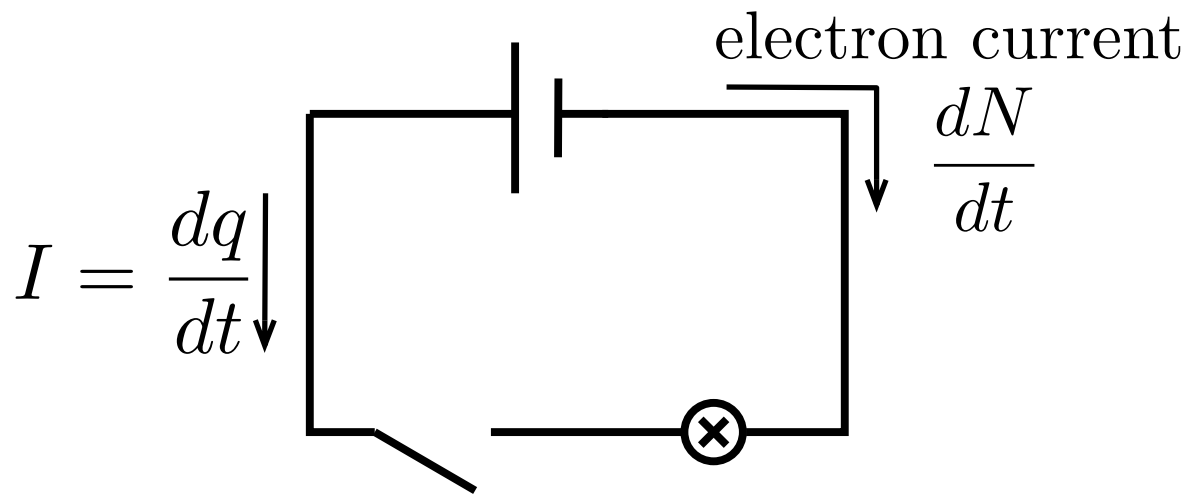


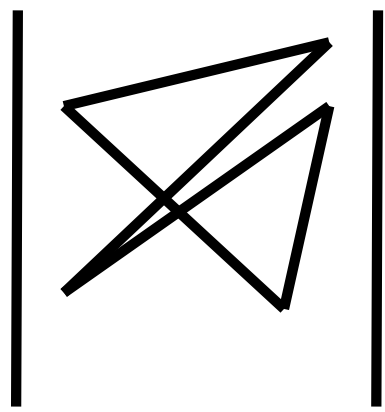
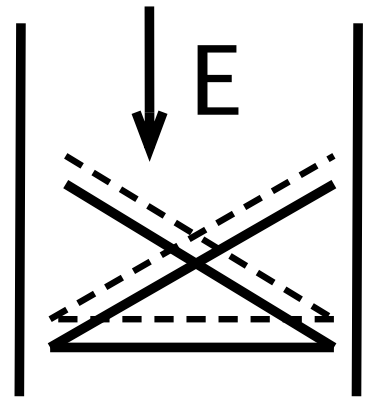
Lecture 23 Single Circuit:



