Lecture 26

Ohm’s Law: OH-1

\[ V = EL, \quad \text{Current density: } J = \frac{I}{A} \]

\[ J = \sigma E \]

micro: \( i = nAuE, \quad I = |q|i \)

\[ \therefore J = \frac{|q|i}{A} = \frac{|q|nAuE}{A} \equiv \sigma E, \quad \sigma = |q|nu \]

Resistivity: \( E = \frac{J}{\sigma} = \rho J \)

Ohm’s Law: OH-2

\[ \frac{V}{L} = \rho \frac{I}{A}, \quad V = \left( \frac{\rho L}{A} \right) I = RI, \quad R = \frac{\rho L}{A} \]
Series Circuits:

\[ V \equiv IR_{+2} \quad V = V_1 + V_2 \]
\[ = IR_1 + IR_2 \]
\[ \therefore R_{12}^{\text{series}} = R_1 + R_2 \]

Parallel Circuits:

\[ i = i_1 + i_2 \]
\[ \frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} \]
\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \]
Example Problem:

Identical Bulbs with R

Parallel connection:

\[ \frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{2}{R} \]

\[ \therefore R_{23} = \frac{R}{2} \]

Interpretation:

\[ R = \frac{\rho L}{A} \quad \text{(double area)} \]

\[ A' = 2A \]

\[ R' = \frac{\rho L}{A'} = \frac{\rho L}{2A} = \frac{R}{2} \]

\[ R_{123} = R + R_{23} = R + \frac{R}{2} = \frac{3}{2}R \]

\[ \therefore I = \frac{\mathcal{E}}{(\frac{3}{2}R)} \]
Brightness -- Power

\[ P = \frac{(\Delta q)V}{\Delta t} = IV = I^2R \]

Compare Brightness

\[ P_1 = I^2R = \left( \frac{2}{3} \frac{\mathcal{E}}{R} \right)^2 R = \frac{4}{9} \frac{\mathcal{E}^2}{R} \]

\[ P_2 = \left( \frac{I}{2} \right)^2 R = \frac{1}{9} \frac{\mathcal{E}}{R} \]
What happens when you “short” a resistor?

Given: $R$ is the resistor of a light bulb.

What happens when $R' \ll R$ is connected to the two terminals of the bulb?

Hint: $I_0 = I' + I''$

\[
V_A - V_B = I' R = I'' R'
\]

\[
\therefore I' = I'' \frac{R'}{R}
\]

\[
I_0 = I'' \frac{R'}{R} + I'' \approx I''
\]

So with the wire connected: $I' \approx 0$  
\[
I'' \approx I_0
\]

The bulb turns dark.
Two Illustrations showing Dissipated Power = Power Delivered

Comments:

Is dissipation greater for parallel connection?
Case I: \( \mathcal{E} - IR_1 - IR_2 = 0 \)
\[
\therefore I_1 \mathcal{E} = I^2 R_1 + I^2 R_2
\]
\[
\therefore \text{Power delivered by the battery is the power consumed by the bulbs.}
\]

Is dissipation the same for both circuits?
check: \( I \mathcal{E} = I_1^2 R_1 + I_2^2 R_2 \)
\[
\text{RHS} = I_1 \mathcal{E} + I_2 \mathcal{E} = (I_1 + I_2) \mathcal{E} = I \mathcal{E}
\]
Again: Power delivered = Power consumed
Part 1: RC Circuits

loop equation: any \( t \)

\[ \mathcal{E} - \frac{q}{C} - IR = 0 \]

\( t = 0, \quad q = 0, \quad I = I_0 = \frac{\mathcal{E}}{R} \).

\( t = \infty \) : fully charged, \( \frac{dq}{dt} = I = 0, \quad q_\infty = \mathcal{E}C \Rightarrow V = \mathcal{E} \)
Analytic solution: Loop equation has 2 variables - $q$, $I$

Convenient to work out with $I$: Take $\frac{d}{dt}$ (loop eqn.)

$$\frac{d}{dt} \left( E - \frac{q}{C} - IR \right) = 0 \Rightarrow - \frac{dq}{dt} \cdot \frac{1}{C} - \frac{dI}{dt} R = 0$$

$\therefore - \frac{I}{C} - \frac{dI}{dt} R = 0$. method of separation of variables $\frac{dI}{dt} = - \frac{I}{RC}$

Rearrange $\frac{dI}{dt} = - \frac{I}{RC} = - \frac{I}{\tau}$

Soluion: $I = I_0 e^{-\frac{t}{\tau}}$

Check: $LHS = - \frac{1}{\tau} I_0 e^{-\frac{t}{\tau}} = - \frac{I}{\tau} = RHS$
Compare two cases: Given: same $\mathcal{E}$, $A$, $s_2 < s_1$
Find: $E_2(t)$ compared to $E_1(t)$

Hint:
$\tau_2 = C_2R$, $\tau_1 = C_1R$
$C_2 > C_1 \Rightarrow \tau_2 > \tau_1$
$C = \frac{\varepsilon_0 A}{s}$
Example Problem: 4 Resistors/1 Capacitor

Battery connected: at equilibrium, there is no current in C

After the battery is disconnected:

\[ |V_C| = |V_A - V_B| = |R_1 i_1 - R_3 i_3| \]

\[ V = V_0 e^{-\frac{t}{\tau}}, \quad \tau = R_{eff} C, \text{ What is } R_{eff}? \]
Part 2: Magnetic Force

Electric Force: \[ \vec{F}_q^E = q \vec{E} \]

source acted on by \( \vec{E} \)

Magnetic Force: \[ \vec{F}_{qv}^B = q \vec{v} \times \vec{B} \]

\[ \Delta Nqv = \frac{\Delta Nq\Delta l}{\Delta t} = I \Delta \vec{l} \]

\[ \therefore \Delta F^B_{I\Delta \vec{l}} = I \Delta \vec{q} \times \vec{B} \]
Consider the setup shown in fig. 26-2, where the magnetic field $B_{\text{ext}}$ is along $\hat{x}$, and the current $I$ flows along $\hat{z}$. Based on $\Delta \mathbf{F} = I \Delta \ell \times \mathbf{B}$, the force on the wire is in the $\hat{y}$ direction.
Denote by $B^I$ the circular magnetic field generated by the current $I$. Both $B^{ext}$ and $B^I$ are illustrated in fig26.2a, where the current is pointing directly towards the viewer.

Define the resultant field to be: $B = B^{ext} + B^I$. Compare the strength of the resultant field $B$ above and below the current.

<table>
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<th>Choice</th>
<th>Which point has a larger resultant field?</th>
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<td>Top</td>
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<tr>
<td>2</td>
<td>Bottom</td>
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Answer to previous slide clicker question:
Resultant field at bottom is stronger. Stronger field region pushed the wire to weaker region.

Intuitively the pattern is strong when the field is wrinkled. Natural tendency is to straighten out the wrinkle. Straight up = wrinkle free.
Magnetic force on two moving charges

Case 1: $q_1 > 0$
Case 2: $q_2 < 0$

<table>
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<tr>
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<th>Direction of Force on $q_2v_2$</th>
</tr>
</thead>
<tbody>
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<tr>
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