Laser Electron Accelerators for Radiation

Medicine: a Feasibility Study

Charles Chiu, Mykhailo Fomytskyi, Franklin Grigsby, Frank Raischel+,
Michael C. Downer and Toshiki Tajima++

University of Texas at Austin, Department of Physics

Austin, TX 78712-1081

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+ Present address: Institut fur Theoretische Physik, Julius-Maximilians-Universitat
  at Wurzburg, Am Hubland, 97074 Wurzburg, Germany.

++ Present address: Kansai Research Establishment, JAERI, 8-1 Umemidai, Kizu,
  Kyoto, 619-0215, Japan.

Abstract

Table-top laser wakefield accelerators (LWFAs), proposed theoretically in 1979, have now generated individual electron bunches in the laboratory with energies of tens of MeV, charge per laser pulse of
> 1 nC and transverse emittance of < 0.1π mm-mrad. The attained electron beam suggests the possibility of medical radiation applications either directly or via conversion to x-rays. However, clinical electron beam applications require selection of a narrow energy bin, ΔE < 7 MeV, for depth control, and a beam expansion to as much as 25cm × 25cm for various tumor radiation treatments. As a result present LFWA sources provide a dose rate which is still at least an order of magnitude lower than the dose rate required for clinical application. We use particle-in-cell (PIC) simulations to evaluate the feasibility of developing an improved LWFA-based medical accelerator. The current LWFA sources require such high peak intensity (> 10^{18} W/cm^2, or squared normalized vector potential a_0^2 > 1) that laser repetition rate is presently restricted to ≤ 10 Hz. A scheme to lower the threshold and increase the repetition rate of efficient LWFA thus appears essential. We show that it should be possible to obtain a medically-promising dose rate using a two-pulse Raman-seeding technique that lowers the threshold for efficient LWFA to a_0^2 ≈ 0.09, thus enabling repetition rate of ~ 100 Hz or more with existing laser technology. As an example, a primary laser pulse with a_0^2 = 0.1225 and pulse duration (FWHM) 100 fs together with a weak seed pulse (0.01a_0^2) shifted downward in frequency by the plasma frequency ω_p, is shown to produce up to ~ 40 MeV electrons with acceptable momentum spectrum. With the appropriate increase in the repetition rate, it has the potential for
medical radiation application.

1 Introduction

The advent of chirped pulse amplification (CPA)\textsuperscript{1} laser technology and of new broadband solid-state laser materials during the early 1990s has enabled laser systems of table-top scale to produce pulses of unprecedented peak power (> 10\textsuperscript{12} W, or 1 terawatt (TW)), with durations \textlesss than \textasciitilde 100fs, which have come to be known as “table-top-terawatt” (“T\textsuperscript{3}”) lasers.\textsuperscript{2} One of the major applications of such lasers has been the experimental realization of laser wakefield acceleration (LWFA),\textsuperscript{3} a technique for compact, high-gradient charged particle acceleration proposed theoretically by Tajima and Dawson in 1979.\textsuperscript{4} In LWFA, a single short (50 < \textless \textless 500 fs), intense laser pulse (typical wavelength \textlambda = 0.8 or 1\textmu m) propagates into a plasma of electron density \textit{n}_\text{e} that ranges typically from \textasciitilde 10\textsuperscript{17} cm\textsuperscript{-3} to 10\textsuperscript{19} cm\textsuperscript{-3}. Consider a pulse with a sufficiently high focused peak intensity, \textit{I} \textasciitilde 2 \times 10\textsuperscript{18} W/cm\textsuperscript{2}. When expressed in a normalized laser strength parameter,\textsuperscript{3} this intensity corresponds to \textit{a}_\text{0} \textasciitilde 1. In SI units, a convention will be followed hereon, \textit{a}_\text{0} \equiv \textit{eE}_\text{y}/(\textit{m}\omega c) = \textit{e}/(\textit{m}\omega c)[2\textit{I}/(\epsilon_0\textit{c})]^{1/2}, where the incident light is polarized in the y-direction. This pulse expels a significant fraction \textit{\delta n}_\text{e}/n_\text{e} of plasma electrons from within its envelope by the pon-
deromotive force along the longitudinal \((x)\) direction,\(^5\) and leaves a “wake” of electron density oscillations of amplitude \(\delta n_e/n_e\) and frequency \(\omega_p\). The picture here is analogous to the wake behind a boat, that propagates at the group velocity \(v_g \sim c\) of the driving pulse. The internal space-charge fields of this wake, which reach values \(10^7 < E_x < 10^9 V/cm\) that are \(10^2\) to \(10^4\) times larger than the breakdown limit of conventional RF accelerators, provide a high-gradient accelerating structure for charged particles that are injected with energy above its capture threshold.

At lower plasma densities \((n_e \sim 10^{17} cm^{-3})\), the natural plasma frequency \(\omega_p = \sqrt{n_e e^2/(m_e \epsilon_0)} \sim 10^{13} s^{-1}\) is close to the reciprocal pulse duration \(\tau^{-1}\) available from \(T^3\) lasers, creating a resonant excitation condition that enables somewhat less intense \((a_0^2 < 1)\) laser pulses to generate high amplitude wakes. This regime was therefore a natural focus of early LWFA experiments.\(^6,7\) Charged particle acceleration in such “standard” wakefields, however, has required external injection of \(\sim 1\) MeV electrons e.g. from a RF linac.\(^8\) Moreover, acceleration has been limited to < 5 MeV,\(^8\) with low yields because the interaction length over which the accelerating field \(E_x \sim e\lambda_p \delta n_e/\epsilon_0 \sim 10^9 V/cm\) acts is limited to the Rayleigh length \(z_R \sim 10^{-2} cm\) of the laser focus in the absence of a guiding mechanism.

