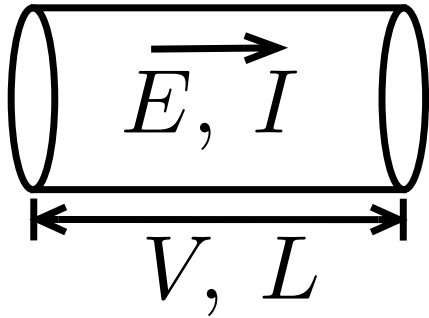


Lecture 26

Ohm's Law: OH-1



$$V = EL, \quad \text{Current density: } J = \frac{I}{A}$$
$$J = \sigma E$$

$$\text{micro: } i = nAuE, \quad I = |q|i$$

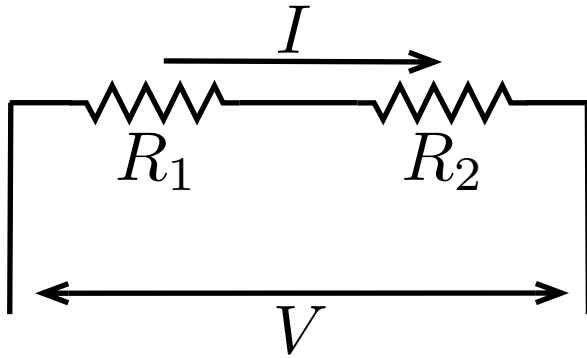
$$\therefore J = \frac{|q|i}{A} = \frac{|q|nAuE}{A} \equiv \sigma E, \quad \sigma = |q|nu$$

$$\text{Resistivity: } E = \frac{J}{\sigma} = \rho J$$

Ohm's Law: OH-2

$$\frac{V}{L} = \rho \frac{I}{A}, \quad V = \left(\frac{\rho L}{A} \right) I = RI, \quad R = \frac{\rho L}{A}$$

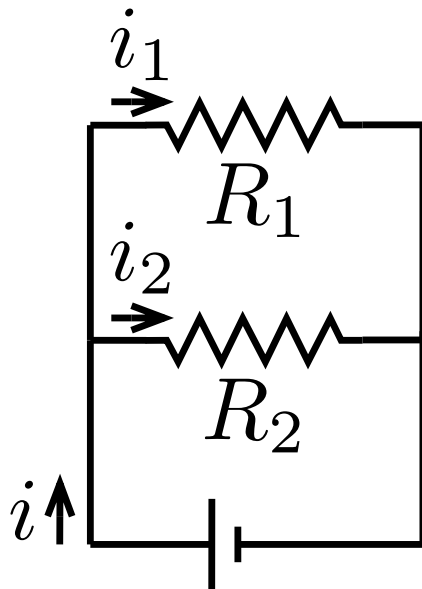
Series Circuits:



$$\begin{aligned} V &\equiv IR_{+2} & V &= V_1 + V_2 \\ & & &= IR_1 + IR_2 \end{aligned}$$

$$\therefore R_{12}^{series} = R_1 + R_2$$

Parallel Circuits:

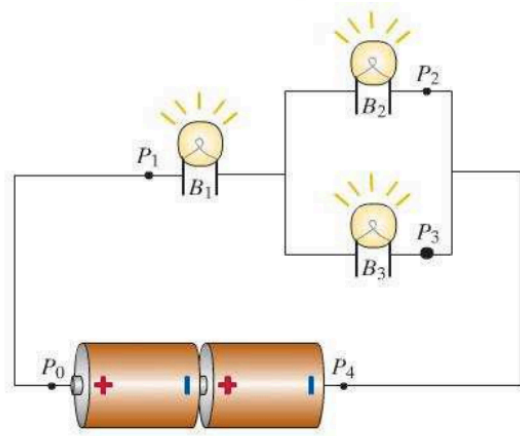


$$i = i_1 + i_2$$

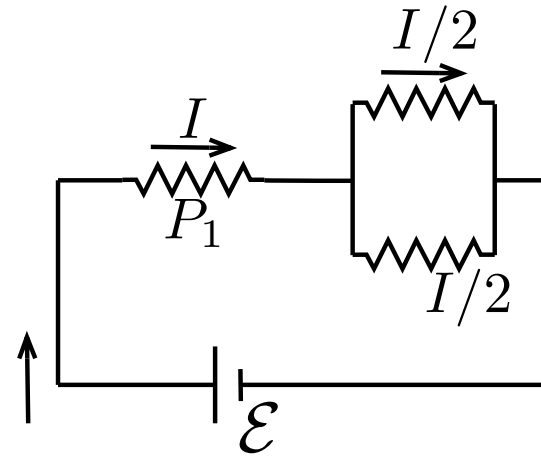
$$\frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Example Problem:



Figure



Identical Bulbs with R

Parallel connection:

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{2}{R} \quad \therefore R_{23} = \frac{R}{2}$$

Interpretation:

$$R = \frac{\rho L}{A} \quad (\text{double area}) \quad A' = 2A \quad R' = \frac{\rho L}{A'} = \frac{\rho L}{2A} = \frac{R}{2}$$

$$R_{123} = R + R_{23} = R + \frac{R}{2} = \frac{3}{2}R \quad \therefore I = \frac{\mathcal{E}}{\left(\frac{3}{2}R\right)}$$

Brightness -- Power

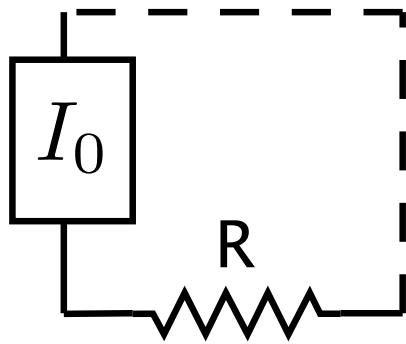
$$P = \frac{(\Delta q)V}{\Delta t} = IV = I^2 R$$

Compare Brightness

$$P_1 = I^2 R = \left(\frac{2}{3} \frac{\mathcal{E}}{R} \right)^2 R = \frac{4}{9} \frac{\mathcal{E}^2}{R}$$

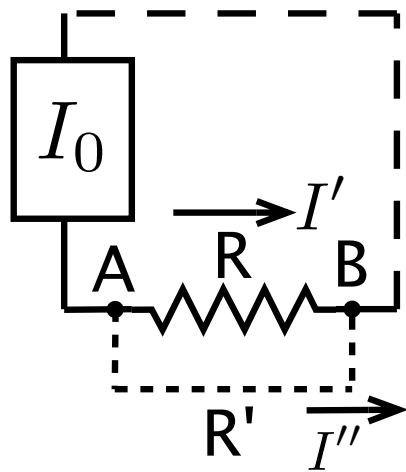
$$P_2 = \left(\frac{I}{2} \right)^2 R = \frac{1}{9} \frac{\mathcal{E}^2}{R}$$

What happens when you “short” a resistor?



Given: R is the resistor of a light bulb.

What happens when $R' \ll R$ is connected to the two terminals of the bulb?



