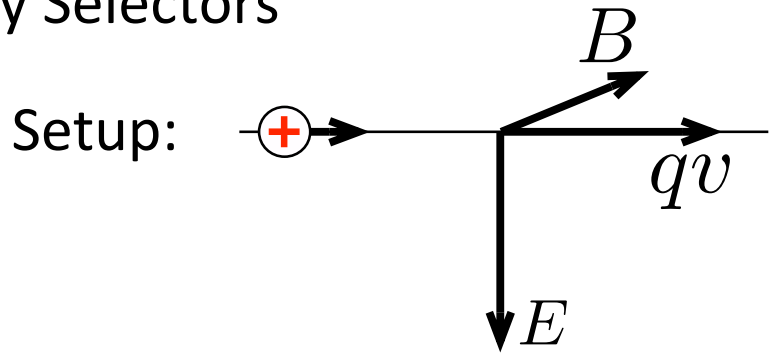


Lecture 29: Velocity Selector, Generators; Revisit Gauss' Law

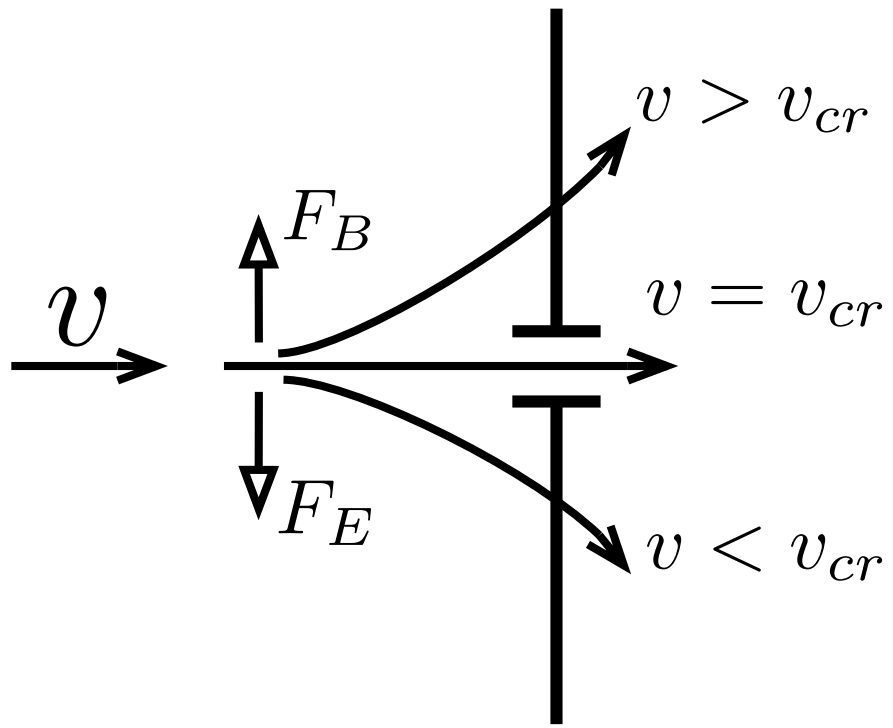
Velocity Selectors



$$F^B = qvB \text{ up}$$

$$F^E = qE \text{ down}$$

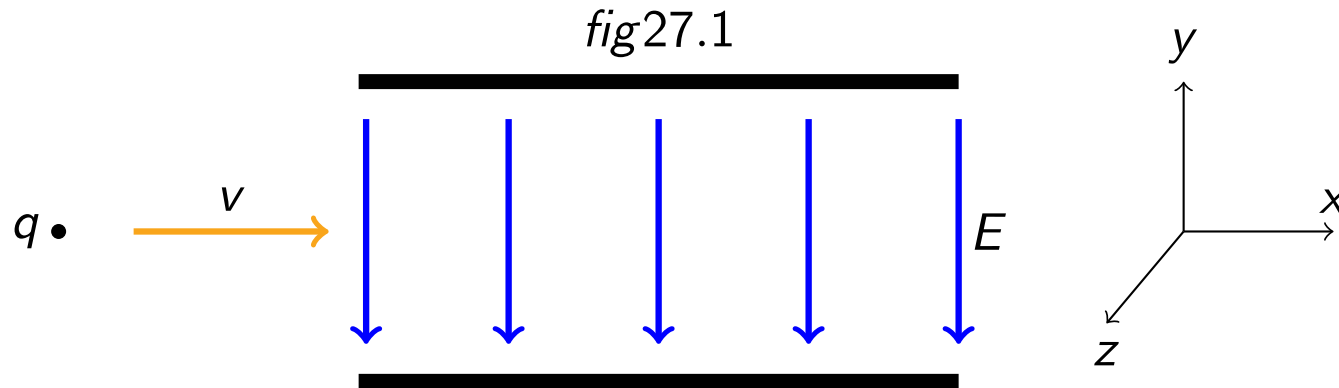
Critical velocity: $F^B = qv_{cr}B = qE, \quad v_{cr} = \frac{E}{B}$



Since E and B are given,
the set up allows only

$v_{cr} = \frac{E}{B}$ to pass through.

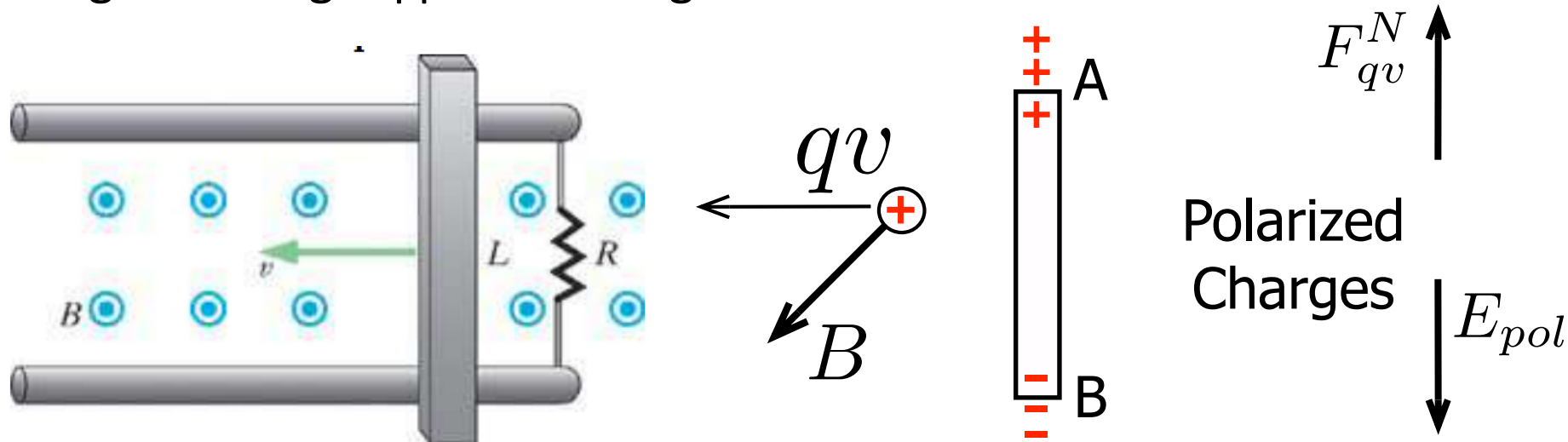
Velocity selector-2



If the sign of the charge q is *negative*, what is the direction of $\mathbf{B}$ in order to reproduce the non-deflection condition?

Choice	Direction of $\mathbf{B}$
1	Into
2	Out

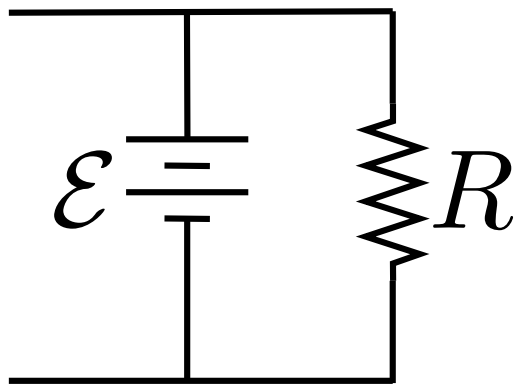
Sliding bar along 2 || conducting rails.