At higher plasma densities \(n_e \sim 10^{19} cm^{-3}\), \(\omega_p >> \tau^{-1}\), creating a far
off-resonant excitation condition that necessitates higher intensity ($a_0^2 > 1$) to modulate the pulse to form a substantial wake. In this regime, usually called the “self-modulated” (SM) LWFA, forward Raman, self-modulation and relativistic self-focusing instabilities dominate laser pulse propagation and wakefield formation. However, once the wakefield forms, its accelerating field $E_x \sim 10^{10} V/cm$ is strong enough that wave breaking can occur, also the field can capture and accelerate electrons from the hot thermal tail of the background plasma, thus circumventing the need for external injection. Moreover, self-focusing extends the interaction length to $\sim 10^{-1} \text{ cm}$, and helps collimate the electron beam. As a result, several groups have observed a high yield ($> 1nC/pulse$) of electrons up to several tens of MeV in a highly collimated beam (transverse emittance $\epsilon_\perp \sim 0.1\pi \text{ mm-mrad}$) using only an intense multi-TW laser pulse focused into a gas jet. While most SM-LWFA demonstrations have been essentially single-shot, repetition rates as high as 10 Hz have been achieved.

Because of its efficient multi-MeV electron yield and simple set-up, the SM-LWFA has become the most promising candidate for near-term LWFA applications. In particular, medical applications have been envisioned, but to our knowledge no concrete analysis has been carried out. The purpose of this paper is to evaluate the feasibility of designing a medical accelerator
for radiation treatment based on existing SM-LWFA performance, and to explore ways of improving this performance for medical applications.

Typically SM-LWFA generates a broad electron energy spectrum ($E \sim 1-40$ MeV) that is in contrast with the beam from a rf-linac source, which is essentially mono-energetic. We make use of following the clinical application guidelines.\textsuperscript{16} The peak energies of clinically useful electron beams range from about 5 to 25 MeV. A beam cross section of $25 \times 25cm^2$ is needed to match current electron radiotherapy units and to allow the gamut of electron beam treatments. A key question is whether the SM-LWFA can provide medically-relevant dose rates ($>4Gy/min$) once energy windowing, beam shaping and transport to the patient are taken into account. Based on this criterion the dose-rate in the $5-25$ MeV range is still at least one order of magnitude below the required dose rate for clinical application for current 10 Hz repetition rate SM-LWFA sources. On the other hand, a two-pulse "Raman-seeded" (RS) LWFA scheme\textsuperscript{17-19} appears promising for lowering the threshold energy, and thus raising the achievable repetition rate of SM-LWFA. Realization of a threshold-lowering scheme such as RS-LWFA may therefore be a key step in realizing medical LWFA applications. In this work we will confine our attention up to the stage of the production of electrons and defer the handling of the output electrons from the accelerator
to the patient to the companion paper\textsuperscript{16} and also an earlier work\textsuperscript{20}.

The remainder of this paper is organized into three sections. Sec. 2 describes design criteria and dimensions of a medical LWFA. Based on a recent SM-LWFA data\textsuperscript{13} we observe that the use of RS-LWFA is an important ingredient in designing the LWFA medical accelerator. Sec. 3 analyzes various features of the accelerator using PIC simulations. Sec. 4 is devoted to discussions, which includes a way to measure Raman seeding effect and some 2D PIC simulation results.

2 Design criteria for a medical laser wakefield accelerator

A schematic diagram of a medical LWFA together with the two laser pulse compressor units is illustrated in Fig. 1. Some of the ingredients of the accelerator have been considered earlier\textsuperscript{15,21,22}. The main pulse from the lower compressor is joined by the Raman seed pulse generated using a known experimental technique\textsuperscript{23} from the upper compressor. The combined pulse is transported by reflecting mirrors and enters into a vacuum chamber through a window made of $MgF_2$ or $LiF$. The pulse is then reflected and focused to the interaction region at the central gas cell. After the interaction region,
energetic electrons emerge from the chamber through the downstream aperture. The remainder of the laser pulse is redirected by a concave mirror to the dump. There is a gas flow through the gas cell. The basic requirements for a laser medical accelerator are: (1) it must be sufficiently compact to provide mobility in a clinical setting, as well as significant savings in cost and space over conventional rf technology; (2) it must provide the energy and dose rate of electrons adequate for radiation therapy. We consider each of the two requirements in turn.

2.1 Layout and dimensions.

Compactness is the chief advantage of high-gradient accelerators over conventional technology. The SM-LWFA accelerates electrons to several tens of MeV in only $\sim 1$ mm path length, as opposed to several meters in conventional rf linacs. However, for a complete evaluation of space requirements, the laser delivery and vacuum systems surrounding the interaction region must also be taken into account.

2.1.1 Vacuum region and gas-cell region

In order to reach intensities sufficient for LWFA, laser pulses from a CPA system must be focused to a spot-size in the range $5 < r_0 < 25 \mu$ into
an atmospheric density gas. A reflective focusing optics, such as an off-axis-parabolic (OAP) mirror, is preferred to prevent distortion of the intense pulse by dispersion, self-focusing or self-phase modulation on passing through a refractive lens. Ionization-induced refraction would prevent the laser pulse from focusing effectively if such a dense gas filled the entire focusing region. Consequently, the OAP must be housed inside a vacuum chamber that maintains background pressure < 1 Torr. The high density gas is restricted to a region ~ 1mm long at the laser focus using either a supersonic gas nozzle or a flowing gas cell with sub-mm entrance and exit orifices for the laser pulse. The latter method allows a greater range and spatial uniformity of gas density. With either method, an efficient (600cfm ~ 280 liters/sec) mechanical pump is required to maintain the pressure difference between the gas-cell or jet and the vacuum region.

