

Lecture 34

Energy in a Capacitor and Inductor:

Power: $P = \frac{dU}{dt} = \frac{\Delta qV}{\Delta t} = IV$

In Capacitors:

$$\left. \begin{array}{c} +Q \\ \hline \\ \hline \\ -Q \end{array} \right\} V = \frac{Q}{C}$$

$$\begin{aligned} U_C &= \int \frac{dQ}{dt} \frac{q}{C} dt \\ &= \frac{1}{C} \int_0^Q q dq \\ &= \frac{Q^2}{2C} \end{aligned}$$

For Inductors:

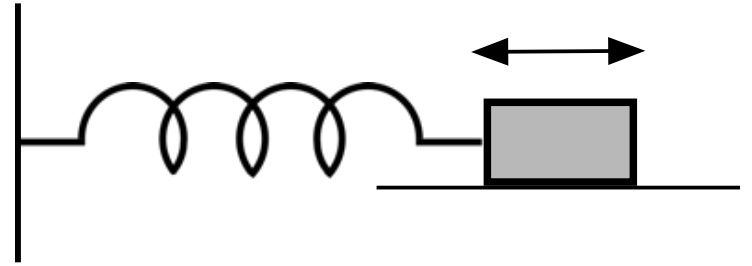


A circuit diagram showing an inductor with three loops. To the right of the inductor is the equation $V = L \frac{dI}{dt}$.

$$\begin{aligned} U_L &= \int I \left(L \frac{dI}{dt} \right) dt \\ &= L \int_0^I I dI \\ &= \frac{LI^2}{2} \end{aligned}$$

Mechanical Analogy: Mass Spring System

$$F = ma = m \frac{d^2 x}{dt^2}$$



$$KE = \frac{1}{2}mv^2$$



Corresponds to:

$$\frac{1}{2}LI^2$$

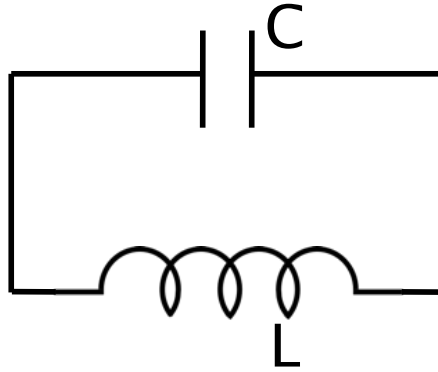
$$PE = \frac{1}{2}kx^2$$



$$\frac{1}{2C}q^2$$

LC circuits oscillate much in the same way a mass spring system oscillates

LC Circuit



Basic Equations:

Loop Equation:

$$V_L + V_C = 0 \qquad L \frac{dI}{dt} + \frac{q}{C} = 0$$

Power Equation:

$$P = I \times \text{loop eqn.} = IV_L + IV_C = 0$$

Conservation of
Energy:

$$0 = I \left(L \frac{dI}{dt} \right) + \frac{dq}{dt} \frac{q}{C}$$

$$U_L + U_C = \text{const.}$$

$$U_{C,max} = U_C + U_L = U_{L,max}$$

Solving the Loop Equation – Analogy to Mass-Spring System

$$m \frac{d^2 x}{dt^2} = -kx$$

Solution: $x = A \cos \omega t$

$$\frac{dx}{dt} = -A\omega \sin \omega t$$

$$\frac{d^2 x}{dt^2} = -A\omega^2 \cos \omega t = -\omega^2 x$$

Comment:

General Solution: $x = x_{max} \cos(\omega t + \delta)$

(δ is the initial phase angle of the oscillation)