Hint: $I_0 = I' + I''$

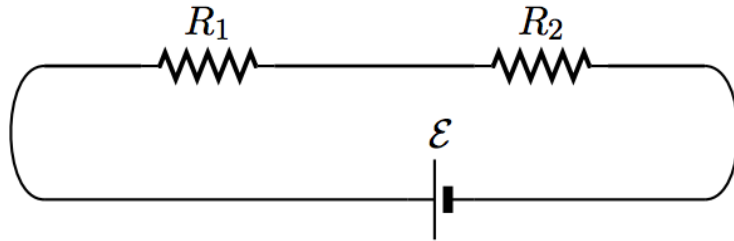
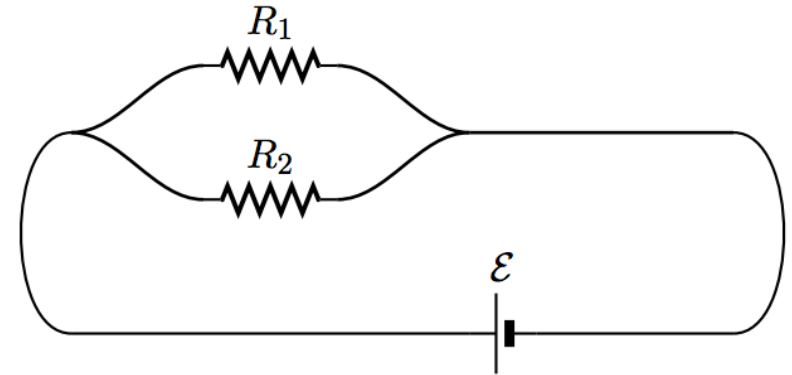
$$V_A - V_B = I' R = I'' R'$$

$$\therefore I' = I'' \frac{R'}{R}$$

$$I_0 = I'' \frac{R'}{R} + I'' \approx I''$$

So with the wire connected: $I' \approx 0$ The bulb turns dark
 $I'' \approx I_0$

Two Illustrations showing Dissipated Power = Power Delivered



Comments:

Is dissipation greater for parallel connection?

Case I: $\mathcal{E} - IR_1 - IR_2 = 0$

$$\therefore I\mathcal{E} = I^2 R_1 + I^2 R_2$$

$\therefore$ Power delivered by the battery is the power consumed by the bulbs.

Is dissipation the same for both circuits?

check: $I\mathcal{E} \stackrel{?}{=} I_1^2 R_1 + I_2^2 R_2$

$$RHS = I_1 \mathcal{E} + I_2 \mathcal{E} = (I_1 + I_2) \mathcal{E} = I\mathcal{E}$$

