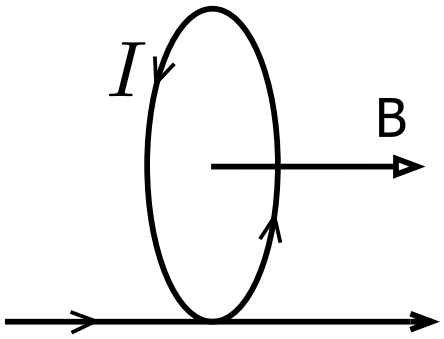
Lecture 31

Inductance



Consider the application of Faraday S Law to a single loop.

$$I \text{ generates } B,\ \phi$$

If I changes $\Rightarrow \phi$ changes. Loop has magnetic inertia, it generates induced B_{in}, an induced flux cancels the change.

This is Faraday's Law: $\mathcal{E}_{ind} = -\dfrac{\Delta\phi}{\Delta t}$

When we work with a circuit, it is convenient to simply write the flux as $\phi = const \times I$. F.L becomes

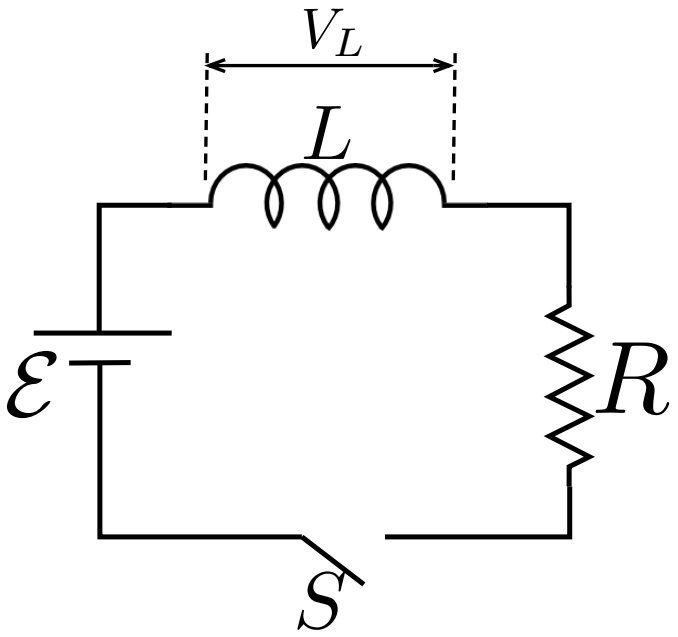
$$\mathcal{E}_{ind} = -const.\frac{dI}{dt} = -\left(\frac{\phi}{I}\right)\frac{dI}{dt}$$

For N loops, the inductance $L = \dfrac{N\phi}{I}$.

This is the proportionality constant which tells us how much emf is induced for a given $\dfrac{dI}{dt}$.

The bigger is L, the larger be the emf induced.

Solenoids are referred to as inductors. It in series with a battery and a resistor forms a simple RL circuit.



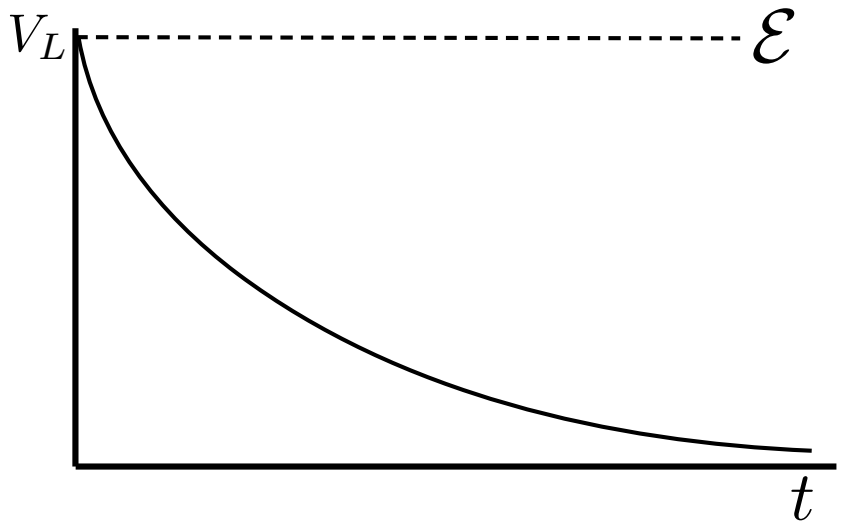
The loop eqn is: $\mathcal{E} - V_L - IR = 0$