2.1.2 Comparision between SM-scheme vs RS-scheme

The SM-LWFA, because it requires such an intense pulse, could not use the window on the vacuum chamber, nor any transit in air between last compressor grating and interaction region, because of nonlinear optical ("B-integral") distortion in the window and air. Consequently the compressor must be attached to the accelerator vacuum chamber, making the clinical
part of the apparatus much bulkier. RS-LWFA, on the other hand, can use substantially weaker pulses, which can go through air and high-band gap window. Consequently the whole laser system can be separated from the smaller accelerator arm, making the clinical part of the apparatus smaller and more mobile. This is a significant secondary argument for threshold-lowering schemes such as RS-LWFA.

2.1.3 Dimension of the accelerator chamber

Since the compactness of the accelerator is an important feature in the present work, we proceed to estimate the chamber length. To maintain the coherence of laser pulse, the angle subtended by the mirror at the interaction region must be less than the diffraction angle, i.e. \( D/f \leq \theta_{diffraction} = \frac{\lambda}{2r_0} \), where \( D \) and \( f \) are the mirror diameter and its focal length, \( \lambda \) and \( w \) are the wavelength and the spot-size of the laser pulse. This imposes a lower limit on the focal length i.e.

\[
f \geq f_{\text{min}} \sim \frac{2r_0}{\lambda} D.
\]

We proceed to estimate \( D \). Assume the damage energy threshold of the mirror to be (with a conservative estimate)

\[
\frac{\Delta U_{\text{damage}}}{\Delta A} = 1 \text{ J/cm}^2.
\]
From Table I (see Sec. 3.4), the values of relevant parameter values for present estimate are: $\lambda = 1 \mu$, $r_0 \sim 18 \mu$ (or the spot-size $w \sim 25 \mu$) and $U_{\text{pulse}} = 170 \text{mJ}$. Setting the pulse energy to be below the damage threshold, one arrives at

$$U_{\text{pulse}} \leq \Delta U_{\text{damage}} = (1 \text{J/cm}^2) \cdot \frac{\pi D^2}{4}. \quad (3)$$

This gives the minimum value $D_{\text{min}} \approx 0.46 \text{cm}$. And from eq.(1), it leads to the maximum value of $f \approx 16.6 \text{cm}$. The $f$-number is $f/D \approx 35$. After the interaction region, we make allowance for a comparable length interval as that before the interaction, this leads to a chamber length of the order of $2f \sim 30 \text{ cm}$.

2.1.4 From the output of the accelerator to the patient

The issue of converting the electron beam outputted by the LWFA into a clinically useful beam incident on the patient remains. This requires beam energy selection, broadening, collimation, and monitoring. The LWFA is a more difficult issue than that of a conventional linac because the former beam is polyenergetic and has a slightly greater angular divergence. In linear accelerators, the beam is typically redirected and energy selected using a 270° achromatic magnet. The beam is broadened using a dual scattering foil system, or occasionally a magnetic scanning system. Raischel et al. 20
have investigated the use of an achromatic magnet system for transporting and broadening a LWFA beam, although the study was restricted to 5-cm diameter beams, too small for standard therapy application but adequate for dosimetric study. The treatment of the broadening and flattening of the beam leading to an acceptable beam at the patient for standard therapy application will be discussed in the companion paper.\textsuperscript{16}

In connection with narrower beams we would like to make a comment here. In principle a narrow beam could be used directly for rastering of the beam as well as for spotting narrow regions of tumor if detectable.

2.2 SM-LWFA data versus medical application requirement

Current radiotherapy machines primarily utilize linear accelerators to produce electron and x-ray beams for radiation therapy. SM-LWFA produces electron beams with a wide energy range, which differ from nearly mono-energetic beams produced by linear accelerators. Therefore, it is necessary to determine the size of the energy window that is acceptable from SM-LWFA for both electron and x-ray beams. A narrow energy width can be expected to produce electron and x-ray beams whose resulting dose distributions are comparable to those of beams produced using linear accelerators. Increasing the energy width increases the dose rate, but at the expense of the quality

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of the depth distribution in the dose rate. A study of the full dosimetric
impact of energy width requires further investigation; however, it is possible
to make a preliminary assessment of the dose rate from SM-LWFA beams
using existing measured data.

In order for a SM-LWFA to be practical in the clinic, it should be able
to produce beams with comparable dose distributions and dose rates. The
Radiation Therapy Committee Task Group report of Purdy et al\textsuperscript{28} has indi-
cated that to produce clinical beams with a dose rate of 4Gy/min, electron
beam currents of 100 nA and 100 µA are required for 6-MeV electron and 6-
MV x-ray beams respectively. Also, electron beam currents of 20 nA and 20
µA are required for 25-MeV electron and 25-MV x-ray beams respectively.

Let us take the recent data of Leemans et al\textsuperscript{13} as an example which
has following experimental numbers. For the laser pulse, the wavelength is
\( \lambda = 0.8 \mu \), the spot-size \( w = 6 \mu \), and the pulse-length \( \tau = 50 \) \( fs \). The pulse
repetition-rate \( f = 10Hz \), with a peak power \( P = 8.3 \) TW. This gives a
peak intensity \( I = 1.5 \times 10^{19} W/cm^2 \), and \( a_0 = 2.63 \). The experimental elec-
tron momentum distribution has the form: \( dN_e/d\rho = f(\rho) = Aexp(-\rho/\rho_0) \)
where \( A = 4.8 \times 10^9 \) electrons/(MeV/c) per pulse, \( \rho \) is given in units of
MeV/c, and \( \rho_0 = 3.3MeV/c \). Denote the number of electrons produced in
the momentum interval \((p_1, p_2)\) by \( F(p_1, p_2) = \int_{p_1}^{p_2} f(\rho) d\rho \). Then the total

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number of electrons produced per pulse \( N_0 = F(0, \infty) = A \rho_0 = 1.6 \times 10^{10} \), giving a total charge produced 2.5 nC per pulse.

Kainz et al\textsuperscript{16} have studied the effect of energy width on the dose distribution of electron beams. They have determined that the most significant effect of a broad energy window is that the falloff of the dose distribution in depth is compromised. A measure of dose falloff is the distance between the depth where dose is 90\% of the maximum dose and the depth where dose is 10\% of the maximum dose, i.e. \( R_{90-10} = R_{90} - R_{10} \). This distance is important because it estimates the minimum dose between the deepest penetration of the tumor and the minimum depth of a normal structure for which the tumor receives full dose and the normal structure minimal dose. At 9 MeV, they assume the maximum tolerable energy-bin width is 4.5 MeV. We apply this criterion to Leemans et al’s data. At 9 MeV, we determined that 8.2\% of the total current of electrons greater than 1 MeV falls in the 4.5-MeV window centered at 9 MeV. Therefore, the data indicate a useful beam current of 0.21 nC per pulse, which for the 10 Hz pulse repetition-rate results in 2.1 nA of useful beam.

The dose rate defined as the product of the “fluence” and the “mass stopping power for water”, is given by

\[
\dot{D} = \left( \frac{\eta I}{cA} \right) \cdot \frac{1}{\rho} \frac{dE}{dx}. \tag{4}
\]
For a water phantom, \((1/\rho)dE/dx \sim 2\text{MeV cm}^2/\text{g}\), and the cross section \(A = \pi r^2\) is the standard fiducial cross section of \(25\text{cm} \times 25\text{cm}\), \(r = (25/\sqrt{2})\) cm. \(\eta\) is the efficiency of beam flattening, i.e. the ratio of the electrons within the clinically useful area of the beam to those in the total area of the beam. Incorporating these values yields

\[
\mathcal{D}(\text{Gy/min}) = 0.122 \times \eta I(nA).
\]  

(5)

The current of the selected electrons produced is \(I = e f \Delta N\), with \(\Delta N\) being the number of electrons produced within the selected energy bin and \(f\) the pulse frequency.

For Loemans et al data, with the quoted current of 2.1nA, using eq.(5), with \(\eta = 0.4\), the corresponding dose rate is 0.1 Gy/min, which is a factor 40 smaller than the minimum required dose rate for clinical application. Our estimate of the factor 40 is also in agreement with that obtained using the beam current criterion reported by Purdy et al.\(^{28}\) More specifically by interpolating the Purdy et al data, at 9MeV, a rate of 4 Gy/min requires a beam current of approximately 87 nA, which is the same factor of 40 greater than the quoted value of 2.1nA.

Although as a precursor to clinical application, at early stage of the accelerator development, one could run tests simulating patient beams using a beam cross section \(5\text{cm} \times 5\text{cm}\), which will gain a factor of 25 in dose rate,
i.e. 2.5 Gy/min at 9 MeV. Still, it is important to explore means to increase the dose rate by many folds in working toward a clinically viable dose rate.

We now turn to the main theme of the present work, i.e. to increase the dose rate by increasing the repetition-rate. As alluded in the Introduction section, Raman-seeding may generate LWFA electrons at a relatively low pulse energy, which will enable the increase of the repetition-rate, and in turn the increase of the dose rate.

3 Physical Process and Optimization

The use of the particles-in-cell (PIC) method to emulate plasma systems and laser-plasma interactions is well known. In this approach, a plasma system is comprised of macro-particle of finite size, where each macro-charged-particle represents a collection of a large number of electrons. Their charges account for long range coulomb force which gives rise to the collective behavior of a plasma, a feature one intends to simulate. Their finite size is an essential ingredient to suppress unphysical high collision rates, which otherwise would occur should they be point-like. Finiteness of particle size implies that the charge density function cannot be resolved finer than the size of a particle, which leads to a coarse-grain description of the space, where the space is divided into cells with the cell-size being comparable
to the particle-size. Computer algorithms have been developed to describe laser-plasma interactions within some small simulation-box, based on the Maxwell equations together with the Lorentz forces.

Depending on setup of the processes involved, approximate symmetry of problems of practical interest may often lead to the “reduction of number of spatial-dimensions”, such as the 1D case and the 2D case. For the 1D case, since $x$– is the only spatial coordinate dependence in the problem, while each vector quantity such as: velocity and fields, has 3-components, a PIC particle in 1D behaves like a charged sheet extending in the $y$– and $z$–dimension. For the 2D case, the PIC particles behave like rods. The bulk of the present work is done in 1D. Some 2D results will also be presented in Sec. 4.

3.1 Use of 1D PIC simulation

It is well known that 2D physical effects such as relativistic self-focusing and self-modulation instability play a central role in SM-LWFA and that 2D simulations are therefore needed for a complete description. However, for sufficiently large spot-sizes, $k_p w >> 1$ for 1D simulation is satisfied at $t = 0$. At $t > 0$, 1D effects will continue to dominate at early times; 2D effects become important later on. We recall Mori’s criteria: The transition from
1D effect dominated by Raman forward scattering, to 2D effect dominated by self-modulation, occurs approximately at:

$$\tau_{\text{transition}} = \frac{4L_R}{c\gamma_p}.$$  

(6)

For example, with Rayleigh length $L_R \approx 1\text{mm}$ and $\gamma_p = 5.8$, $\tau_{\text{transition}} = 760\omega_p^{-1}$. Except in Sec 4, we will confine our attention to 1D PIC simulations of wakefield acceleration up to $\tau_{\text{transition}}$. Our study of 2D fluid model confirms the 1D dominance for spot-size $\sim 20\mu$ and $t < 500\omega_p^{-1}$. With this spot-size the wakefield generated by the 2D-fluid model is essentially the same as that generated by 1D-PIC code. There is an additional reason why 1D gives approximate estimate of the number of energetic electrons produced. The Raman seed accelerates the initial LWF growth so greatly that most of the relevant pick up and acceleration phase has taken place before the 2D effect becomes important. We will defer 2D PIC simulation results to Sec. 4.