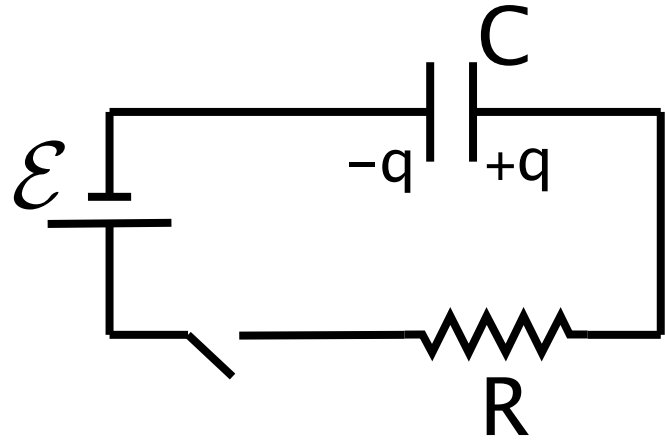
Again: Power delivered = Power consumed

Part 1: RC Circuits

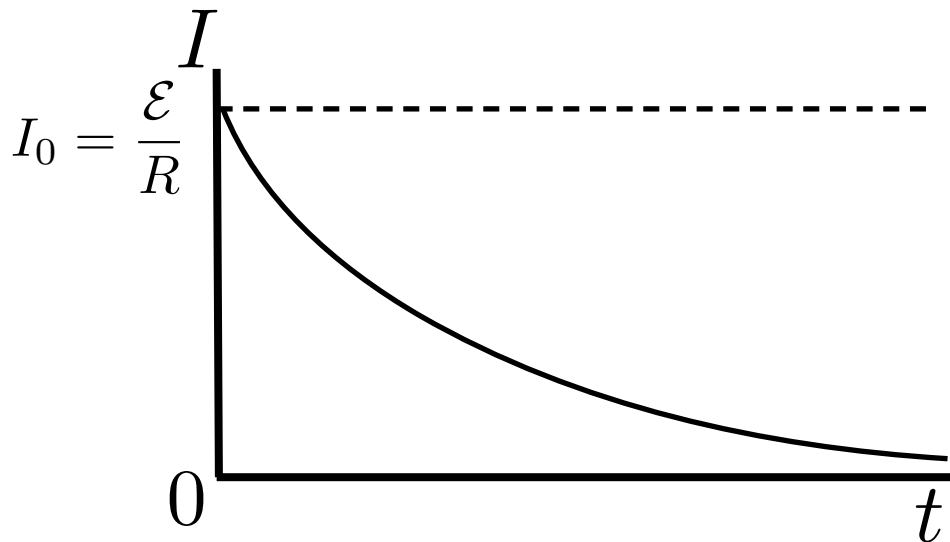
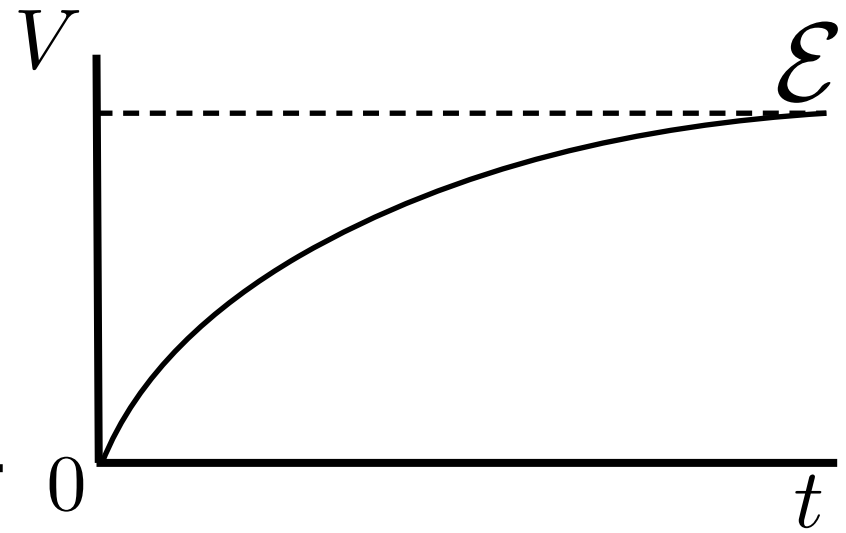
loop equation: any t

$$\mathcal{E} - \frac{q}{C} - IR = 0$$

$$t = 0, \quad q = 0, \quad I = I_0 = \frac{\mathcal{E}}{R}.$$



$$t = \infty : \text{fully charged, } \frac{dq}{dt} = I = 0, \quad q_{\infty} = \mathcal{E}C \Rightarrow V = \mathcal{E}$$



Analytic solution: Loop equation has 2 variables - q , I

Convenient to work out with I : Take $\frac{d}{dt}$ (loop eqn.)

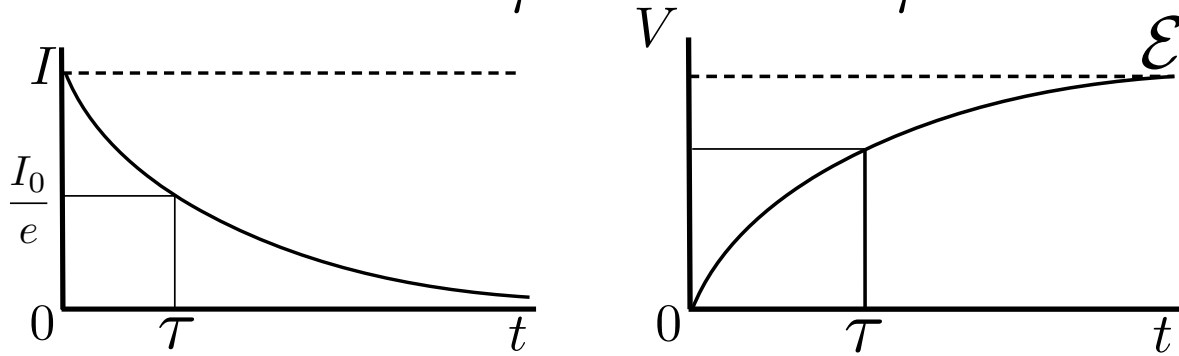
$$\frac{d}{dt} \left(\mathcal{E} - \frac{q}{C} - IR \right) = 0 \Rightarrow -\frac{dq}{dt} \cdot \frac{1}{C} - \frac{dI}{dt} R = 0$$

$$\therefore -\frac{I}{C} - \frac{dI}{dt} R = 0. \text{ method of separation of variables } \frac{dI}{dt} = -\frac{I}{RC}$$

$$\text{Rearrange } \frac{dI}{dt} = -\frac{I}{RC} = -\frac{I}{\tau} \quad \frac{dI}{I} = -\frac{dt}{RC}$$

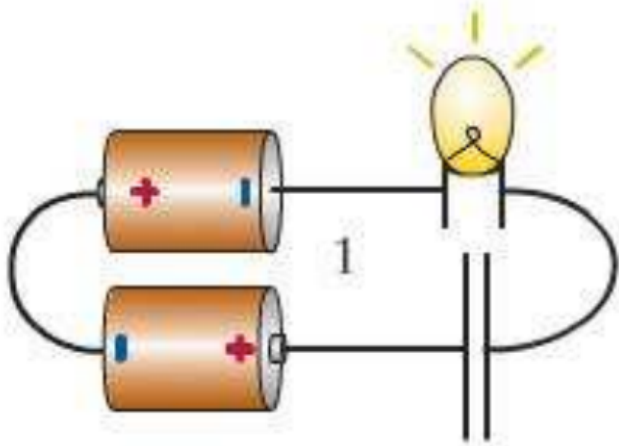
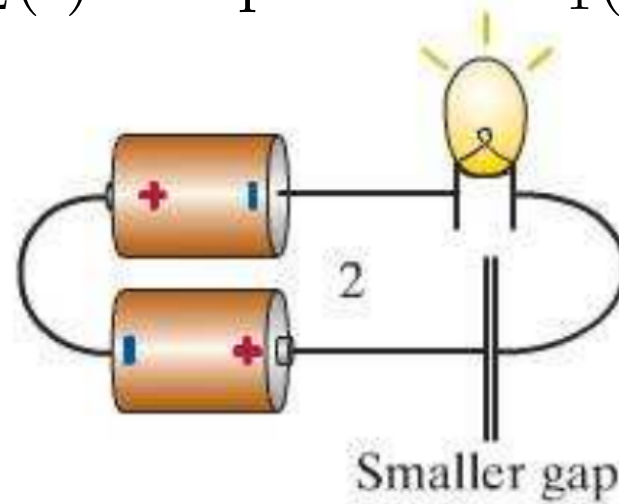
$$\text{Solution: } I = I_0 e^{-\frac{t}{\tau}}$$

$$\text{Check: } LHS = -\frac{1}{\tau} I_0 e^{-\frac{t}{\tau}} = -\frac{I}{\tau} = RHS$$