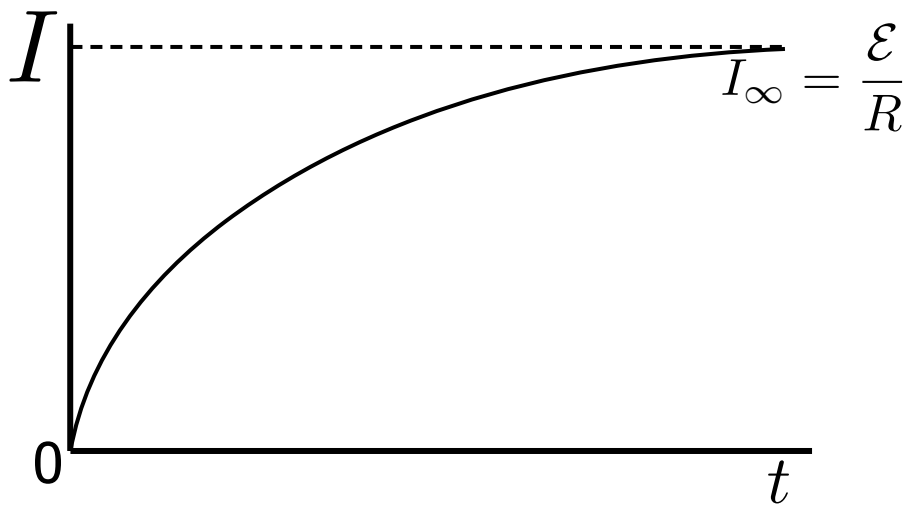
where $V_L = \dfrac{dI}{dt}$

Faraday's Law says: $\mathcal{E}_{ind} = -L\dfrac{dI}{dt}$

Close s at $t = 0$. Due to magnetic inertia of the solenoid, at $t = 0^+$, $I = 0$, $V_L = \mathcal{E}$.

at $t = \infty$, equilibrium is reached, $V_L = L\dfrac{dI}{dt} = 0$, $I = \dfrac{\mathcal{E}}{R}$





Analogous to RC case: $V_L = \mathcal{E} e^{-\frac{t}{\tau}}$ $\qquad I = I_\infty \left(1 - e^{-\frac{t}{\tau}}\right)$

Notice the derivative of the loop equation gives:

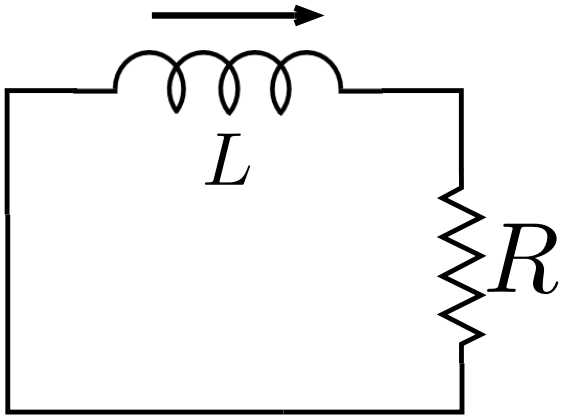
$$\frac{d}{dt}\left[\mathcal{E} - V_L - IR\right] = -\frac{dV_L}{dt} - \frac{dI}{dt}R = \frac{dV_L}{dt} - \left(L\frac{dI}{dt}\right)\frac{R}{L} = 0$$

Rearrangement gives: $\dfrac{dV_L}{dt} = -\dfrac{V_L}{(L/R)}$ $\qquad L\dfrac{dI}{dt} = V_L$

L/R has the dimension of time and

τ is the characteristic time of the circuit.