For the simulation we follow the standard 1D PIC algorithm, using a total number of cells: $2^{11}$. Number of PIC particles per cell is about 10. The main and seed pulses propagate in the x-direction, and are linearly polarized in the y-direction, with the corresponding fields given by

$$E_y = E_0 f(x - x_c, \sigma_x, k), \text{ where } f(\Delta x, \sigma, k) = \exp \left[ \frac{-\Delta x^2}{2\sigma^2} \right] \sin kx,$$  

(7)
\[ E_y' = E_0' f(x - x'_c, \sigma'_x, k'), \] with \( \omega' = \omega - \omega_p \) and \( k' \approx k - k_p \). \hfill (8)

Here \( x_c, \sigma_x \) and \( k = 2\pi/\lambda \) are respectively the location of the center, the width and the wave number of the pulse. \( k_p \) and \( \omega_p \) satisfy the dispersion relation: \( \omega^2 = \omega_p^2 + (ck_p)^2 \). The primed quantities are those for the seed pulse.

3.2 Effect of Raman seeding in the growth of wakefield

The Raman scattering at or close to the forward direction has been identified as mainly due to the 4-waves instability which is the result of recursive interaction among the incident waves (with frequency \( \omega_0 \)), the plasma waves (with frequency \( \omega_p \)), Stokes waves (with \( \omega_- = \omega_0 - \omega_p \)) and the anti-Stokes waves (with \( \omega_+ = \omega_0 + \omega_p \)). There is also the 3-waves instability due to recursive interaction among those three waves excluding the anti-Stokes waves. This instability becomes more and more pronounce as the scattering angle increases. We refer the reader to an article by Decker et al\textsuperscript{10} for a comprehensive discussion on the temporal evolution of various types of Raman instabilities. To illustrate how Raman instability growth shows up in our present PIC simulation, we make a somewhat oversimplified ansatz, assuming that at early time, the instability growth is entirely due to the 4 waves-instability.
Denote the seedling-strength parameter by: $b = \left( \frac{a_{\text{seed}}}{a_0} \right)^2$. Including the seed contribution, the original expression for an asymptotic growth of the wakefield amplitude due to 4-waves-instability at the tail of the pulse, of Decker et al, becomes,

$$
\delta E_x \propto a_{\text{noise}} \frac{\exp(z)}{\sqrt{2\pi z}}, \quad \text{where } z = G \sqrt{(t - t_1)t_1 \omega_p^2}, \quad \text{with } G = \frac{a_0(1 + \sqrt{b})}{\sqrt{2\gamma_p}}.
$$

Here $t_1 = \text{pulselength}/c$. Fig. 2 is a 1D PIC simulation output for the wavelength of the main pulse $\lambda = 0.8\mu$, $a_0 = 0.5$ and $\gamma_p = 6$. $a_{\text{noise}}$ is the noise amplitude. When the seed contribution dominates, the noise amplitude becomes $a_{\text{seed}}$.

Fig.2a shows the wakefield amplitude as a function of the normalized strength of the seed $a_{\text{seed}}/a_0 \equiv \sqrt{b}$, at $t=100$. Throbbbbughout this work the units of the time variable, unless otherwise specified is always given in units of $\omega_p^{-1}$. Notice that even for the $\sqrt{b} = 0$ case, due to the finite pulse length, there is some noise-contribution to the wake field.10 So for the present laser-plasma system, the $\sqrt{b} = 0$ case defines the background noise level. It is only when the strength of the seed is sufficiently large, would the seed contribution begin to dominate. In this region, $a_{\text{noise}}$ is to be replaced by $a_{\text{seed}}$. Here the wakefield is expected to rise linearly (see eq(9)), until other effects take over. Fig 2a suggests that at $t=100$, this linear region begins at around $\sqrt{b} \sim 0.04$ extending to at least 0.1. Fig 2b shows how
the maximum wake field varies as a function of time. The points connected by the thick-lines indicate the time behavior of the maximum wakefield for the seed strength parameters: $\sqrt{b} = 0$, 0.01, 0.05, and 0.1. Notice that as the strength of the seed increases, the corresponding maximum wake-field curve occurs earlier and earlier in time. The shape of the growth curves as predicted by eq.(9) for the cases $\sqrt{b} = 0$, 0.05 and 0.1 are shown by the light-curves, for comparison. The normalizations of the $\sqrt{b} = 0$ and $\sqrt{b} = 0.05$ cases are arbitrary. Based on the normalization of the $\sqrt{b} = 0.05$ case, the curve of the $\sqrt{b} = 0.1$ case is predicted by eq.(9). In the small $t(< 100)$ region, the prediction agrees with the PIC results reasonably well.

Since $b << 1$, from eq.(9) one expects that the G-factors for all cases illustrated are very similar. Despite this similarity, one sees that the parameter $b$ controls the threshold behavior, which in turn controls the overall growth pattern. Apparently the exponential rise predicted by eq.(9) is applicable only over a relatively small time interval. After some rise, evidently due to other competing effects, the growth stops.

3.3 Simulation Results

For the simulation results below, the following parameters are used. The main pulse has $\lambda = 1\mu$ pulse length $\sim 2\sigma_x = 100fs$, the seed pulse $\sigma'_x = \sigma_x$
and \( x'_e = x_e + 0.5 \sigma_x \). The plasma has density \( n_e = 3.3 \times 10^{19} \text{cm}^{-3} \), which gives \( \lambda_p = 5.8 \mu \), and \( \gamma_p = \lambda_p/\lambda = \omega/\omega_p = 5.8 \mu \).

