

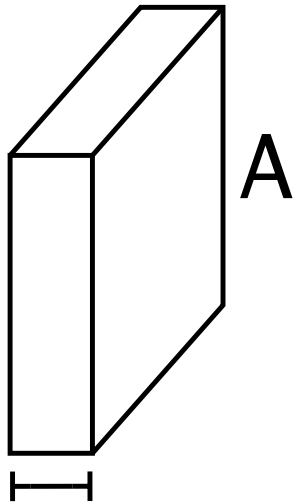
Lecture 38 Radiation Energy Density

$$U_E = \frac{1}{2}\epsilon_0 E^2 \qquad U_B = \frac{1}{2\mu_0} B^2$$

EM Wave: $E = cB, \quad c^2 = \frac{1}{\mu_0\epsilon_0} \rightarrow U_E = U_B$

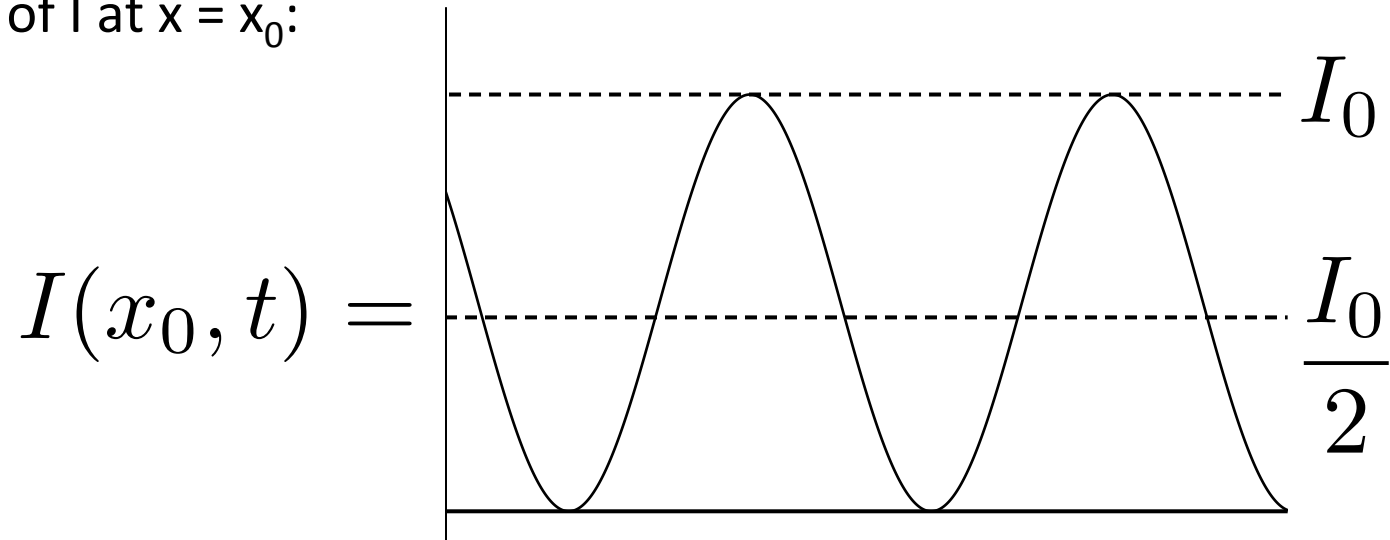
Equal partitions: $\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 c^2 B^2 = \frac{1}{2}\epsilon_0 \frac{1}{\epsilon_0\mu_0} B^2 = \frac{1}{2\mu_0} B^2$

Intensity:



$$\begin{aligned} I &= \frac{\Delta U}{A\Delta t} = \frac{\Delta U}{\Delta A \left(\frac{\Delta x}{c} \right)} = cu \\ &= 2cu_E \\ &= c\epsilon_0 E^2 \end{aligned}$$

Time dependence of I at $x = x_0$:



Time averaged value:

$$\bar{I} = c\epsilon_0 E_{max}^2 \overline{\cos^2(\omega t - kx)}$$

$$\therefore \bar{I}^2 = \frac{c\epsilon_0 E_{max}^2}{2}$$

$$E_{rms} = \sqrt{\bar{E}^2}$$

Intensity Vector (Poynting Vector)

Show that: $\frac{EB}{\mu_0} = cu = c\epsilon_0 E^2$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

Energy-Momentum Relationship: Relativistic Kinematic

For a particle will mass m :

$$\begin{cases} \text{Energy} & U = mc^2\gamma \\ \text{Momentum} & p = mv\gamma \end{cases}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = \frac{v}{c}$$

Light particle: photon

$$m = 0, \quad v = c$$
$$\frac{U}{p} = \frac{mc^2\gamma}{mv\gamma} = \frac{c^2}{c} = c, \quad \text{or } U = pc$$

Radiation Pressure

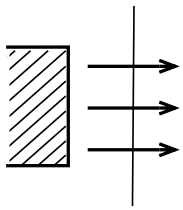
$$Pressure = \frac{F}{A} = \frac{dp}{dt} / A \begin{cases} absorptive : & \frac{dU/C}{dtA} = \frac{\Delta U}{\Delta t c A} = u \\ reflective : & 2u \end{cases}$$

Geometry Dependent A: Reflective Example

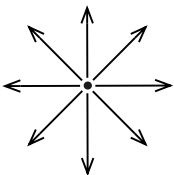
$$F = 2uA$$

Geometric consideration:

$$I = \frac{\Delta U}{\Delta t A} = \frac{Power}{A}$$

Light beam shining on a book:  $I = \frac{Power}{A}$

The diagram shows a rectangular surface with diagonal hatching on the left. Three horizontal arrows point from the surface to the right, representing a parallel light beam. To the right of the arrows is the symbol $\parallel \Delta A$.

Bulb shining on a book:  $I = \frac{Power}{\Delta A} \frac{\Delta A}{4\pi R^2}$

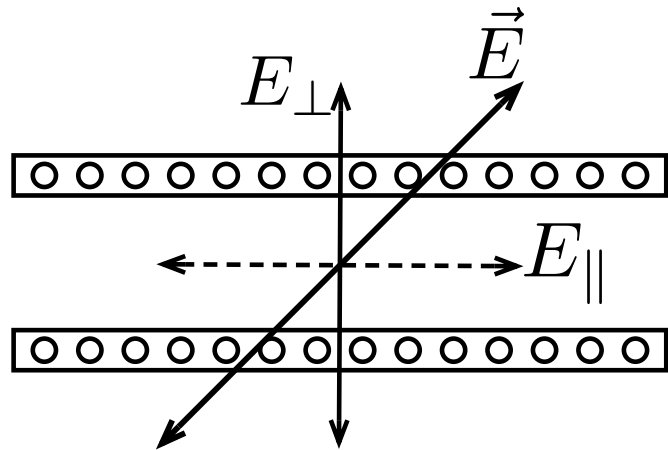
The diagram shows a central point with eight arrows radiating outwards in different directions, representing isotropic radiation from a bulb. To the right of the arrows is the symbol $\parallel \Delta A$.

$$I_{book} = \left(\frac{Power \Delta A}{4\pi R^2} \right) / \Delta A$$

Polarization

Define: Direction of polarization = direction of oscillation of E

Metal perpendicular strips:



E-parallel drives electron oscillation

Large induced current

Energy absorbed by medium

E-perp. Negligible induced current

This component can be transmitted

The strip setup serves as a polarizer.