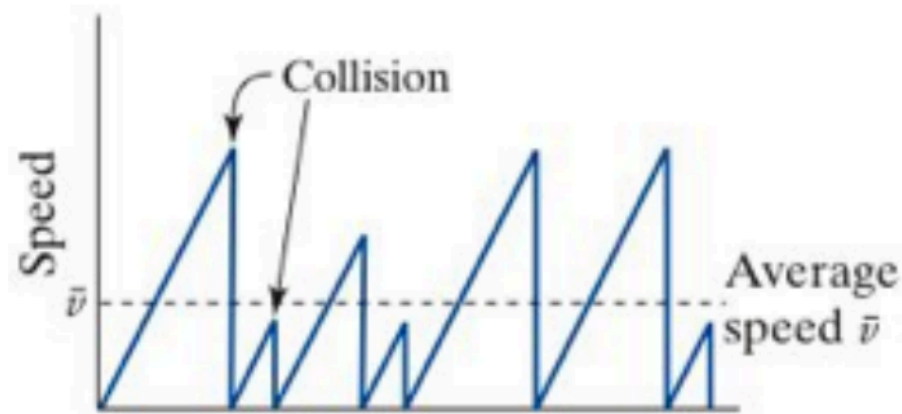
Drift Velocity

$E = 0$ Random Motion

$E > 0$ Steady Drift



Electron drift



Direction of force on electron: $-e\vec{E}$
 and direction of $\vec{a}$, $\vec{v}$, i ← Are all opposite to direction of E

$$\vec{v} = \vec{a}t_c = -\frac{e\vec{E}}{m}t_c = \left(\frac{et_c}{m}\right)(-\vec{E})$$

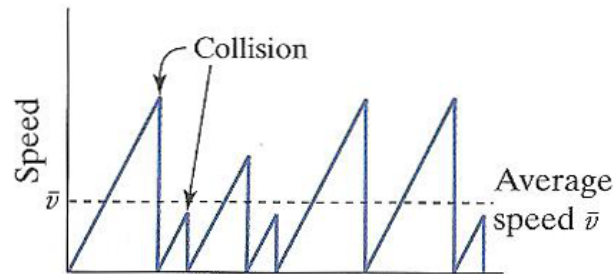
$$v = uE, \text{ where } v = |\vec{v}| \text{ and } E = |\vec{E}|$$

↑
mobility

Electric circuit based on the microscopic point of view

The Drude model

Fig15.26



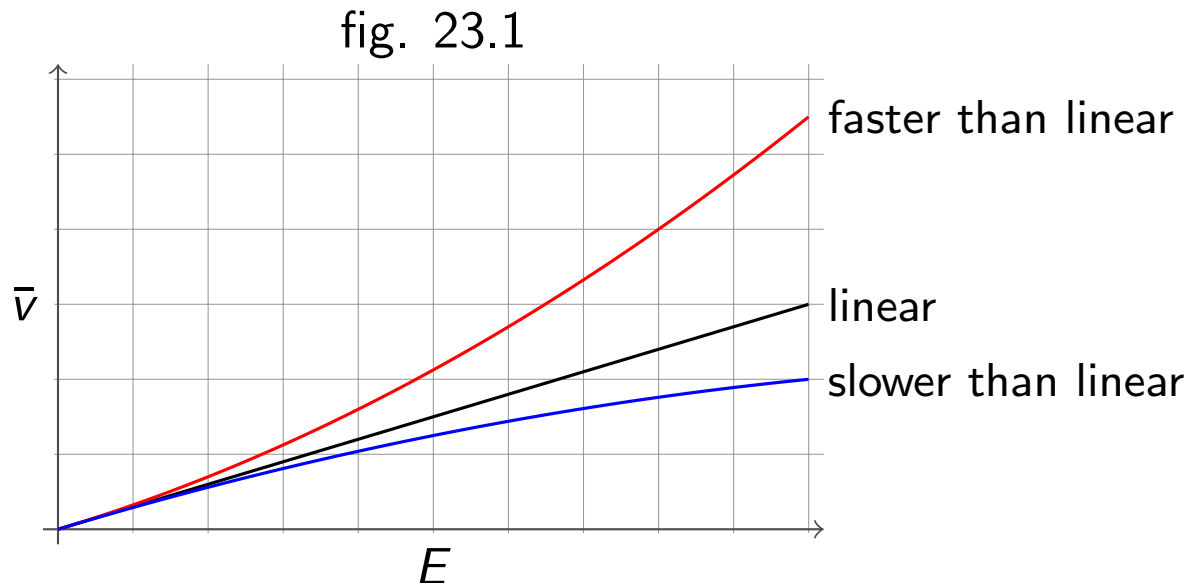
The Drude model describes the motion of mobile electrons in a metal medium in the presence of electric field.