The unseeded and the seeded cases given below are for the case where the main pulse amplitude \( a_0 = 0.35 \) and seed amplitude \( a_0' = 0 \) or \( a_0' = 0.1 a_0 \). The evolutions of unseeded and seeded pulse are shown in Fig. 3. Notice how the seeded pulse is being "modulate" with relatively even spacings, and the lack of the comparable pattern for the unseeded case. The evolutions of the two pulses in the Fourier space are shown in Fig. 4. For the seeded pulse, notice there are the anti-Stokes and Stokes side bands at \( t=100 \). As \( t \) increases there is also the appearance of higher order harmonics. This is to be compared with the corresponding much subdued sidebands for the unseeded pulse. The modulated pulse in turn causes the growth of the wakefield, the Raman instability growth. This is displayed in Fig. 5. The time evolution of the phase space (\( p_x \ vs \ x \)) distribution of the electrons is shown in Fig. 6. Again notice the contrast in the phase space development between the seeded case and that of the unseeded case. The seeded case leads to the eventual production of energetic electrons, and for the unseeded case, the lack of their production.

In Fig. 7, we show the time evolution of (a) the normalized wakefield potential \( \phi = eE_x/(m\omega_p c) \) and (b) the maximum electron momentum \( p^\text{max}_e \).
for $a_0 = 0.35$ and unseeded and seeded cases. For the unseeded case, $\phi$ never
exceeds 0.1, whereas for the seeded case $\phi$ reaches a maximum of 0.6 at
t = 300\omega_p^{-1} < \tau_{\text{transition}}$, sufficient to trap electrons.\(^{31}\) For the seeded case,
with $\phi = 0.6$ at $t \approx 300$, the maximum kinetic energy can be determined
to be\(^3\) $KE_{\text{max}} = 4\gamma_p^2\phi_0 mc^2 = 4 \times (5.8)^2 \times 0.6 \times 0.5 \approx 40$ MeV. The peak
in $KE_{\text{max}}$ is expected to be at one dephasing time later than the peak in
$\phi$, with $t_{\text{dephase}} = 2\pi\gamma_p^2/\omega_p \approx 221$. This is consistent with the plot with
max KE (notice in the relativistic domain $p_x/cm \approx KE/mc^2$) reaches its
maximum at $t \sim 500$ as seen in the Fig7b.

Figure 8 shows how (a) $\phi_{\text{max}}$ (b) $KE_{\text{max}}$ and (c) the percent electron
production scale with $a_0$ for seeded SM-LWFA, while maintaining seed am-
plitude $a_0 = 0.1a_0$. The threshold for electron trapping and acceleration to
MeV energies is near $a_0 \sim 0.28$. $KE_{\text{max}}$ rises steadily up to $a_0 \approx 0.35$, then
saturates, despite continuing increase of $\phi$, indicating deviation from the
expectation from a linear theory. To keep the $a_0$ value as low as possible,
but at the same time to produce adequate hot electrons ($KE > 1\text{MeV}$) with
the appropriate energy spectrum for medical application, we found that the
optimal case is to have $a_0$ take on a near threshold value, e.g. $a_0 \sim 0.35$,
where the corresponding production of hot electrons turned out to be about
0.15\%. This near threshold production level is indicated by a short-dashed
line in (c), which is a the linear plot, and also in (d), which is a semi-log plot.

3.4 Energy Spectrum

To go from the Lorentz-factor spectrum $dN/d\gamma$ of the PIC particles to the energy spectrum of electrons $dN_e/dKE$, we make use of the following expression

$$\frac{dN_e}{dKE} = \frac{f}{m_e c^2} \left[ \frac{dN}{d\gamma} \right] , \text{ where } f = \frac{N_e}{N_T} = \frac{n_e AL}{N_T} . \tag{10}$$

Here $N_e$ and $N_T$ are respectively the total number of electrons and the total number of PIC particles within the co-moving window. The symbol $n_e$ is the electron density. As mentioned earlier it is taken to be $n_e = 3.3 \times 10^{19}/\text{cc}$. The co-moving frame window width is taken to be $L = 184\mu$.

In the 1D case, the laser spot-area does not explicitly enter in a simulation. Here one assumes that the laser spot-area is sufficiently large so that the 1D approximation is valid. From 2D fluid model simulation, we have verified that both the pulse modulation and the wakefield enhancement for the 1D case are similar to those of the 2D case at least down to $r_0 = 18\mu$. This led us to take the spot-area to be $A = \pi r_0^2 = 1000\mu$. Notice that $r_0$ here is taken to be a disc-radius. If one follows the convention of defining the spot-size $w$, by a Gaussian radial dependence of the field, i.e. $a = a_0exp[-(r/w)^2]$, the corresponding effective area will be given by
\[ A = \pi w^2/2. \] The present disc-like-radius is related to the spot-size by:

\[ w = \sqrt{2}r_0 = 25\mu. \]

The final energy spectrum obtained for \( a_0 = 0.35 \) and \( t = 600 \) is shown in Fig. 9. Notice that there is a steep drop in the energy distribution in the small \( KE \) region, up to \( 1 - 2 \text{MeV} \). The variation of the spectrum beyond several MeVs becomes gradual. There is about 0.13 \% of hot electrons \( (KE > 1 \text{MeV}) \) produced among the total plasma electrons within the simulation window.

The number of hot electrons per pulse and the corresponding charges are given by

\[
N^e_{\text{out}} = \frac{N_{\text{out}}}{N_T} n_e AL, \quad \text{and} \quad Q^e_{\text{out}} = e N^e_{\text{out}}. \tag{11}
\]

For the present case, \( N^e_{\text{out}} \approx 6.07 \times 10^{12} \frac{N_{\text{out}}}{N_T} = 0.79 \times 10^{10}, \ Q^e_{\text{out}} = 1.3\mu\text{C}. \) Here \( n_e = 3.3 \times 10^{19}/\text{cm}^3, \ A = 1000\mu\text{m}^2, \ L = 184\mu, \ N_{\text{out}}/N_T = 0.13\% \) and \( e = 1.6 \times 10^{-19}\text{C} \) were used. Our optimal parameter set together with some simulation results are given in Table I.

### 3.5 Comparison between the beat-wave and the Raman-seeding schemes

The Raman-seeded SM-LWFA superficially resembles the plasma beat-wave accelerator (PBWA).\(^3\) However, the two approaches differ markedly in the
ratio \( b = I_{\text{seed}}/I_{\text{main}} \), which is 0.01 for the simulations above, and \( \sim 1 \) for the PBWA. This, in turn, leads to two important distinctions between the two methods. First, the smaller ratio \( b \sim 0.01 \) is much easier to realize experimentally, since only a small fraction of the main pulse need be converted to the Stokes wavelength. Secondly, the growth of the Raman-seeded wakefield is much less sensitive than PBWA to detuning \( \Delta \omega = \omega_{\text{seed}} - (\omega - \omega_p) \) of the seed pulse frequency from its optimum value \( \omega_{\text{seed}} = (\omega - \omega_p) \), as illustrated in Fig. 10. This plot compares \( KE_{\text{max}} \) vs \( \Delta \omega/\omega_p \) for the Raman-seeded LWFA and PBWA. For this comparison, we assumed a common primary pulse with intensity \( I_0 \) was split into a \textit{main} pulse with \( I_{\text{main}} = (1 - \alpha)I_0 \) and a \textit{seed} pulse with \( I_{\text{seed}} = \alpha \eta I_0 \), where \( \eta \) is the conversion efficiency into the Stokes frequency. Thus \( b = \eta \alpha/(1 - \alpha)I_0 \) and \( a_0 = 0.356 \) for the Raman-seeded LWFA example (using \( \alpha = 0.032, \eta = 0.3 \)), which yields the electron spectrum shown in Fig. 9 and 1.3 nC/pulse as discussed above, while \( b = 1 \) and \( a_{0,\text{main}} = a_{0,\text{seed}} = 0.171 \) for the PBWA example (using \( \alpha = 0.769, \eta = 0.3 \)). As shown in Fig. 10, \( KE_{\text{max}} \) is consistently higher for the seeded case than for the PBWA. More importantly, the seeded LWFA is more stable against deviation from the Stokes frequency \( \Delta \omega \). Evidently, the detuned seed pulse self-corrects more quickly as self-modulation evolves when \( b \) is small. Thus the Raman-seeded LWFA is much more compatible
than PBWA with plasmas of nonuniform density, such as those present in a gas jet. As $b$ decreases from unity, the maximum KE plot evolves continuously from the narrow plot shown for the PBWA to the broader seeded LWFA curve. Of course, $b$ cannot decrease indefinitely below 0.01. Eventually the production rate of energetic electrons would fall to the level of the unseeded SM-LWFA.

4 Discussion

Raman seeding effect and the advantage factor: We have seen that Raman seeding leads to speeding up of the self-modulation of a pulse, which causes the enhancement of wake field generation, and in turn the enhancement of electron production. We recall that to keep the $a_0$ value as low as possible, at the same time to produce adequate hot electrons for medical application, in Sec. 3.3 we chose to work with $a_0$ value near the threshold of electron production. As an illustration, for the seeded case, we showed the simulation result with $a_0 \sim 0.35$ and the near threshold hot-electron production level of $\sim 0.15\%$.

Raman seeding effect in general varies with experimental parameters. To quantify this effect for a given set of experimental parameters, we find it
is relevant to look at the time taken for a pulse to reach the level of near-threshold hot electron production, such as \( \sim 0.15\% \). We will refer this time as the critical time, \( T_c \) for the experiment in question.

Take Leemans et al's experiment parameters as an example. Here the laser wavelength is \( \lambda \sim 0.8\mu \), the pulse length is \( \sim 50fs \), \( a_0 = 2.63 \) and the plasma density is \( 2.5 \times 10^{19}/cm^3 \). Based on 1D PIC simulation, we found that, for the present case where there is no seeding involved, \( T_c = 200 \). By varying \( a_0 \) value leaving other experimental parameters fixed, one may proceed to determine the \( T_c \)'s for various \( a_0 \)'s.

Fig 11a is the \( T_c \) vs \( a_0 \) plot for the unseeded case for the parameters quoted except leaving \( a_0 \) as a variable. Notice there is a general trend that as \( a_0 \) decreases, \( T_c \) increases. This trend is expected, since the lesser the laser intensity is, the slower the instability growth is, in turn it takes longer for the pulse to self-modulate to reach the level of the near-threshold production.

Next, we examine the seeded case which is the counterpart of the unseeded case at \( a_0 = 2.63 \). We will keep the same time, i.e. \( t = 200 \). For the seeded case, as \( a_0 \) varies, the hot electron production level varies also. Here using 1D PIC simulation, we have determined that the threshold production level for the seeded case is reached at \( a_0=0.6 \). This led us to introduce an equivalent \( a_0 \) for the seeded case, with which the seeded pulse takes the same
amount of time to reach the threshold production level as the corresponding unseeded case. For the present case, the equivalent pair has $a_0(\text{seeded}) = 0.6$ and $a_0(\text{unseeded}) = 2.63$, with $T_c = 200$. A figure of merit of the Raman seeding effect may be given in terms of the advantage ratio defined by

$$A = \left( \frac{a_0(\text{unseeded})}{a_0(\text{seeded})} \right)^2 \quad (12)$$

For Leemans et al’s setup, $A = (2.63/0.6)^2 \sim 20$. Fig. 11b is a plot of the equivalent pairs, $a_0(\text{seeded})$ versus the corresponding $a_0(\text{unseeded})$, where each pair shares a common $T_c$. A closer inspection on Fig.11b reveals that this advantage factor decreases as the $a_0$ of the unseeded pulse decreases. For instance at $a_0 = 1$, this factor is reduced to $A = (1/0.44)^2 \sim 5$.