Magnetic Force is pushing positive test charge uphill

$$\therefore emf = \frac{W_{A \rightarrow B}}{q} = \frac{qvBh}{q} = vBh$$

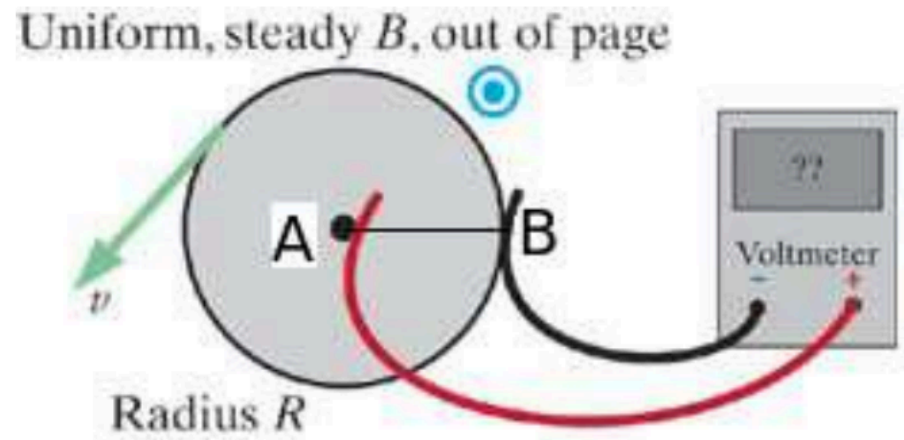
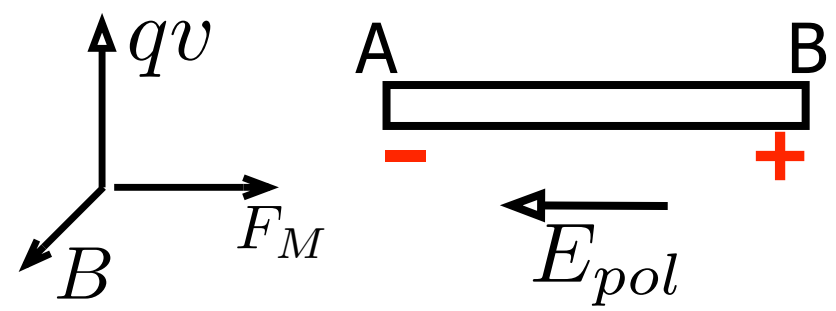
Mechanical Power = Fv $\stackrel{?}{=}$ Power of dissipation = $\mathcal{E}I$



$$(IhB)v = (vBh)I$$

Conservation of Energy

Rotating Disk

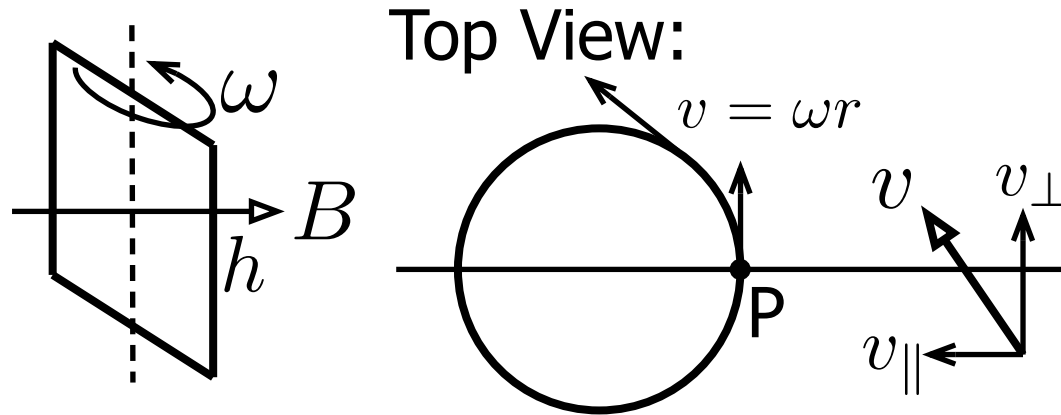


$\vec{E}_{pol}$: Define the direction of down hill to be from B to A.
Magnetic Force F_M pushes positive charge from A to B.