When an external field is applied, mobile electrons are accelerated in the direction opposite the field: $\mathbf{a} = \mathbf{F}/m = -e\mathbf{E}/m$. This motion is in addition to the random thermal motion of the electrons, and imparts a slow “drift velocity” to the mobile electrons opposite the direction of $\mathbf{E}$.

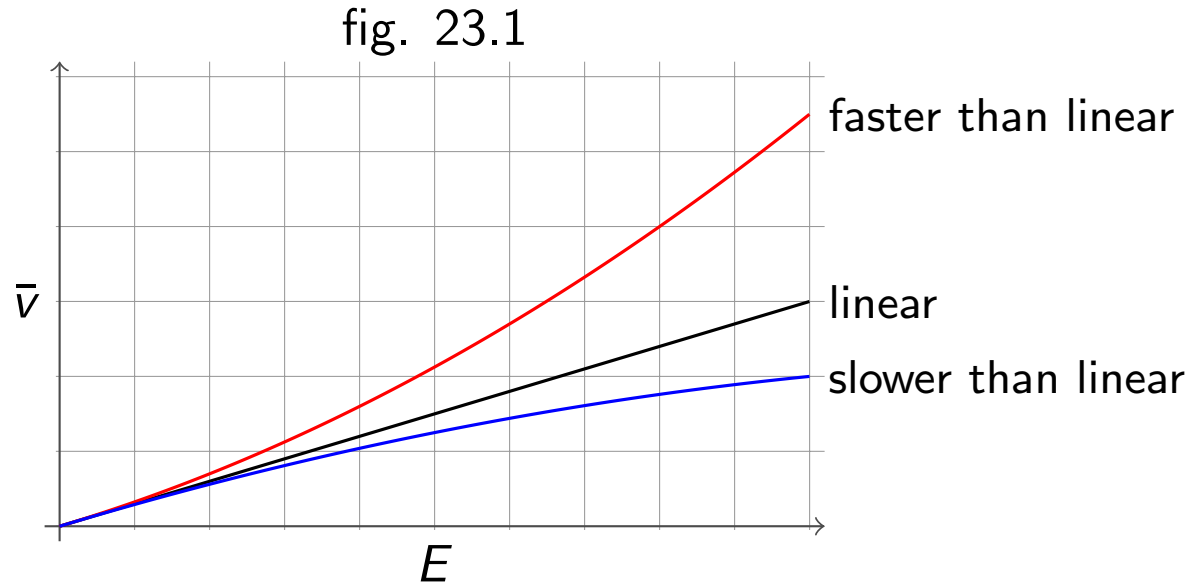
The mobile electrons do not accelerate freely — they collide with the atomic cores in the metal lattice. The Drude model assumes that the drift velocity $\rightarrow 0$ after each collision, as shown in fig. 15.26. This model allows us to calculate an average velocity $\bar{v}$ — the drift velocity — as a function of the average time between collisions $\overline{\Delta t_c}$.

The average drift speed is given by $\bar{v} = a\overline{\Delta t_c}$; $\overline{\Delta t_c}$ is a physical property of the material. The average collision time $\overline{\Delta t_c}$ is constant at a given temperature. For a given material, the values of $\overline{\Delta t_c}$ can be looked up in a table at various temperatures.

Here is a question for you: at a given temperature, $\overline{\Delta t_c}$ is constant — one expects to find a monotonic relationship between E and $\bar{v}$. Fig. 23.1 shows several possible monotonic curves for the $\bar{v}$ vs E plot. Which curve best describes $\bar{v}$ vs E ? Hint: Use $a = eE/m$.
(Choices on next page) (Note: E is the magnitude of the field)



Which of the curves is correct? Hint: Use $a = eE/m$.