**2D PIC simulation** In this work so far the bulk of our analysis has been based on 1D PIC simulation. 1D simulation can capture the main mechanisms involved in wakefield creation and particle acceleration such as raman forward scattering, longitudinal wave breaking, particle trapping, etc. Since it takes little time to run 1D codes we can investigate dependencies on different parameters. Although 1D is a valid and useful tool, in order to make quantitative statements one should make 2D simulations to account for so-called purely 2D phenomena, such as relativistic self-focusing, transverse self-modulation, and transverse wave-breaking etc. Moreover, 2D simula-
tion allows us to make quantitative statements about electron production and transverse characteristics of the beam, which are of practical importance. While quantitative analysis on the 2D PIC simulation data is beyond the scope of the present paper, we would like to report some Raman seeded 2D PIC simulation results here.

Our 2D PIC simulation work is based on the Vorpal code. For the run discussed below, the parameters are as follows. The dimensions of the x-y simulation box is $200\mu \times 50\mu$, and the corresponding gridlines $2000 \times 300$. Number of PIC particles per cell is 5. The laser pulse begins outside of the simulation box. At $t = 0$, it starts to travel toward the box. At the moment the pulse arrives at the center of the window, the pulse-moving frame takes over, and the pulse stays at the center of the window for the rest of the simulation.

2D results: Fig 12 illustrates the results of a typical 2D PIC run, where the main pulse intensity is associated with $a_0 = 0.5$, or the peak intensity $I_0 = 5.3 \times 10^{17} W/cm^2$. The seed intensity is 1% of the main pulse intensity. For the main pulse, the pulse-length is $\tau_{\text{pulse}} = 100fs$, its spot-size, $w = 6\mu$. We recall in terms of the spot-size $w$, the transverse cross section is given by (see Sec. 3.4) $A = 0.5\pi w^2$. And the corresponding pulse energy is $U_{\text{pulse}} = A\tau_{\text{pulse}}I_0 \sim 30mJ$. The density of the medium is such that $\gamma_p = 6$.  

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This gives a Rayleigh length \( L_R = 141 \mu \) or the corresponding “Rayleigh time” \( t_R \omega_p = L_R k_p = 184 \). Since the ratio \( P/P_c = \left( \frac{\omega_0}{2 \gamma_p} \cdot \frac{\vec{k}}{\vec{k}_p} \right)^2 \approx 1.0 \), the relativistic focusing effect is non-negligible here.

In Fig. 12, the top-plot illustrates the modulated pulse at \( t=200 \), which is the duration for the pulse to travel about 1 Rayleigh length, and mid-plot the corresponding wakefield at the same time. The bottom-plot shows the normalized momentum spectrum at \( t=400 \), i.e. after the pulse has traveled about 2 Rayleigh lengths. The production rate of hot electrons (i.e. with \( KE > 1 \text{ MeV} \)) is \( 1.1 \text{nC} \) per pulse. Notice that the energy spectrum extends over the medical application energy range, i.e. 5 to \( \sim 20 \text{ MeV} \). Thus one arrives at following important conclusion:

- RS-LWFA leads to the production of energy spectrum suitable for medical application, where the pulse energy could be as low as 30 mJ. This implies, for instance, that a 1W-laser may generate medical application relevant energy spectrum, with a repetition rate as high as \( 1W/30mJ = 330 \text{Hz} \).

This pulse energy of 30 mJ is only about 1/6 of the pulse energy of 170 mJ estimated from our 1D analysis, see Table 1. Apparently our 1D estimate, while valid is conservative. For instance the 2D results shown in Fig. 12, allows an increase in the repetition rate by another factor of 6 from that
implied by the 1D value.

We have also run the 2D PIC simulation for the unseeded cases with 
\( a_0 = 0.8 \) and \( a_0 = 1 \). The advantage factors deduced from these 2D runs 
are similar to those deduced from Fig. 11b, based on 1D simulation. This 
similarity together with qualitative features presented in Fig. 12 provides a 
posteriori evidence to support the approximate validity of 1D PIC analysis 
of the present work.

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References


Table I. Operation Parameters and Simulation Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Laser power</td>
<td>1.7 TW</td>
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<tr>
<td>Repetition Rate</td>
<td>10 Hz</td>
</tr>
<tr>
<td>Pulse Length ($\sim 2\sigma_s$)</td>
<td>100 fs</td>
</tr>
<tr>
<td>Wavelength</td>
<td>$1\mu$</td>
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<td>beam-diameter at mirror</td>
<td>5mm</td>
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<tr>
<td>focal length of the mirror</td>
<td>17 cm</td>
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<tr>
<td>spot-size: $w$, or disk-radius: $r_0$</td>
<td>$w \sim 18\mu$, or $r_0 \sim 25\mu$</td>
</tr>
<tr>
<td>Pulse intensity</td>
<td>$1.7 \times 10^{17} W/cm^2$, (or $a_0 = 0.35$)</td>
</tr>
<tr>
<td>Rayleigh length ($L_R = \pi r_0^2/\lambda$)</td>
<td>1mm</td>
</tr>
<tr>
<td>Electron density</td>
<td>$3.3 \times 10^{19}/\alpha$</td>
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<tr>
<td>Plasma waves</td>
<td>$\lambda_p = 5.8\mu m$, $\omega_p = (3.1\text{fs})^{-1}$</td>
</tr>
<tr>
<td>Wakefield: seeded/unseeded at $0.6L_R \sim 0.6\text{mm}$</td>
<td>$\approx 6$ fold enhancement</td>
</tr>
<tr>
<td>Number of electrons ($KE &gt; 1$ MeV, at $\sim 0.6\text{mm}$)</td>
<td>$\sim 0.8 \times 10^{10}$, or $\sim 1.3$ nC per pulse</td>
</tr>
<tr>
<td>Maximum electron energy (at $\sim 0.6\text{mm}$)</td>
<td>$\sim 40$ MeV</td>
</tr>
<tr>
<td>Dose rate: 35 MeV with an 8MeV-bandwidth</td>
<td>$D \approx 4.5$ Gy / min</td>
</tr>
</tbody>
</table>