Compare the potentials: V_A vs. V_B

1. $V_A < V_B$
2. $V_A > V_B$

Rotating Loop induces emf in the loop



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$|F| = qv_{\perp} B$$

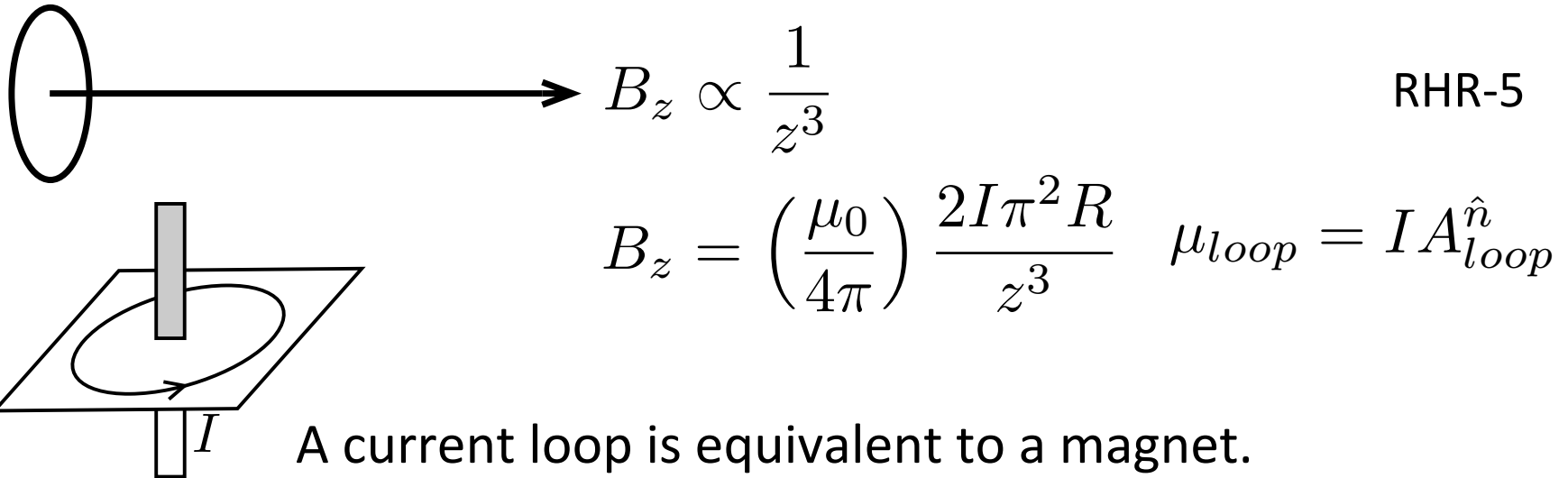
Max occurs at P: $F_{max} = qvB$

$$emf_{max} = \frac{F_{max}h}{q} = vBh = \omega RBh.$$

$$\omega \text{ is in units of } \frac{\text{rev}}{\text{min}} = \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi}{\text{rev}}$$

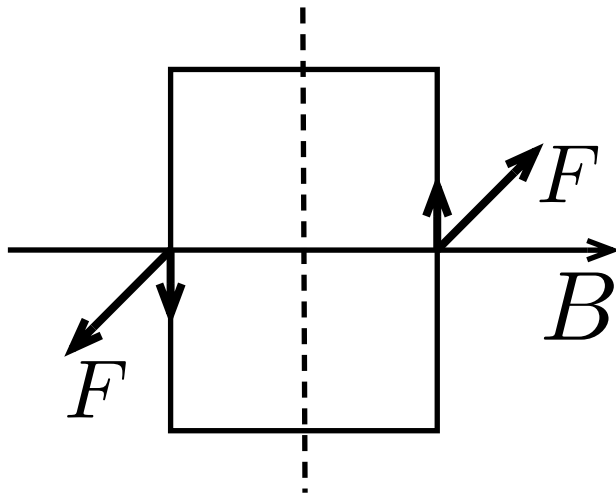
Current loop in motors and generators

We recall that a current loop generates a magnetic field.



A current loop is equivalent to a magnet.

Loop-magnet, density

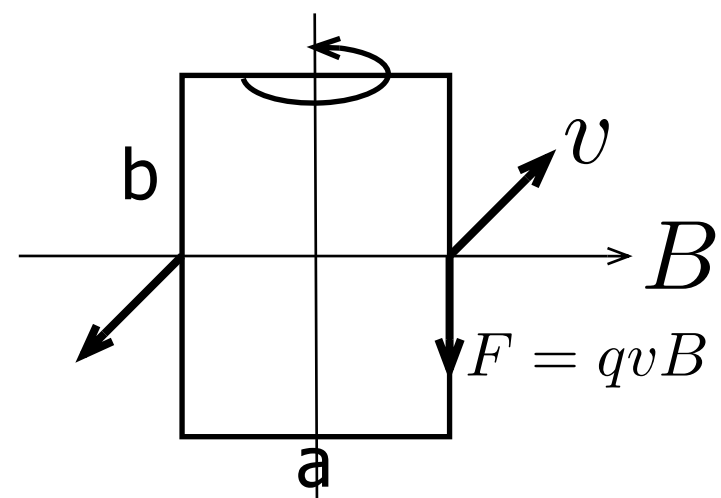


B exerts a torque on the emf loop

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

With a split ring arrangement, the loop can rotate endlessly – this is the principle behind a motor.