For decay case:

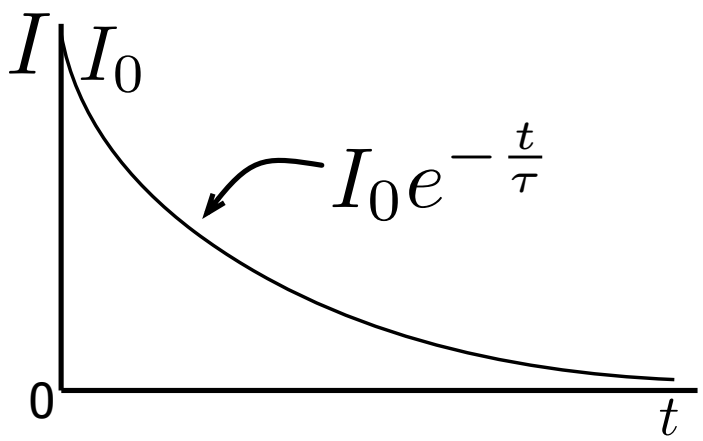


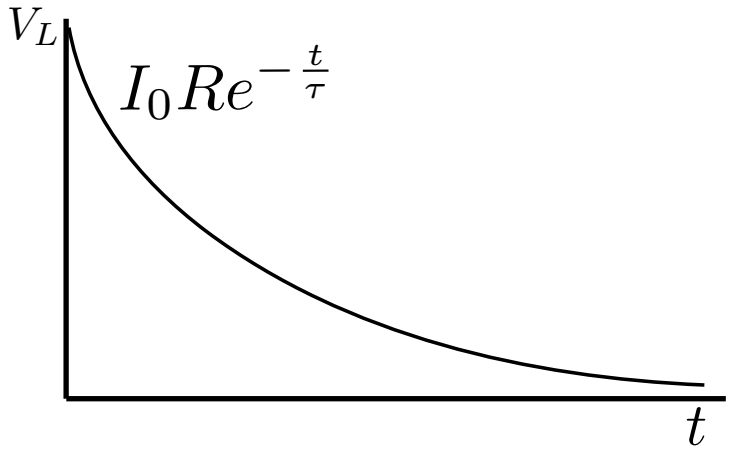
Here, the battery is removed.

Loop Equation: $-V_L - IR = 0$

At $t = 0$, let $I = I_0$, $-V_L = I_0 R$.

At $t = \infty$, equilibrium is reached. $I = 0$, $V_L = L\dfrac{dI}{dt} = 0$.





Energy Stored in C and L

$$\text{Power} = P = \frac{dU}{dt} = \frac{dqV}{dt} = IV$$

$$U_C = \int IV_C dt = \int \left(\frac{dq}{dt}\right)\left(\frac{q}{C}\right) dt = \frac{L}{C}\int_0^Q qdq = \frac{Q^2}{2C}$$

$$U_L = \int IV_L dt = \int IL\frac{dI}{dt}dt = L\int IdI = \frac{LI^2}{2}$$

Express Energy Stored in terms of B

$$LI = \left(\frac{N\phi}{I}\right) I = N\phi = NBA$$

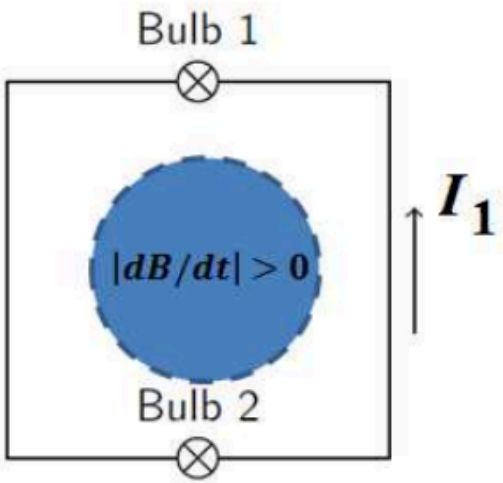
$$B = \mu_0 \frac{N}{l} I = N\phi = NBA$$