Compare two cases: Given: same $\mathcal{E}$, A , $s_2 < s_1$

Find: $E_2(t)$ compared to $E_1(t)$

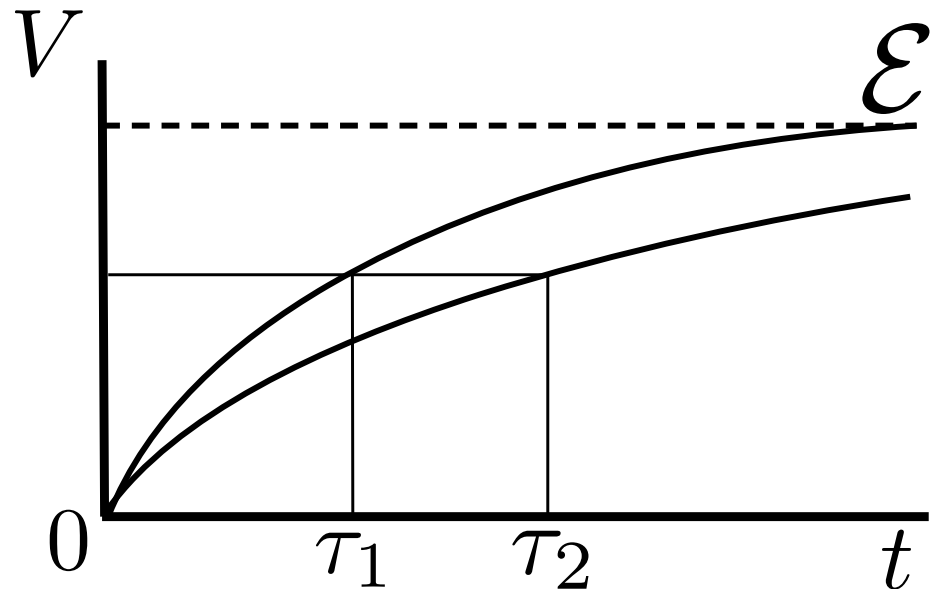


Hint:

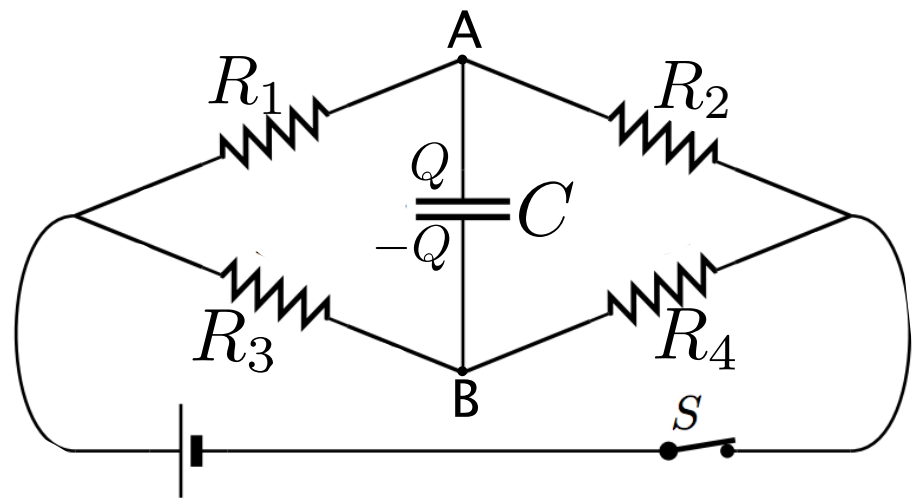
$$\tau_2 = C_2 R, \quad \tau_1 = C_1 R$$

$$C_2 > C_1 \Rightarrow \tau_2 > \tau_1$$

$$C = \frac{\epsilon_0 A}{s}$$



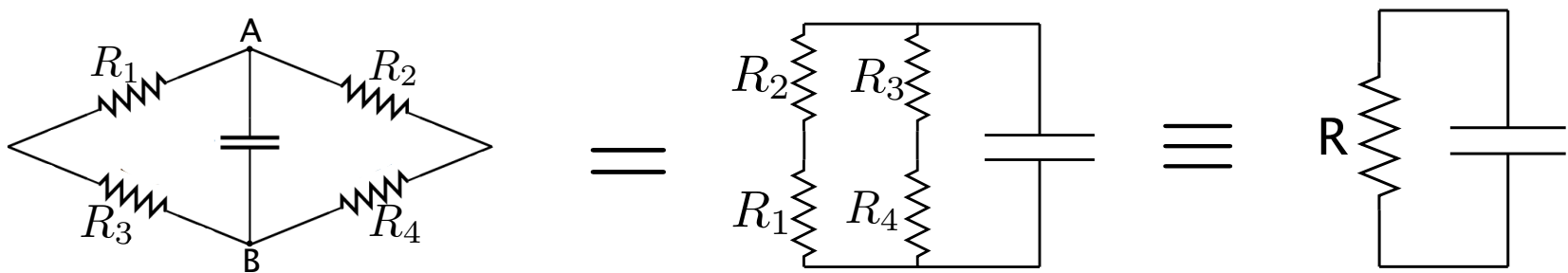
Example Problem: 4 Resistors/1 Capacitor



$$\begin{aligned} |V_C| &= |V_A - V_B| \\ &= |R_1 i_1 - R_3 i_3| \end{aligned}$$

Battery connected: at equilibrium, there is no current in C

After the battery is disconnected:

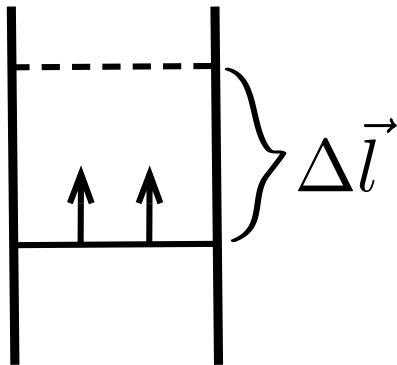


$$V = V_0 e^{-\frac{t}{\tau}}, \quad \tau = R_{eff} C, \text{ What is } R_{eff}?$$

Part 2: Magnetic Force

Electric Force: $\vec{F}_q^E = q\vec{E}$
 source acted on by $\vec{E}$

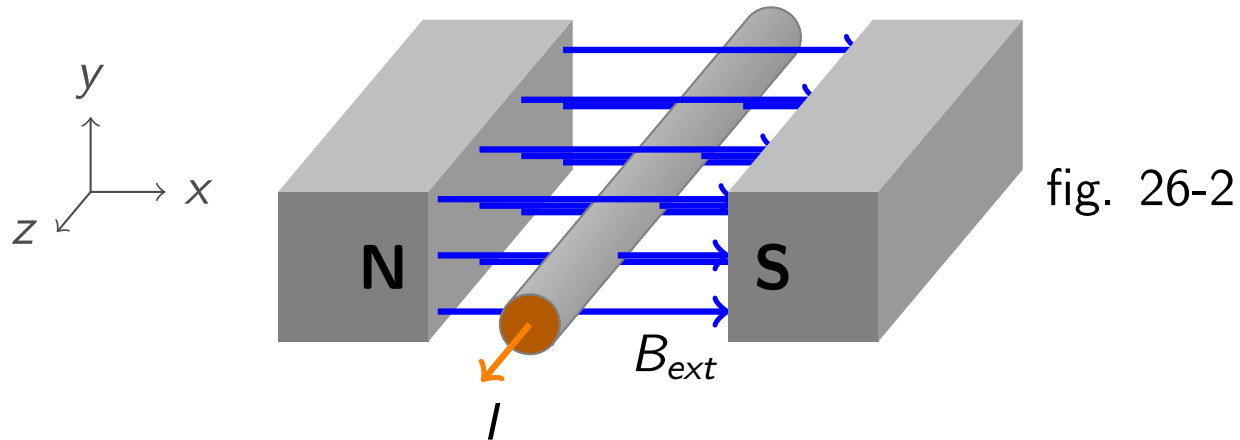
Magnetic Force: $F_{q\vec{v}}^B = q\vec{v} \times \vec{B}$



