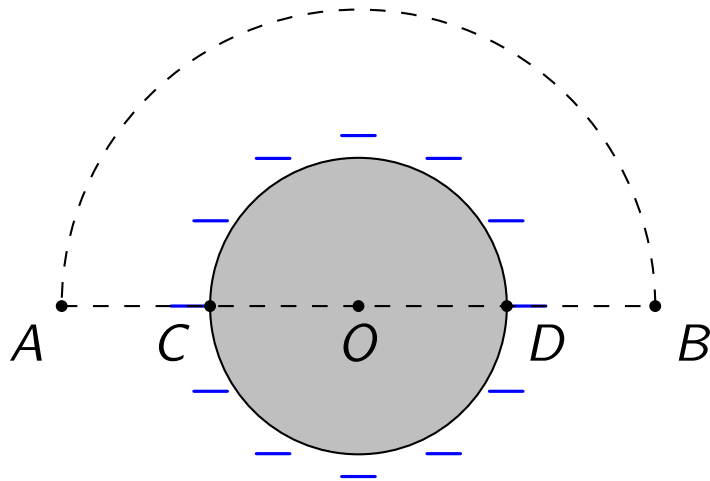
$e [\Delta V + V(R)]$, where $V(R) = \frac{kZe}{R}$

②
①

Fig(clicker) 16-1

Path independence of potential diff., energy in a capacitor

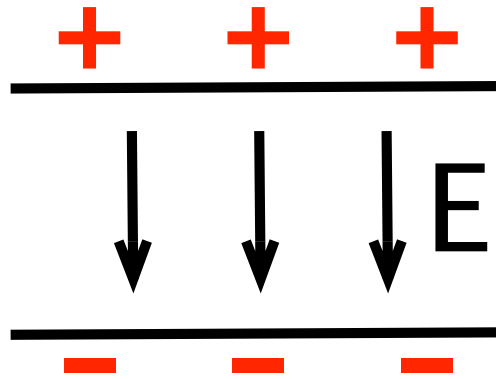
Consider the path of a line and a circular arc.



Which of the following quantities is zero?

1. $V_C - V_A$
2. $V_B - V_O$
3. $V_D - V_B$
4. $V_A - V_B$

Energy Stored in a Capacitor



Capacitance: $C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\epsilon_0 A}{d}$

$$E = \frac{Q/A}{d}$$

An arrow points from the Q/A term in this equation to the Q in the denominator of the Ed term in the equation above.

Mechanical Separation:

$$U = F_{\text{attract}} \times d$$
$$= Q E_Q^{1-\text{plate}} d = Q \times \left(\frac{Q/A}{2\epsilon_0} \right) d = \frac{Q^2}{2C}, \quad C = \frac{\epsilon_0 A}{d}$$

Charging: $V = \frac{q}{C}$

$$\Delta U = V \Delta q, \quad U = \int_0^Q \frac{q}{C} dq = \left. \frac{q^2}{2C} \right|_0^Q = \frac{Q^2}{2C}$$