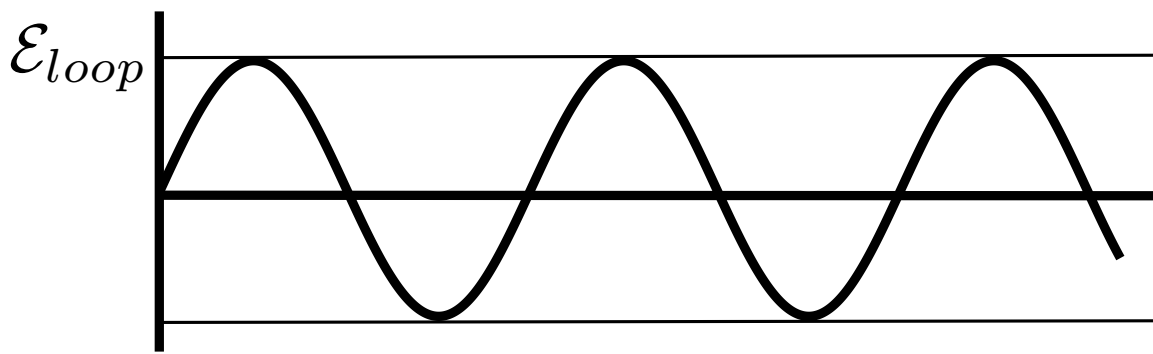
Rotating loop generates emf in a loop



Max emf is when B aligns with the plane of the loop (rectangle $a \times b$)

$$\begin{aligned}\mathcal{E}_{max} &= 2 \frac{qvBb}{q} \\ &= 2vBb \\ &= 2 \frac{a}{2} Bb \\ &= \omega abB\end{aligned}$$

$$\mathcal{E} = (\omega abB) \sin \omega t$$

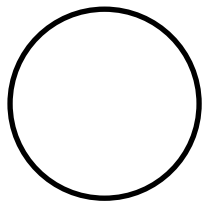


Beyond symmetric application of Gauss Law and Ampere's Law

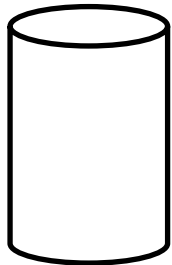
Gauss Law: Point charge $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}, \quad 4\pi r^2 E = \frac{Q}{\epsilon_0}$

General Form: $\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_S}{\epsilon_0}$

S can be arbitrary, Q_S any charge distribution enclosed by S



$$4\pi r^2 E = \frac{Q}{\epsilon_0}$$

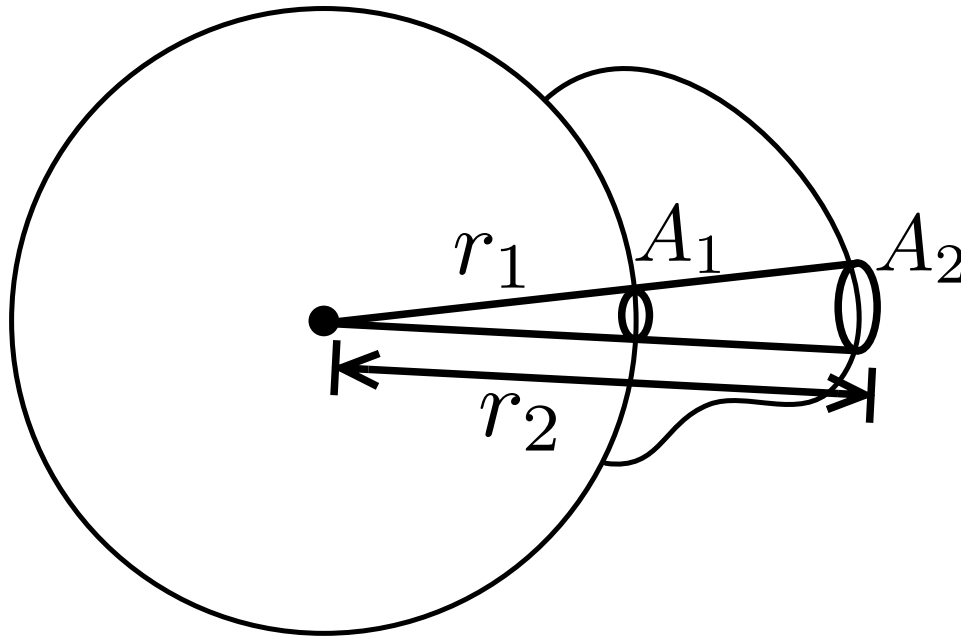


$$2\pi r h E = \frac{Q}{\epsilon_0}$$



$$2AE = \frac{Q}{\epsilon_0}$$

Begin with 1 point charge q with arbitrary surface S which encloses q



$$E_1 A_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_1^2} \hat{r} \cdot A_1$$

