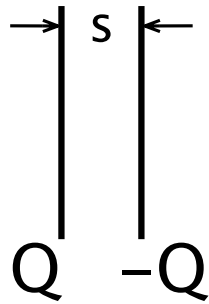


Lecture 25 Circuit Elements

Capacitors – **parallel plate**, we have discussed earlier

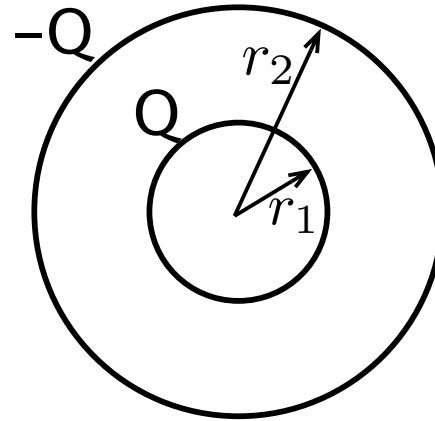
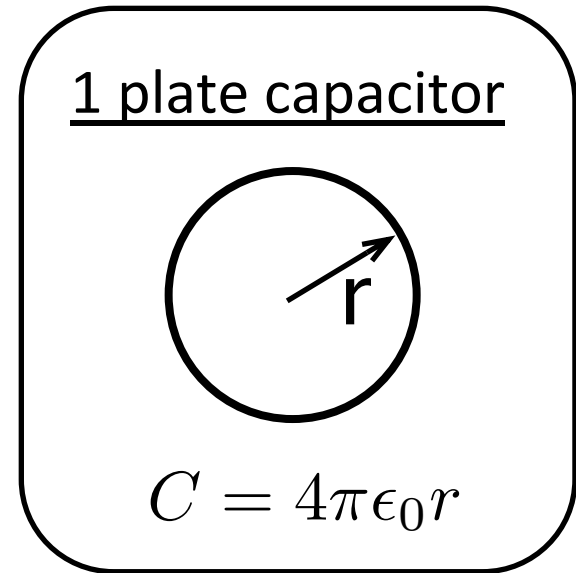


$$C = \frac{Q}{V} = \frac{Q}{Es} = \frac{Q}{\left(\frac{Q/A}{\epsilon_0}\right)s} = \frac{\epsilon_0 A}{s}$$

– **spherical capacitor**

$$V \equiv \Delta V \equiv \frac{kQ}{r_1} - \frac{kQ}{r_2}$$

$$C = \frac{Q}{V} = \frac{1}{\frac{-k}{r_1} - \frac{k}{r_2}}$$



Large r , $s \ll r$. $r_1 = r$, $r_2 = r + s$

$$C = \frac{1}{\left(\frac{k}{r}\right)} \cdot \frac{1}{1 - \frac{1}{(1+\epsilon)}} \approx \frac{1}{\frac{ks}{r^2}} \approx \frac{A\epsilon_0}{s} \quad (\text{same as } || \text{ plate capacitor})$$

Energy Stored

$$\Delta q \left(V \frac{q}{-q} \right) = \frac{q + \Delta q}{-q - \Delta q}$$

$$\int_0^U \Delta U = V \Delta q = \int_0^Q \frac{q}{C} \Delta q = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

$$\text{Energy Density: } u = \frac{U}{As} \Rightarrow U = \frac{1}{2} \frac{A\epsilon_0}{S} \cdot s^2 E^2$$
$$= \frac{1}{2} \epsilon_0 (As) E^2$$

$$\therefore u = \frac{U}{As} = \frac{1}{2} \epsilon_0 E^2$$

Insert a dielectric into the gaps:

$$C' = \frac{Q}{V'} = \frac{Q}{\frac{V}{\kappa}} = \kappa C$$

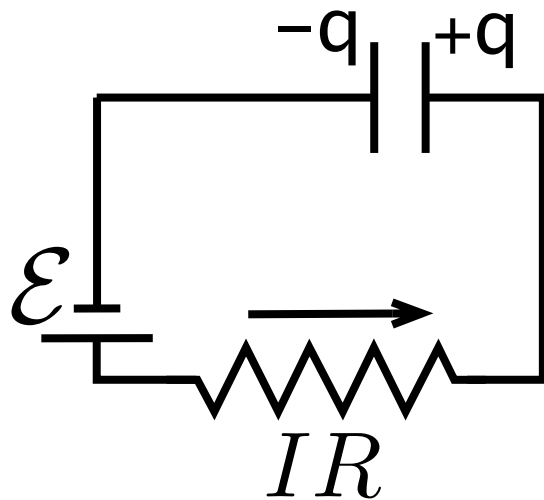
Fixed Q case:

$$U' = \frac{Q^2}{2C'} = \frac{u}{\kappa}$$

Fixed V case:

$$U' = \frac{1}{2}C'V^2 = \kappa u$$

RC Circuit

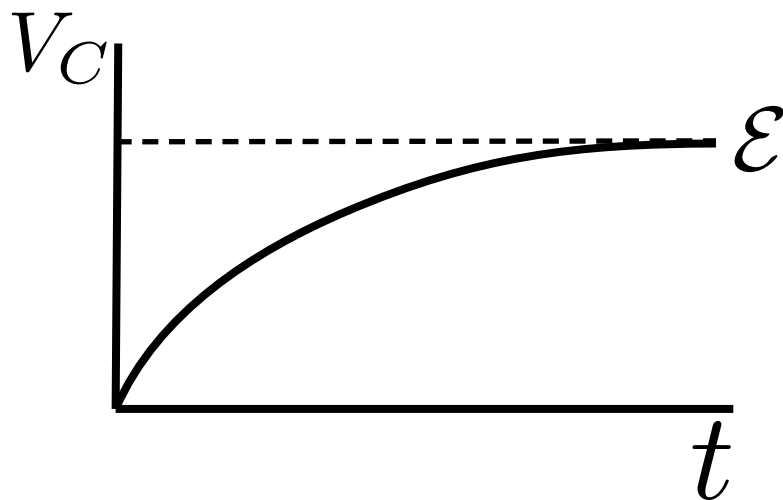
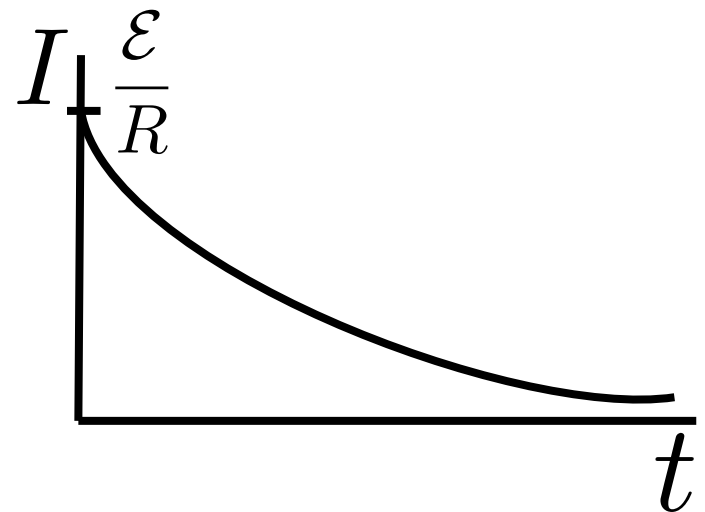


Loop equation: $\Delta V = 0$

$$\mathcal{E} - IR - \frac{q}{C} = 0$$

$$t = 0^+ : \quad q = 0, \quad I = \frac{\mathcal{E}}{R}$$

$$t = \infty : \quad I = 0, \quad q = \mathcal{E}C$$



Example Problem: Setup:

Case 1:

$$\frac{Q_0, V_0}{C_0}$$

Case 2:

$$\frac{Q' \quad Q''}{V_1 \quad C_1}$$

Given: C_0, C_1, V_1

$$Q' + Q'' = Q_0$$

Find: V_0

$$\text{Hint: } V_0 = \frac{Q_0}{C_0} = \frac{Q' + Q''}{C_0}$$

This is the situation of 2 equations and 2 unknowns. Solve Q' and Q'' first, then find V_0 .

Cut a wire into N equal segments, then make || connection.

$$L' \Rightarrow \frac{L}{N} \qquad A' \Rightarrow NA$$

$$\begin{aligned} \therefore R' &= \rho \frac{L'}{A'} \\ &= \rho \left(\frac{L}{N} \right) \frac{1}{(NA)} \\ &= \left(\frac{\rho L}{A} \right) \frac{1}{N^2} \\ &= \frac{R}{N^2} \end{aligned}$$