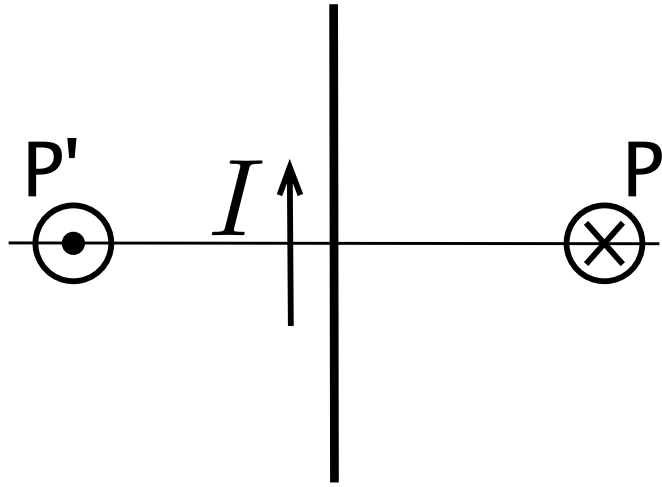


Lecture 21

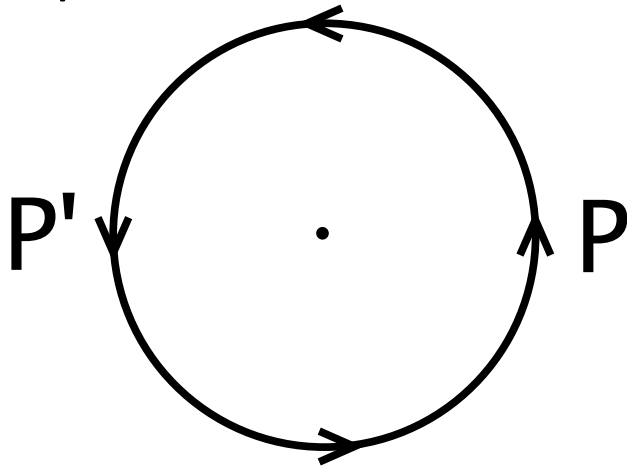


For a long wire with current I , we have found at P a distance r from the wire:

$$B = \frac{\mu_0 I}{2\pi r} \quad (1)$$

B has a curly field pattern, it is into the page at P and out of the page at P'.

Top View:



Rewrite (1) as $2\pi r B = \mu_0 I$

$$LHS = \oint_S \vec{B} \cdot d\vec{l}$$

circle

Ampere's law in it's simplest case says for cylindrically symmetric current, choose "Amperian loop" to be a circle

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$$\oint_S B dl = \mu_0 I_S$$

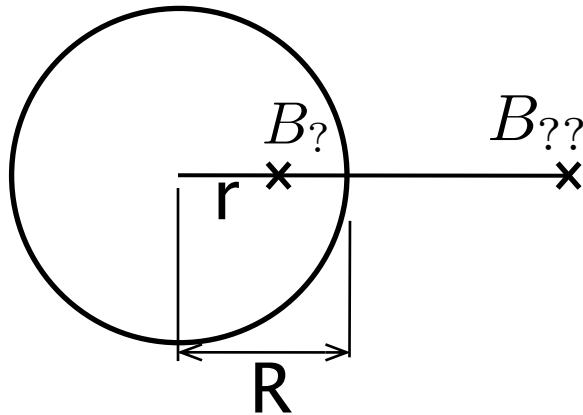
Denote:

B_ϕ at P. Let S be the circular loop, $LHS = B_\phi \oint dl = 2\pi r B_\phi$

$$RHS = \mu_0 I_S = \mu_0 I$$

$I_S =$ current enclosed by S

Long, thick wire: Assume current density is uniform.



Notations:

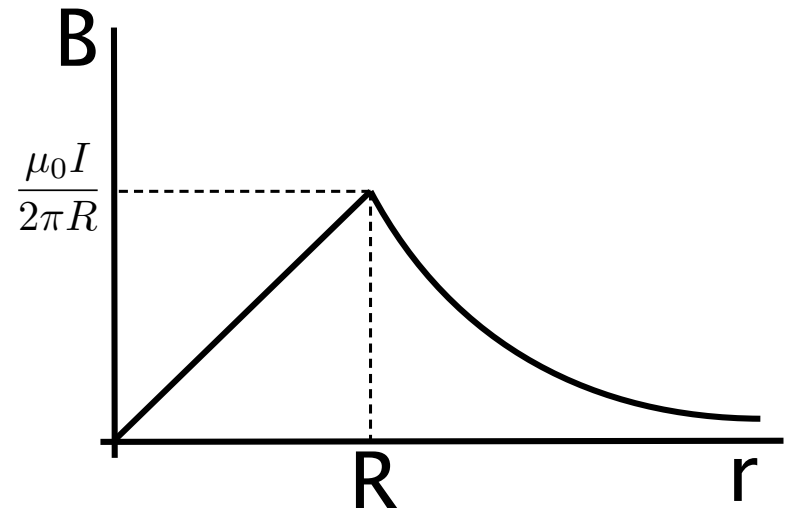
for $r < R$, where $B = B_?$

for $r > R$, where $B = B_{??}$

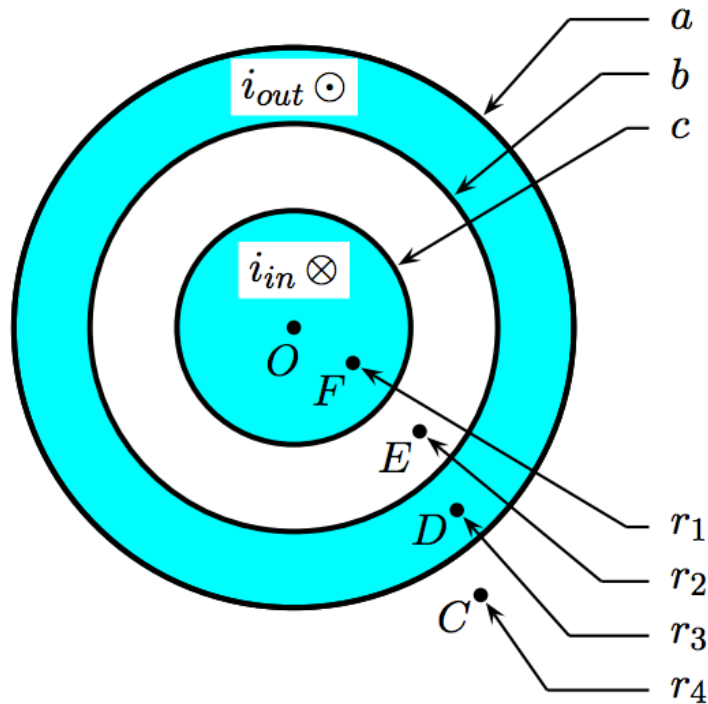
$$\text{Define: } J = \frac{\Delta I}{\Delta A} = \frac{I}{\pi R^2}$$

$$r < R : \quad 2\pi r B_? = \mu_0 (J\pi r^2) \\ \therefore B_? \propto r$$

$$r > R : \quad 2\pi r B_{??} = \mu_0 I \\ \therefore B_{??} \propto \frac{1}{r}$$



Coaxial Cable (example problem):



Let $I_{in} = I_{out} = I_0$

Find: $B?$ at D , where $r = r_3$.

Hint: Ampere's Law

$$LHS = \oint_L B dl = 2\pi r_3 B$$

$$RHS = \mu_0 I_L = \mu_0 J A_{b \rightarrow r_3}^{ring}$$

$$J = \frac{I_0}{A_{b \rightarrow a}^{ring}}$$

$$A_{r_1 \rightarrow r_2}^{ring} = \pi r_2^2 - \pi r_1^2$$

$$\therefore RHS = \mu_0 I \left[\frac{A_{b \rightarrow r_3}^{ring}}{A_{b \rightarrow a}^{ring}} \right]$$

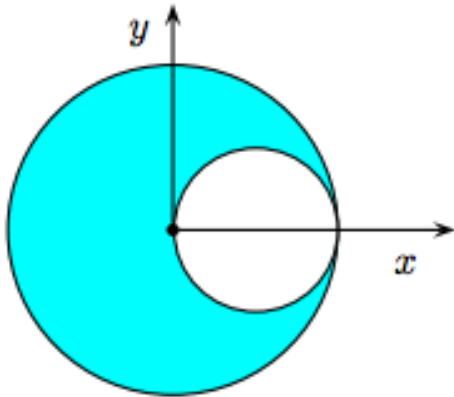
Discussion on example problem

Given: Total current I_0 flows through a long conductor with a hole.

Off Centered Hole

016 10.0 points

A total current of 47 mA flows through an infinitely long cylindrical conductor of radius 5 cm which has an infinitely long cylindrical hole through it of diameter r centered at $\frac{r}{2}$ along the x -axis as shown.