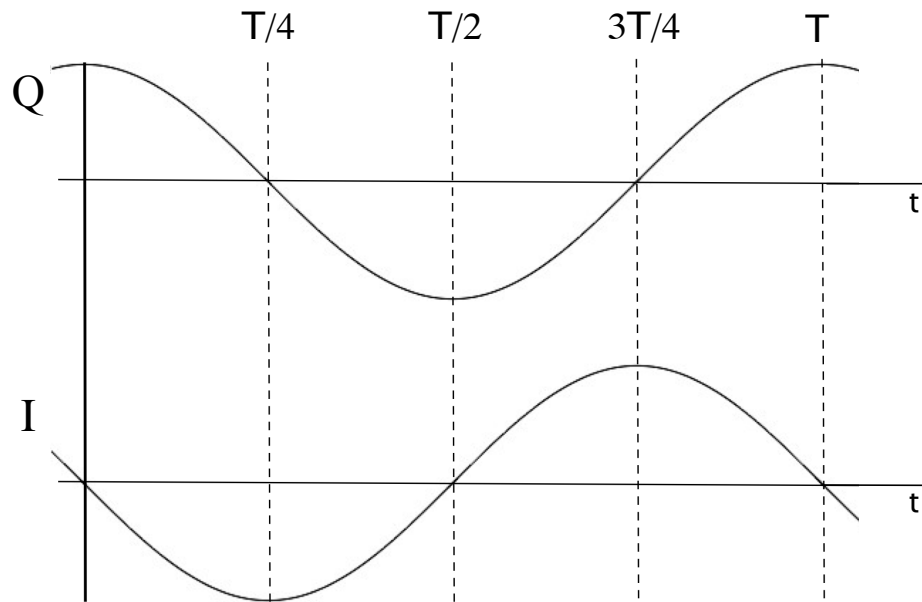
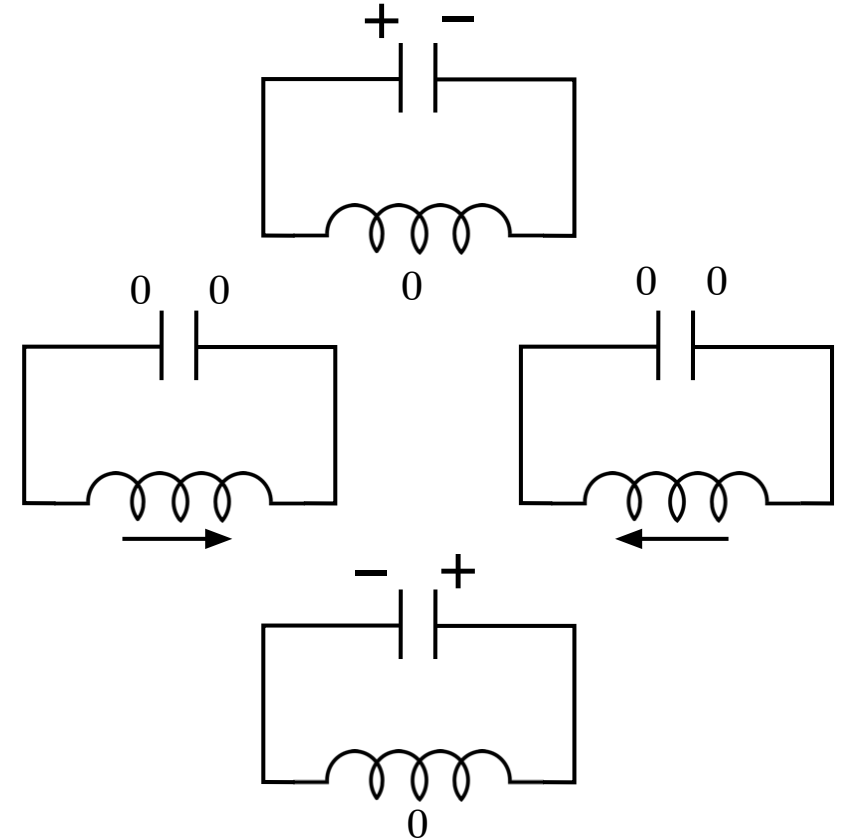
Often set $\delta = 0$, where at $t = 0$, the initial condition is chosen to be $x = x_{max}$

LC Circuit Analogy

$$q = q_{max} \cos \omega t$$

$$I = \frac{dq}{dt} = -q_{max} \omega \sin \omega t$$

$$= -I_{max} \sin \omega t$$



q charge on left plate. at $t = 0$, $q = q_{max} \cos (\omega 0) = q_{max}$

I current in coil. At $t = \frac{T}{4}$, $I = -I_{max} \sin \left(\frac{\omega T}{4} \right) = -I_{max} \sin \left(\frac{2\pi}{4} \right) = I_{max}$

Check: $U_L + U_C = U_0$

$$LHS = \frac{1}{2}L (I_{max} \sin \omega t)^2 + \frac{1}{2C} (q_{max} \cos \omega t)^2$$

$$u_0 = \frac{1}{2}L I_{max}^2$$

$$= \frac{1}{2}L \omega^2 q_{max}^2$$

$$= \frac{1}{2}L \frac{1}{LC} q_{max}^2$$

$$u_0 = \frac{1}{2C} q_{max}^2$$

$$LHS = u_0 \sin^2 \omega t + u_0 \cos^2 \omega t = u_0$$

$$\text{where } \sin^2 \omega t + \cos^2 \omega t = 1$$

RLC Circuit – Loop Equation

$$-L \frac{dI}{dt} - \frac{q}{C} - IR = 0$$

$$\text{Power Eqn.} = I \times (\text{loop eqn})$$

$$-\frac{d}{dt} \left[\frac{L}{2} I^2 + \frac{q^2}{2C} \right] - I^2 R = 0$$

$$+LI \frac{dI}{dt} + \frac{q}{C} \frac{dq}{dt} + I^2 R = 0$$

$$\underbrace{P_L + P_C}_{\text{Each term can be positive or negative.}} + \underbrace{P_R}_{\text{Always positive}} = 0$$

Each term can be positive or negative.

Always positive

$P_L + P_C$ stays negative

Graph of Q and I of Dampened Oscillation:

