

Lecture 30

Motional emf is a special case of Faraday's Law

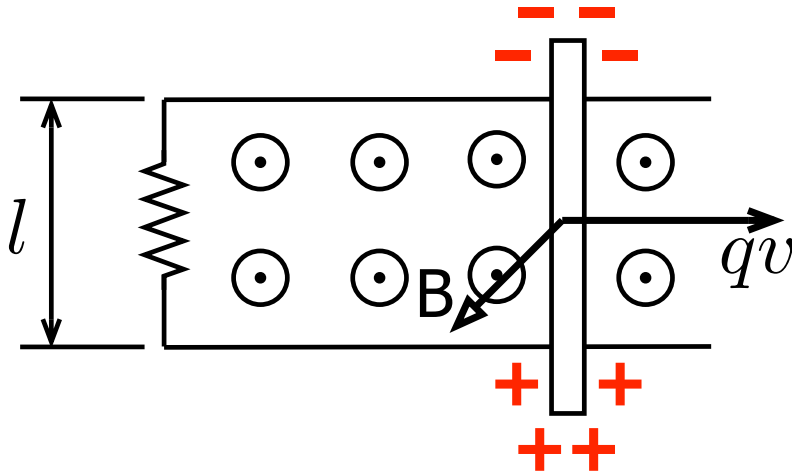
General form of Faraday's Law:

$$\mathcal{E} = \int_{path} \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi_{path}}{dt} \quad \text{The path is the Faradian loop.}$$

$$RHS = -\frac{dBA}{dt} = -\frac{dB}{dt}A - B\frac{dA}{dt} \quad (1)$$

Check: Moving rod along two || railing setup gives: $\mathcal{E} = vBl$
 which can be derived based on Faraday's Law

From motional emf: $\mathcal{E} = \frac{qvBl}{q} = vBl \quad (2)$



Lower end has higher potential:
 $\mathcal{E}$ is CW. (2')

From RHS of F.L., i.e. eq(1) $\left| -\frac{BdA}{dt} \right| = \left| -\frac{Bdhx}{dt} \right| = Bh \frac{dx}{dt}$

Lenz law implies B_{in} is $\otimes$, or E_{in} CW. (3') $= Bhv \quad (3)$

Lower end has higher potential:

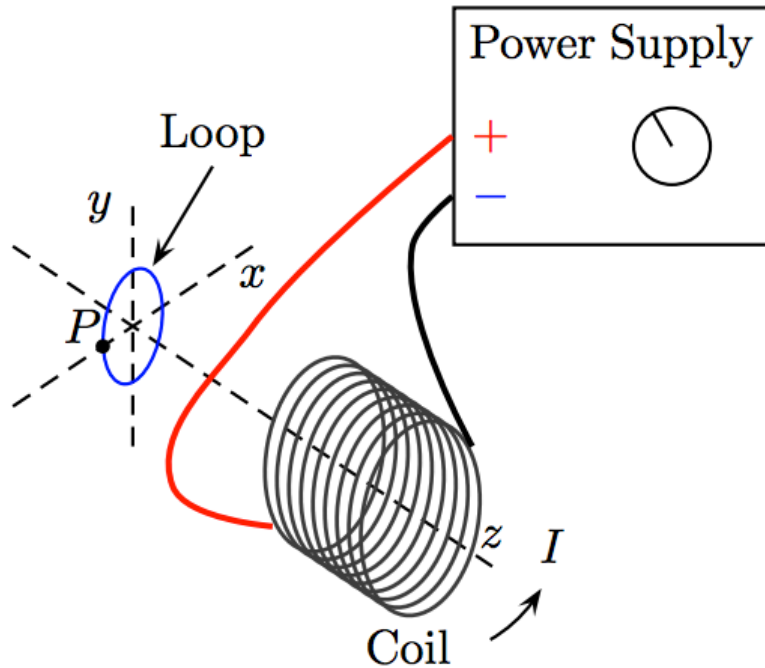
$\mathcal{E}$ is CW. (2')

We see (2) and (2') agree with (3) and (3'). Our motional emf is a spherical case of Faraday's law.

