For a long wire with current $I$, we have found at $P$ a distance $r$ from the wire:

$$B = \frac{\mu_0 I}{2\pi r} \quad (1)$$

$B$ has a curly field pattern, it is into the page at $P$ and out of the page at $P'$.

Rewrite (1) as $2\pi r B = \mu_0 I$

$$LHS = \oint_S \vec{B} \cdot d\vec{l}$$

circle

Ampere’s law in it’s simplest case says for cylindrically symmetric current, choose “Amperian loop” to be a circle
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\[ \oint_S B \, dl = \mu_0 I_S \]

Denote:

\( B_\gamma \) at P. Let S be the circular loop, \( LHS = B_\gamma \oint dl = 2\pi r B_\gamma \)

\[ RHS = \mu_0 I_S = \mu_0 I \]

\( I_S = \) current enclosed by S
Long, thick wire: Assume current density is uniform.

Notations:
for $r < R$, where $B = B_?$
for $r > R$, where $B = B_??$

Define: $J = \frac{\Delta I}{\Delta A} = \frac{I}{\pi R^2}$

$r < R$:
$2\pi r B_? = \mu_0 \left( J \pi r^2 \right)$
$\therefore B_? \propto r$

$r > R$:
$2\pi r B_?? = \mu_0 I$
$\therefore B_?? \propto \frac{1}{r}$
Coaxial Cable (example problem):

Let \( I_{in} = I_{out} = I_0 \)

Find: \( B \) at \( D \), where \( r = r_3 \).

Hint: Ampere’s Law

\[
LHS = \oint_L B \, dl = 2\pi r_3 B
\]

\[
RHS = \mu_0 I_L = \mu_0 J A^{ring}_{b \rightarrow r_3}
\]

\[
J = \frac{I_0}{A^{ring}_{b \rightarrow a}}
\]

\[
A^{ring}_{r_1 \rightarrow r_2} = \pi r_2^2 - \pi r_1^2
\]

\[
\therefore RHS = \mu_0 I \left( \frac{A^{ring}_{b \rightarrow r_3}}{A^{ring}_{b \rightarrow a}} \right)
\]
Given: Total current \( I_0 \) flows through a long conductor with a hole.

A total current of 47 mA flows through an infinitely long cylindrical conductor of radius 5 cm which has an infinitely long cylindrical hole through it of diameter \( r \) centered at \( \frac{r}{2} \) along the \( x \)-axis as shown.

\[
A_{hole} = \pi \left( \frac{R}{2} \right)^2 = \frac{A_1}{4}
\]

\[
A_{cyl} = \pi R^2 = A_1
\]

\[
A_0 = \text{Area of conduction medium} = A_{cyl} - A_{hole} = \frac{3A_1}{4}
\]
Assume $I_0$ flows out of the page, current density is constant.

Magnitude: $I_{cyl} = JA$, $I_{hole} = J \frac{A_1}{4}$

$I_{cyl} = \frac{I_0}{A_0} A_1 = \frac{4}{3} I_0$  (out)

$I_{hole} = \frac{I_0}{A_0} \frac{A_1}{4} = \frac{1}{3} I_0$  (into)

Find: $B$ at $P$

$B = B_{cyl} - B_{hole}$

$B_{cyl} = \frac{\mu_0 I_{cyl}}{2\pi x}$

$B_{hole} = \frac{\mu_0 I_{hole}}{2\pi \left(x - \frac{R}{2}\right)}$
Solenoid Discussion

\[ LHS = 0 + 0 + 0 + B_?d \]

\[ RHS = \mu_0 I_s = \mu_0 \Delta dI \]

\[ LHS = RHS = B_?d = \mu_0 \frac{N}{L} dI \]

\[ \therefore B_? = \mu_0 \frac{N}{L} I \]