$$\Delta N_{qv} = \frac{\Delta N_q \Delta l}{\Delta t} = I \Delta \vec{l}$$

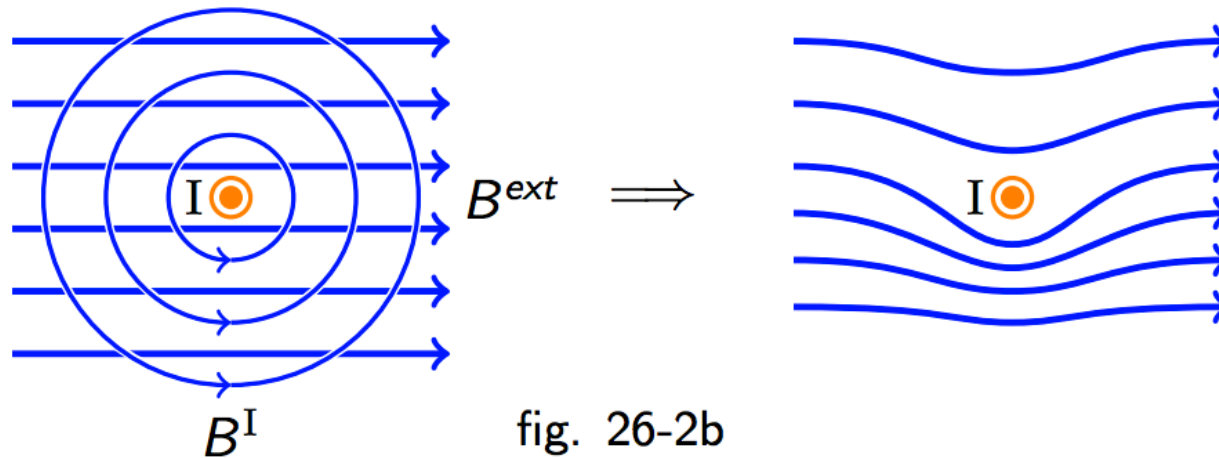
$$\therefore \Delta F_{I\Delta \vec{l}}^B = I\Delta \vec{q} \times \vec{B}$$

Understanding the direction of magnetic force



Consider the setup shown in fig. 26-2, where the magnetic field B^{ext} is along $\hat{x}$, and the current I flows along $\hat{z}$. Based the $\Delta \mathbf{F} = I \Delta \ell \times \mathbf{B}$, the force on the wire is in the $\hat{y}$ direction..

Denote by $\mathbf{B}^I$ the circular magnetic field generated by the current I . Both $\mathbf{B}^{ext}$ and $\mathbf{B}^I$ are illustrated in fig26.2a, where the current is pointing directly towards the viewer.



Define the resultant field to be: $\mathbf{B} = \mathbf{B}^{\text{ext}} + \mathbf{B}^{\text{I}}$. Compare the strength of the resultant field $\mathbf{B}$ above and below the current.

Choice	Which point has a larger resultant field?
1	Top
2	Bottom

Answer to previous slide clicker question:

Resultant field at bottom is stronger. Stronger field region pushed the wire to weaker region.

Intuitively the pattern is strong when the field is wrinkled.
Natural tendency is to straighten out the wrinkle.
Straight up = wrinkle free.

Magnetic force on two moving charges

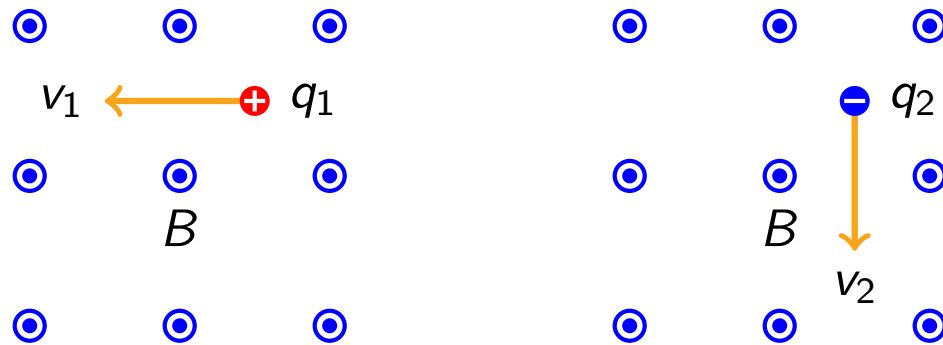


fig. 26-3

Case 1: $q_1 > 0$

Case 2: $q_2 < 0$

Choice	Direction of Force on $q_1 \mathbf{v}_1$	Direction of Force on $q_2 \mathbf{v}_2$
1	Up	Left
2	Down	Left
3	Up	Right
4	Down	Right