If the incident light is unpolarized (i.e. polarization is uniformly distributed in the azimuthal direction) the outgoing light will be polarized in the vertical direction – direction of E-parallel

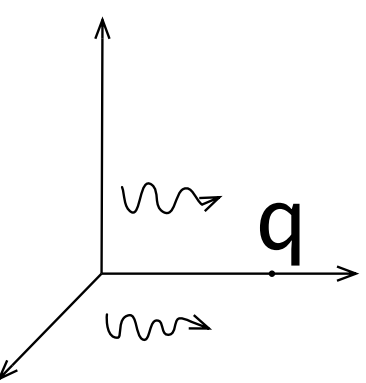
If the incident light is polarized along E where there is angle θ between E-perp and E , then $E_{\text{perp}} = E \cos\theta$,

Outgoing Intensity: $I_{out} = I_{in} \cos^2 \theta$ This is Malus' law.

Metal Strip Analyzer

- Rotating the strip can check polarization of the incident light.
- Unpolarized incident light, if no variation in intensity
- Unpolarized light may be represented by two equal weight mutually perpendicular polarized lights
- Two mutually perpendicular analogues can fully block out an polarized light

Radiation from a charged particle initially at rest:



$$\langle \vec{v} \rangle = \langle q \vec{E} \rangle$$

$$\vec{F} = q \vec{v} \times \vec{B}$$

$$q > 0 \text{ case, } \langle \vec{F} \rangle = \langle q \vec{v} \times \vec{B} \rangle = \langle \vec{v} \times \vec{B} \rangle = \langle \vec{E} \times \vec{B} \rangle$$

Direction of magnetic force on q initially at rest:

	q > 0	q < 0
1)	Left	Left
2)	Right	Left
3)	Left	Right
4)	Right	Right

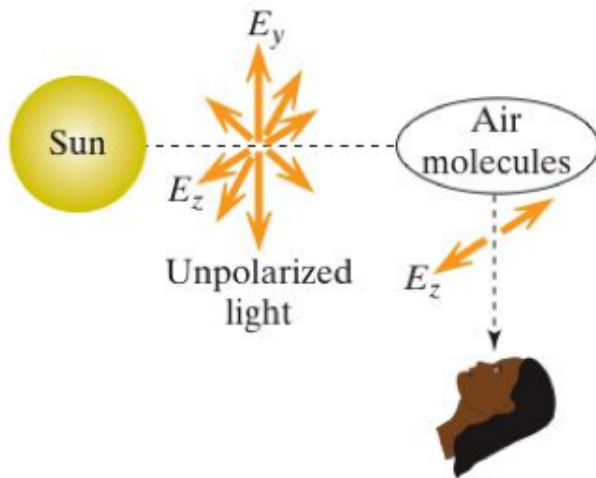
HINT:
$$\vec{F} = q \vec{v} \times \vec{B}$$

$$\langle v \rangle = \langle q E \rangle$$

Notice:
$$\langle F \rangle = q^2 \langle E \times \vec{B} \rangle$$



The polarized sky light



Fig(mi) 24.51

Sunlight: Unpolarized light

Rescattered light observed by ground observer is polarized along z

Intensity of scattered light:

$$I = cu = c(u_E + u_B) = c \cdot 2u_E = c\epsilon_0 E^2, \quad E \propto a_{\perp} \propto \omega^2 z$$

Compare Intensity of rescattered light with frequencies ω_1 and ω_2 :

$$\frac{I_1}{I_2} = \frac{E_1^2}{E_2^2} = \left(\frac{a_{\perp 1}}{a_{\perp 2}} \right)^2 = \left(\frac{\omega_1^2 z}{\omega_2^2 z} \right)^2 = \left(\frac{\omega_1}{\omega_2} \right)^4 = \left(\frac{\lambda_2}{\lambda_1} \right)^4$$

$$\frac{I_{blue}}{I_{red}} = \left(\frac{\lambda_r}{\lambda_b} \right)^4 \frac{\lambda_r}{\lambda_b} = \frac{700 \text{ nm}}{400 \text{ nm}} = 1.75, \quad \frac{I_b}{I_r} = (1.75)^4 \approx 9$$