$$\Delta\Omega = \frac{\hat{r}}{r^2} \cdot A$$

$$E_2 A_2 = \frac{1}{4\pi\epsilon_0} q \Delta\Omega$$

Over enclosing surface: $LHS = \frac{1}{4\pi\epsilon_0} q \oint \Delta\Omega = \frac{q}{\epsilon_0} \oint \Delta\Omega = 4\pi$

Let: $Q_s = \sum_k q^{(k)}$

Super position: $\oint E^{(k)} \cdot dA = \frac{\sum^k q_k}{\epsilon_0}$

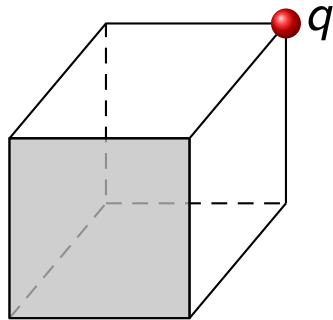


fig. 29-1

A charge q is placed at the corner of the cube shown in fig29.1. Find the electric flux through the shaded area.

Choice	ϕ_{shaded}
1	$q/(6/\epsilon_0)$
2	$q/(12/\epsilon_0)$
3	$q/(24/\epsilon_0)$
4	$q/(36/\epsilon_0)$

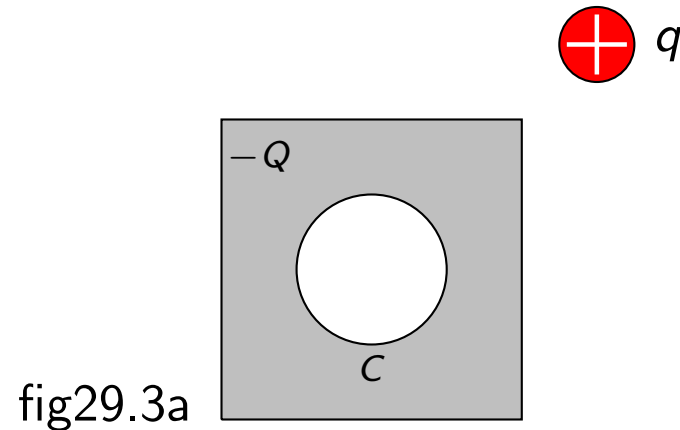
$$\text{Total flux: } \frac{q}{\epsilon_0}$$

$$\text{Through 1 cube: } \times \frac{1}{8}$$

$$\text{Through 1 face: } \times \frac{1}{3}$$

$$\therefore \text{Flux through shaded square face: } \frac{q}{\epsilon_0} \times \frac{1}{24}$$

Beyond symmetric charged distributions-III



A metal block containing a cavity and carrying a net charge $-Q$ is located near a positive charge q as shown in fig. 29.3a. For a static situation, can there be charges on the surface C of the cavity?