Choice	How should $\bar{v}$ increase with E ?
1	$\bar{v}$ increases linearly with E
2	$\bar{v}$ increases faster than linearly with E
3	$\bar{v}$ increases slower than linearly with E

Electron number current

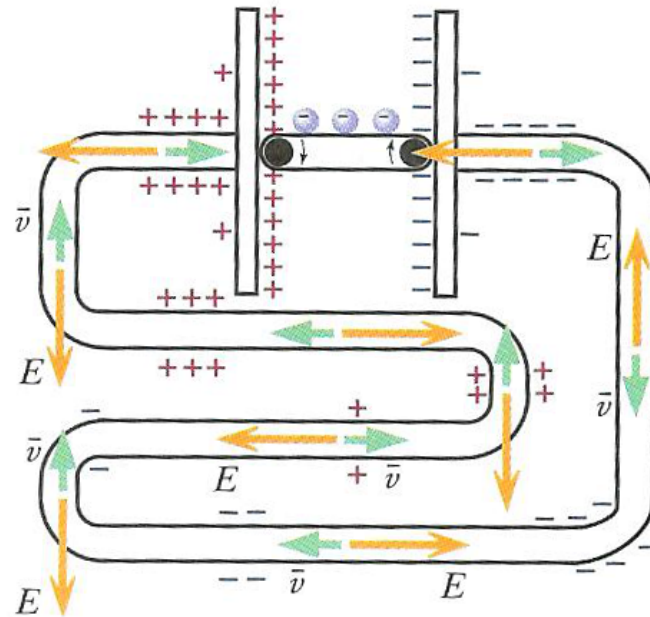
$$i = \frac{dN}{dt} = \frac{nA\Delta l}{\Delta t} = nAv = nAuE$$

Drude model: $i = nAuE$

- Assume $i > 0$, denote $E = |\vec{E}|$
- $\vec{i}$ is in the same direction as the force, $\vec{F}_{\text{on } e} = -e\vec{E}$
- $\vec{E}$ is opposite to $\vec{i}$
- i is proportional to E , magnitude of the field

Surface charge distribution in a steady state circuit

Fig19.17



We begin with a simple circuit consisting of a battery and a wire of cross section A , length L , and uniform mobility u . The battery maintains a potential difference $\mathcal{E}$, referred to as the emf. The electron number current is defined by:

$$i = \frac{\text{number of electrons}}{\text{time}} = \frac{\Delta N}{\Delta t} = \frac{nA\Delta\ell}{\Delta t} = nA\bar{v} = nAuE. \quad E = |\vec{E}|$$

Current is parallel to the wire and constant throughout the circuit; since n , A , and u are properties of the wire, we conclude E must be also be constant and locally parallel to the wire.

E is generated by source charges. The wire is electrically neutral — charges in the wire cannot drive the current. Charges on the plates of the battery cannot generate constant, locally parallel E within the wire. The source charges reside on the *surface* of the wire. We will see that surface charges with a steadily decreasing charge density can create the constant, locally parallel E within the wire. Such a surface charge distribution is illustrated schematically in MI-Fig19.17. Let us look at the situation in one segment, which is shown in the next figure.

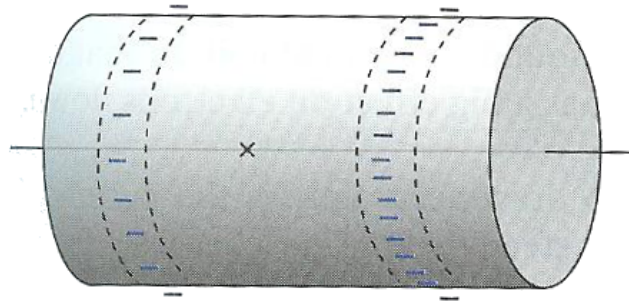


Fig19.19

In the figure above, negative surface charge density increases from left to right, as can be seen from the two representative rings of surface charge.

What is the direction of the resultant field at x ?