One can show in the case of current loop from both consideration one finds:

$\mathcal{E} = \mathcal{E}_{max} \sin \omega t$, where for N turn coils,

$$\mathcal{E}_{max} = NvBl$$



Find: ϕ of the loop

$$\phi_{loop} = B_{loop}^{sol} \pi r^2$$

$$B = B_{loop}^{solenoid} = \frac{\epsilon_0}{4\pi} \frac{2\mu^{sol}}{d^3}$$

$$\mu_{sol} = N \mu^{ring} = N I A = N I \pi R^2$$

$$\phi_{loop}^{sol} = \text{const. } I$$

$$L = \text{cost} = \frac{\phi}{I} = \left(\frac{\mu_0}{4\pi} \right) \left(\frac{2N\pi R^2}{d^3} \right) \pi r^2$$

What is E_{in} at P when $I = \text{const.}$?

Choices: $|E_{in}| = 0$, or > 0

at t_2 as $I \downarrow$, find $\langle E_{in} \rangle$

Choices: $\langle E_{in} \rangle = \uparrow$ or $\downarrow$

Find $\frac{\Delta\phi}{\Delta t}$

$$\left| \frac{\Delta\phi}{\Delta t} \right| = \left| \frac{\Delta LI}{\Delta t} \right| = L \frac{\Delta I}{\Delta t}$$

$$|emf|_{inloop} = \left| \frac{\Delta Q}{\Delta t} \right|$$

Remove the ring from $B = B_1$ region to the region where $B = 0$ in t_1 . Assume ring resistance is R .

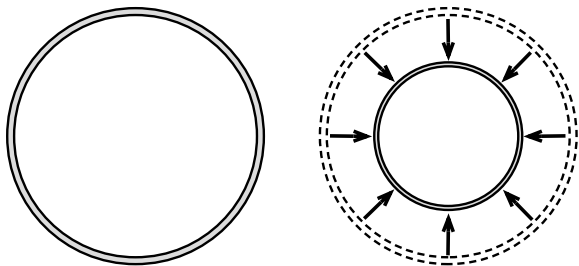
Find: Average $\mathcal{E}$, Energy consumed

$$\mathcal{E}_{avg} = \frac{(B_1 A - 0 A)}{t_1 - 0} = \frac{B_1 A}{t_1} \text{ volts}$$

$$\text{Energy consumed} = P \Delta t = I \mathcal{E} \Delta t = \frac{\mathcal{E}^2}{R} \Delta t$$

$$|\mathcal{E}| = \left| \frac{d\phi}{dt} \right| \quad \text{Answer same } \mathcal{E} \text{ throughout } t\text{-interval}$$

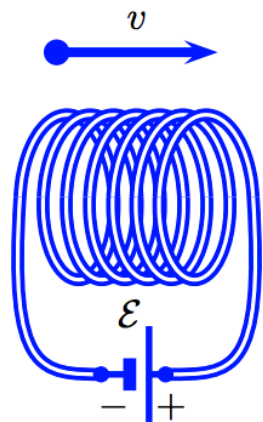
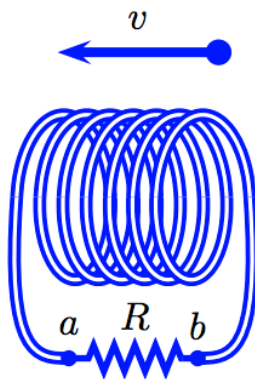
What if the loop is shrinking down too?



shrinking down to 0

$$\text{Energy} = P \Delta t = \frac{\mathcal{E}^2}{R} \Delta t$$

Example Problem:



Determine the direction of the magnetic field in the coil with the battery attached.