Choice	Charges on the surface C ?
1	Yes
2	No

Beyond symmetric charged distributions-IV

fig29.4

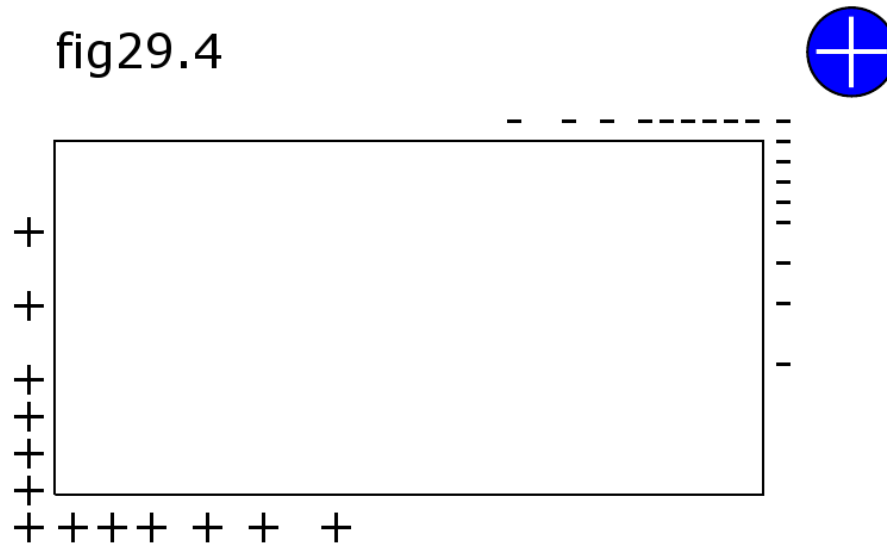
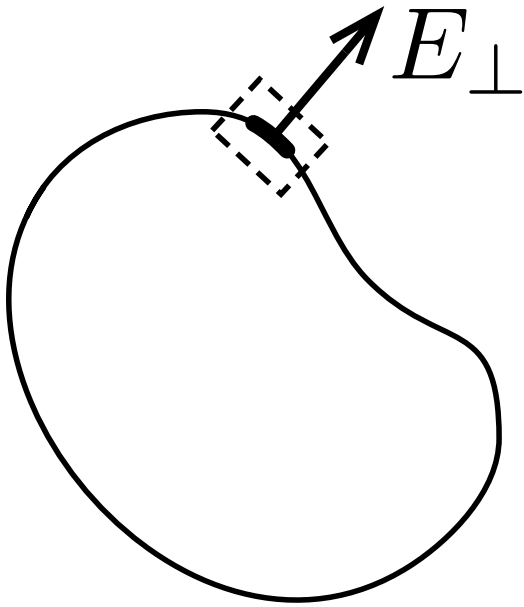


Fig. 29.4 shows an empty metal box located near a positive charge q . q causes the metal box to polarize, producing the surface charge distribution shown. Determine the electric field at a point inside of the metal box near the upper right hand corner.

Choice	E-field near the right-upper corner of the block
1	E is pointing towards from $+q$
2	E is pointing away from $+q$
3	$E = 0$

Field near the surface of a conductor



Gauss Law:

$$E_{\perp} \Delta A = \frac{\Delta Q_{surface}}{\epsilon_0}$$

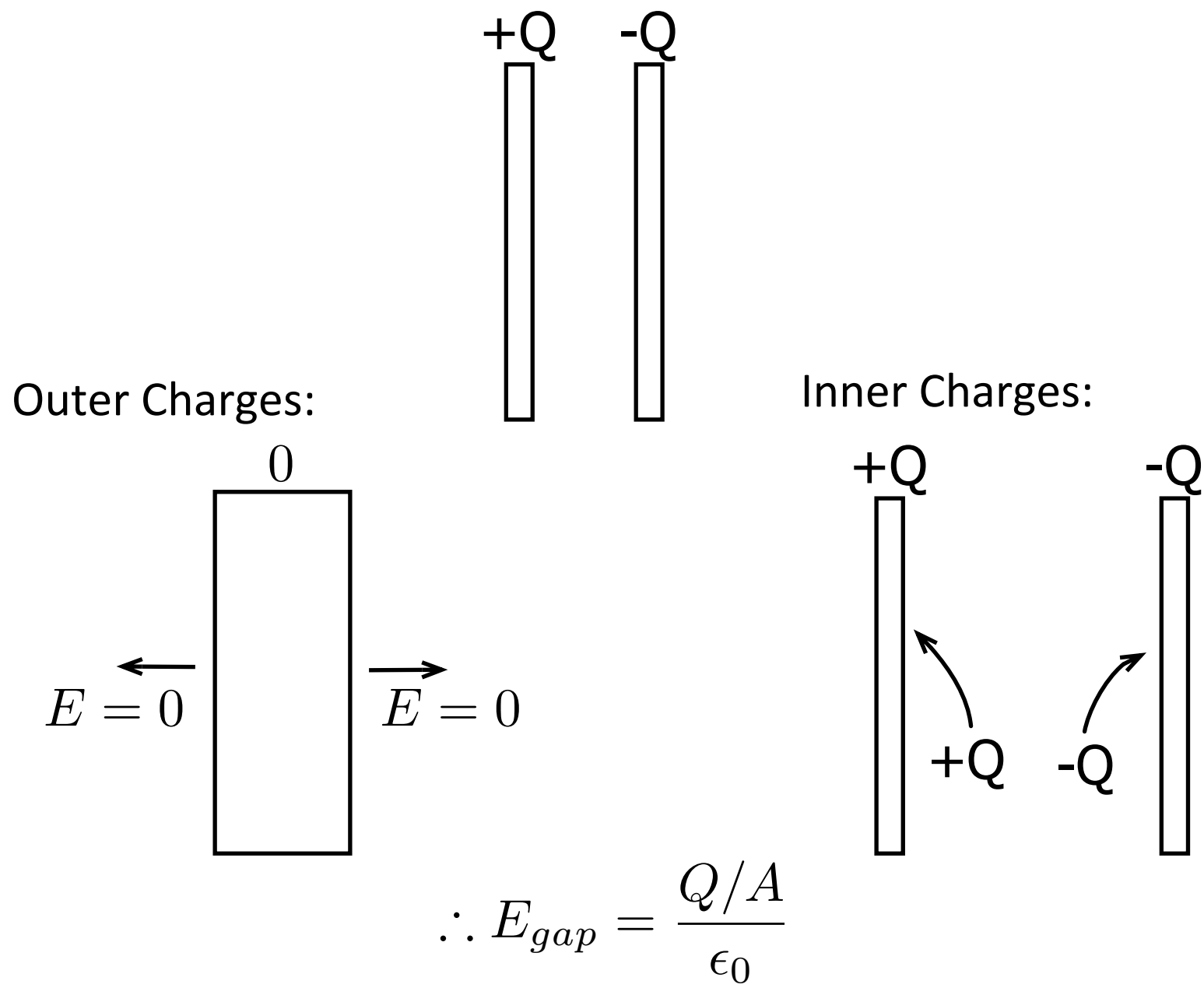
$$E_{\perp} = \frac{\sigma_{surface}}{\epsilon_0}$$