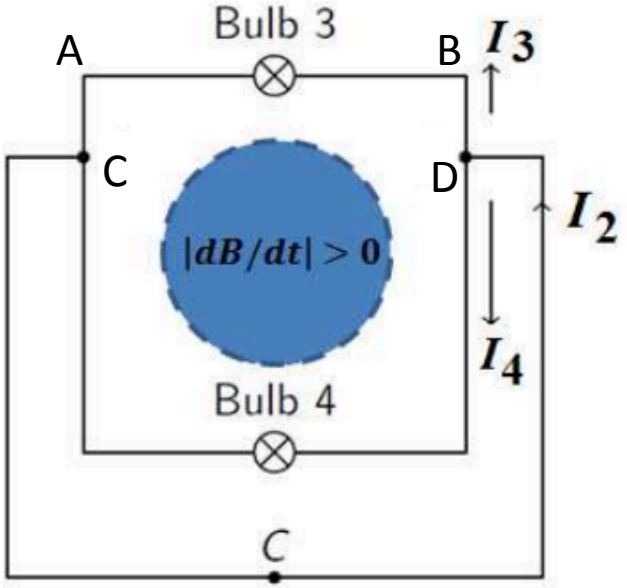
$$\therefore U_L = \frac{1}{2}(LI)(I) = \frac{1}{2}(NBA)\frac{Bl}{\mu_0 N} = \frac{1}{2\mu_0}B^2 Al$$

Energy Density

magnetic $$\mu_B = \frac{U_L}{AL} = \frac{1}{2\mu_0} B^2.$$

electric $$u_E = \frac{1}{2}\epsilon_0 E^2$$



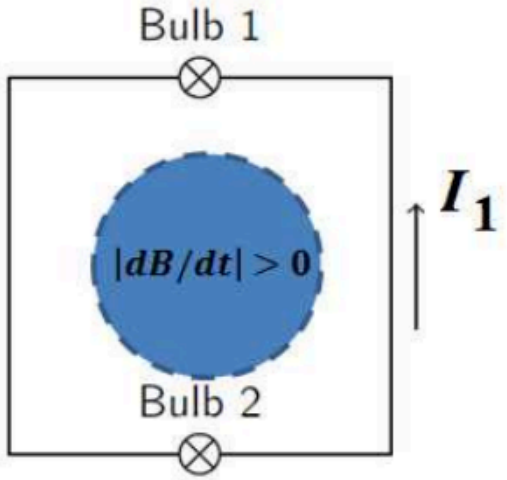


$$R_{bulb} = R$$

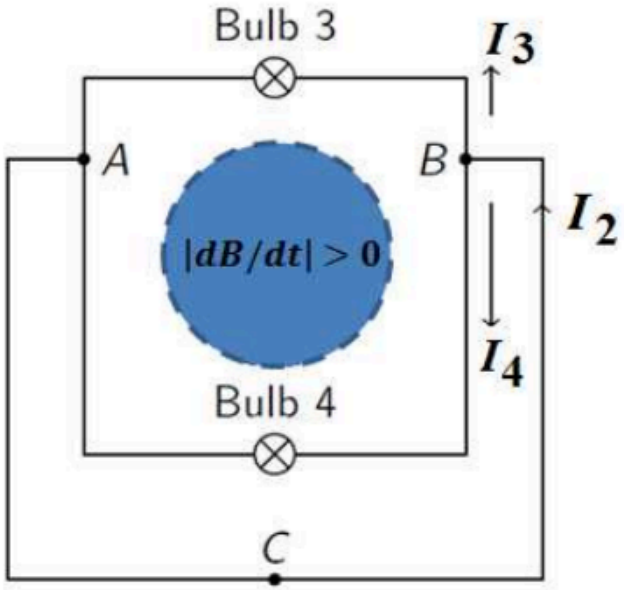
Left Diagram:

Loop eqn: $emf - I_1 R - I_2 R = emf - 2IR = 0$

$$I = \frac{emf}{2R} \quad , \; P_1 = I^2 R = \frac{emf^2}{4R}$$



$$R_{bulb} = R$$



Right Diagram:

Loop A3B4A: $emf - I_3R + I_4R_{wire} = emf - I_3R$

$$I_3 = \frac{emf}{R} \qquad P_3 = 4P_1$$

Loop C(wire)D4C: $0 - I_5R_{wire} - I_4R = 0$

$$I_4R = -I_5R_{wire} = 0$$

$$\therefore I_4 = 0, \;\; P_4 = 0$$