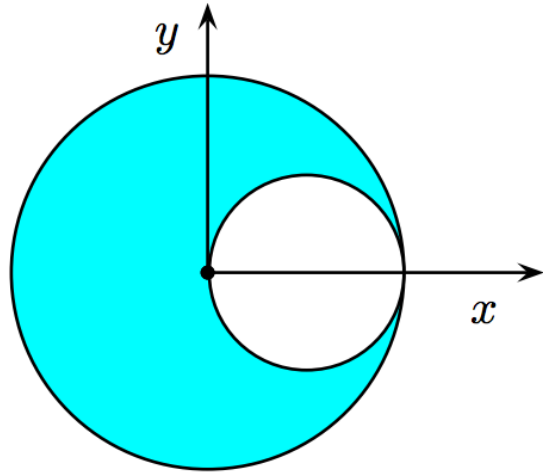
What is the magnitude of the magnetic field at a distance of 10 cm along the positive x -axis? The permeability of free space is $4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$. Assume the current density is constant throughout the conductor.

$$A_{hole} = \pi \left(\frac{R}{2} \right)^2 = \frac{A_1}{4}$$

$$A_{cyl} = \pi R^2 = A_1$$

$$\begin{aligned} A_0 &= \text{Area of conduction medium} \\ &= A_{cyl} - A_{hole} = \frac{3A_1}{4} \end{aligned}$$

Assume I_0 flows out of the page, current density is constant.



$$J = \frac{I_0}{A_0}$$

Magnitude: $I_{cyl} = JA$, $I_{hole} = J \frac{A_1}{4}$

$$I_{cyl} = \frac{I_0}{A_0} A_1 = \frac{4}{3} I_0 \quad (\text{out})$$

$$I_{hole} = \frac{I_0}{A_0} \frac{A_1}{4} = \frac{1}{3} I_0 \quad (\text{into})$$

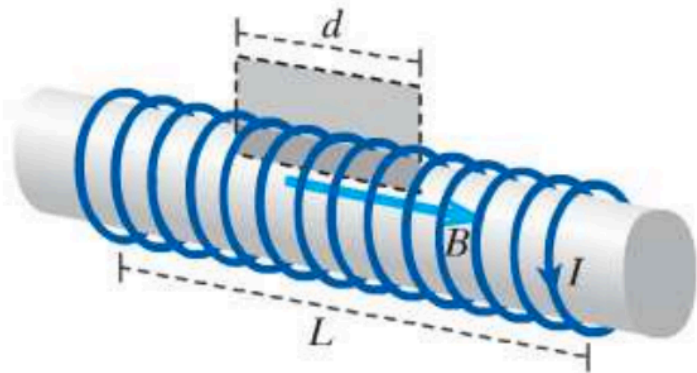
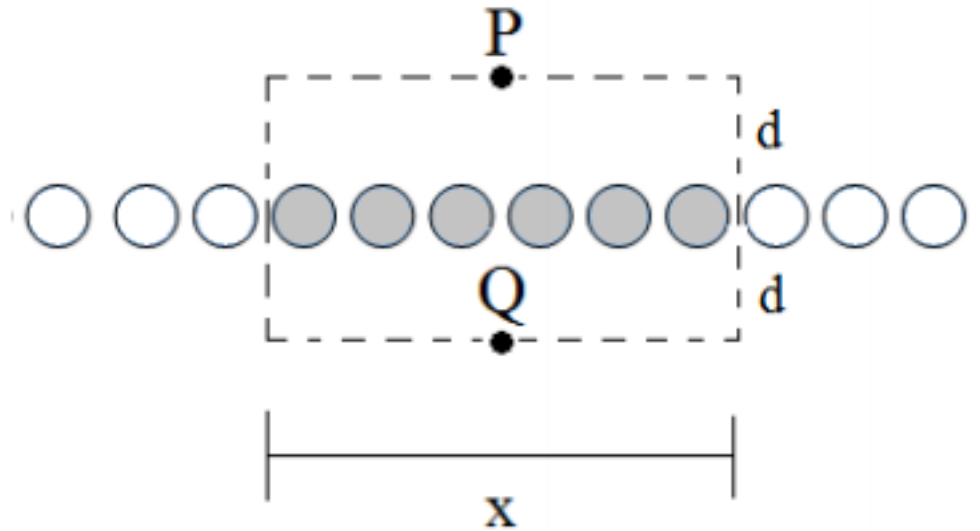
Find: B at P

$$B = B_{cyl} - B_{hole}$$

$$B_{cyl} = \frac{\mu_0 I_{cyl}}{2\pi x}$$

$$B_{hole} = \frac{\mu_0 I_{hole}}{2\pi \left(x - \frac{R}{2}\right)}$$

Solenoid Discussion



$$LHS = 0 + 0 + 0 + B?d$$

$$RHS = \mu_0 I_S = \mu_0 \Delta d I$$

$$LHS = RHS = B?d = \mu_0 \frac{N}{L} d I$$

$$\therefore B? = \mu_0 \frac{N}{L} I$$