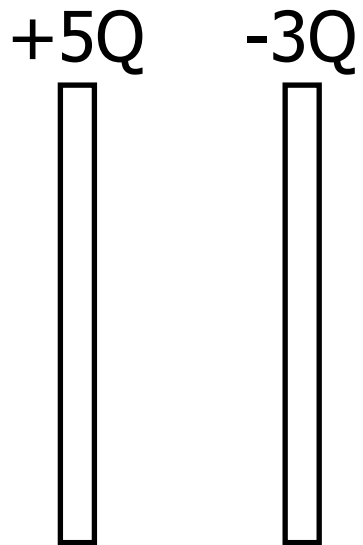
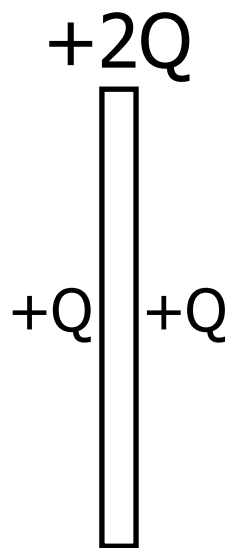
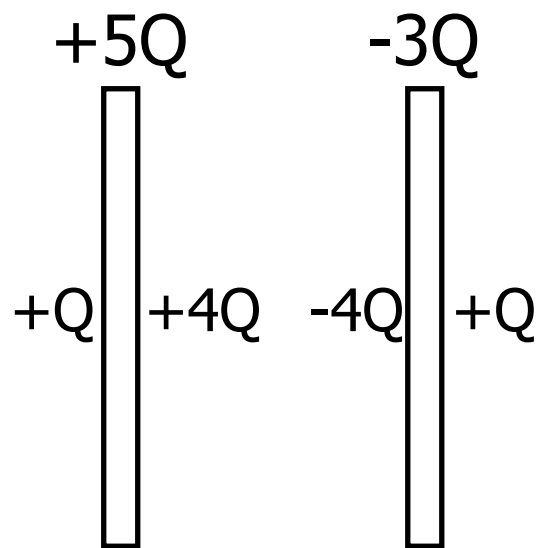
$$\text{Theorem: } E_{\perp} \Leftrightarrow \frac{\Delta Q_{surface} / \Delta A}{\epsilon_0}$$

So once E-perpendicular is given, the corresponding surface charge density is determined.

Conversely, once the surface charge density is given E-perpendicular can be determined.

Application: Determine surface charge by reflection





$$\therefore E_{gap} = \frac{4Q/A}{\epsilon_0}$$