Determine the direction of the magnetic field in the coil with the resistor attached

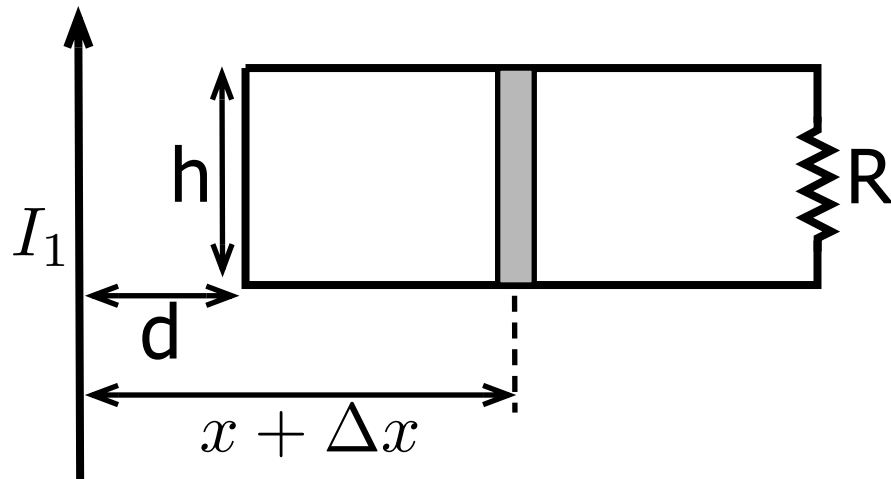
Determine The direction of the induced current in the resistor

Hint: Go to the rest frame of the right-coil.

Here $B_{\text{at the right coil}}^{\text{due to the left coil}}$ is to the right.

and is increasing B_{induced} must point to the left
i.e. to oppose the change.

Example Problem:



If I_1 is decreasing, find direction of emf_{ind} in the loop

Choices

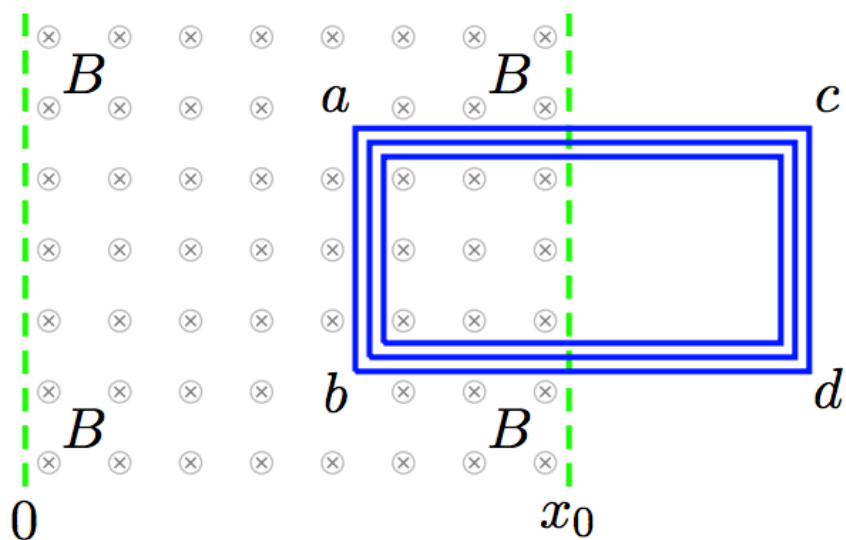
1 – CW

2 – CCW

Find: $|emf|$. Hint: $|emf| = \left| \frac{d\phi}{dt} \right|$

$$\begin{aligned}
 emf &= \frac{d}{dt} \int dx dy B = \frac{d}{dt} h \int dx \frac{\mu_0}{2\pi x} I_1 \\
 &= \frac{h\mu_0}{2\pi} \frac{dI_1}{dt} \int_d^{d+W} \frac{dx}{x}
 \end{aligned}$$

Example Problem



Determine the direction of the force on the segment AB as B is decreasing.

Hint: The force on ab is given by:

$$F_{I_{in}h}^{B_{in}} = I_{in}h \times B_{in}$$

First determine the direction of B_{in} (should it be into or out)

The correct response is to keep flux within the loop constant