Choice	Direction of E at x
1	To the left
2	To the right

Surface charge distributions:

Gradient in the surface charge distribution leads to an electric field inside of the wire

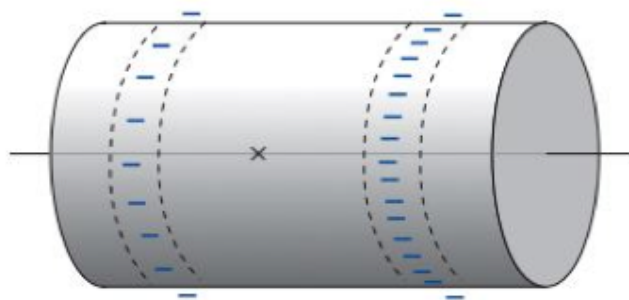


Figure 19.19 Two rings of surface charge on a wire, with different charge density. What is the field direction at location $\times$?

If $i = \text{constant} \Rightarrow E = \text{constant}$ along the wire.

There is a surface distribution with a gradient throughout the circuit

$$i = nAv = nAuE$$

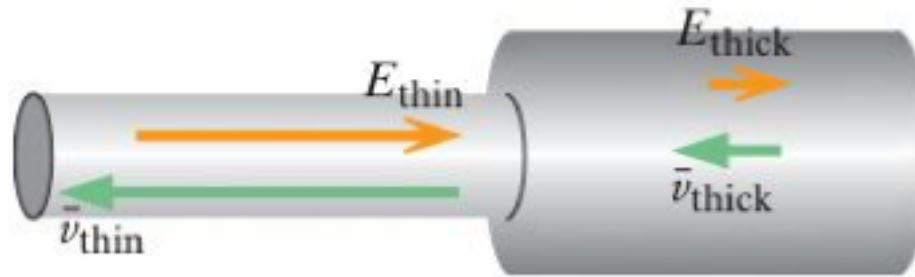


Figure 19.7 Electric field at locations inside the thin and thick wires.

For steady i , with same material $nu = \text{constant}$

$$AE = \text{constant}$$

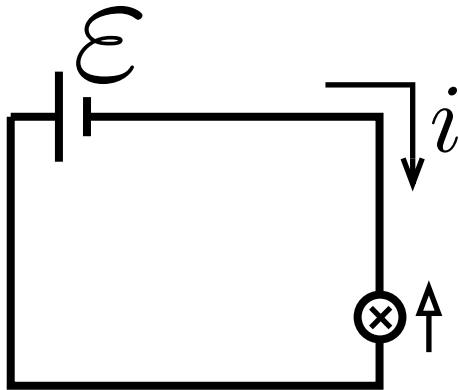
Smaller in A , Bigger is E if $A_1 \ll A_2$
 $E_1 \gg E_2$

In circuits, assume $A_{\text{wire}} \gg A_{\text{filament}}$, ΔV_{wire} is negligible.

Circuit setup:

Given: Battery emf and circuits, solve for E and i in each element of the circuit.

Example 1:

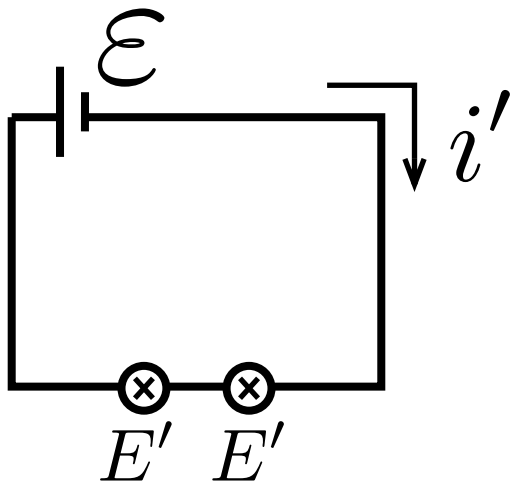


Loop equation:

$$\mathcal{E} - EL = 0, \quad E = \frac{\mathcal{E}}{L} = E_0$$

$$i = nAv = nAuE = nAu \frac{\mathcal{E}}{L} \equiv i_0$$

Example 2:



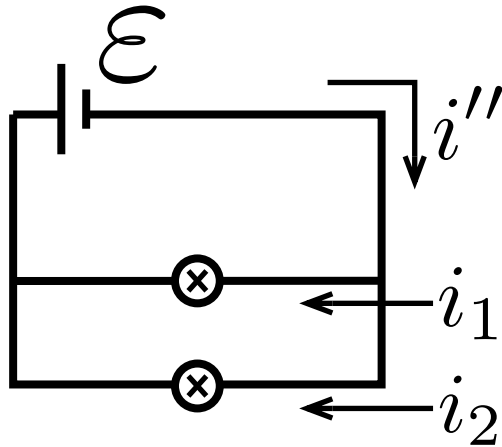
$$\mathcal{E} - E' L - E' L = 0$$

$$\therefore E' = \frac{\mathcal{E}}{2L}$$

$$i' = nAuE' = nAu \frac{\mathcal{E}}{2L} = \frac{i_0}{2}$$

$$\therefore E' = \frac{E_0}{2}, \text{ and } i' = \frac{i_0}{2}$$

Example 3:



$$\mathcal{E} - E_1 L = 0$$

$$\mathcal{E} - E_2 L = 0$$

$$i_1 = nAuE_1 = nAu\frac{\mathcal{E}}{L} = i_0$$

$$i_2 = i_0$$

$$\text{Note: } i'' = i_1 + i_2 = 2i_0, \quad E_1 = E_2 = E_0 = \frac{\mathcal{E}}{L}$$

Exercise:

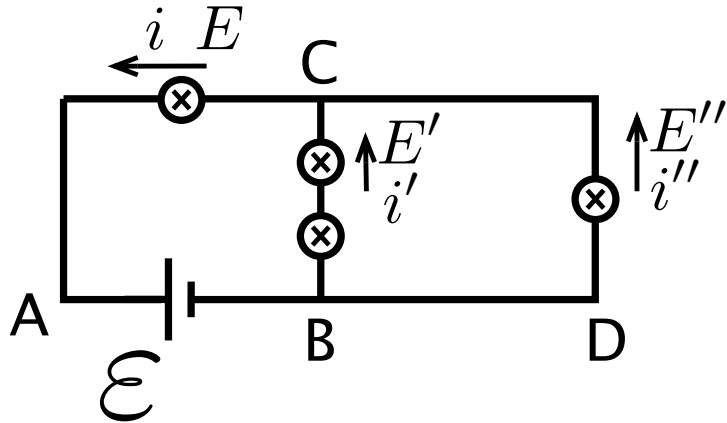
What will be the node equation if $A'' = A_1 = A$ and $A_2 = 2A$

Hint:

$$i'' = nAuE'', \quad i_1 = nAuE_1 \text{ and}$$

$$i_2 = n2AuE_2, \text{ or } E'' = E_1 + 2E_2$$

Example 4:



$$\text{ABCA: } \mathcal{E} - 2E'L - EL = 0 \quad (1)$$

$$\text{ABDCA: } \mathcal{E} - E''L - EL = 0 \quad (2)$$

$$\text{Node eqn: } i = i' + i'' \quad (3)$$

$$i = nAuE$$

$$i' = nAuE'$$

$$i'' = nAuE''$$

$$\therefore \text{Node Eqn. } \Rightarrow E = E' + E''$$

3 equations and 3 unknowns

Solve for E' , E'' , E Express all E 's in terms of E''

$$2E' = E'', \quad E' = \frac{E''}{2} \qquad E = E' + E'' = \frac{3}{2}E''$$

$$\mathcal{E} - E''L - \frac{3}{2}E''L = \mathcal{E} - \frac{5}{2}E''L = 0, \quad E'' = \frac{2}{5} \frac{\mathcal{E}}{L}$$

$$E' = \frac{E''}{2} = \frac{1}{5} \frac{\mathcal{E}}{L}.$$

$$E = E' + E'' = \frac{3}{2}E'' = \frac{3}{5} \frac{\mathcal{E}}{L}$$

Exercise:

Denote i_0 to be i . Express i' and i'' in terms of i_0