Laser Electron Accelerators for Radiation Medicine: a Feasibility Study

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Abstract

Table-top laser wakefield accelerators (LWFAs), proposed theoretically in 1979, have now generated individual electron bunches in the laboratory with a significant number of electrons having energies up to 10 MeV and beyond with the maximum energy reaches tens of MeV and charge per laser pulse of > 1 nC. The attained electron beam properties have stimulated discussion about possible applications of LWFAs to medical radiation treatment, either directly or via conversion to x-rays. The purpose of this paper is to analyze whether or not such applications are feasible, or can be made feasible with existing laser technology. Clinical electron beam applications require selection of specific electron energies in the range of 5-25 MeV with a narrow energy bin (\(\Delta E < 5\) MeV) for depth control, and a beam expansion to as much as 25 cm \(\times\) 25 cm for various tumor radiation treatments. As a result, we show that present LWFA sources provide a dose rate that falls short of the requirements for clinical application by at least an order of magnitude. We then use particle simulations to evaluate the feasibility of developing an improved LWFA-based medical accelerator. Current LWFA sources require such high peak intensity that laser repetition rate is restricted to \(\leq 10\) Hz. A scheme to lower the threshold and increase the repetition rate of efficient LWFA thus appears essential. We analyze one such scheme. We show that by “seeding” the
primary laser pulse with a second, hundred-fold less intense pulse that is shifted downward in frequency by approximately the plasma frequency $\omega_p$, LWFA leads to a comparable dose rate as that provided by present source, except that primary pulse energy is now more than one order of magnitude lower than that in current LWFA. This enables repetition rate of $\sim 100$ Hz or more with existing laser technology, which is one step closer toward medical radiation applications.

1 Introduction

The advent of chirped pulse amplification (CPA) laser technology and of new broadband solid-state laser materials during the early 1990s has enabled laser systems of table-top scale to produce pulses of unprecedented peak power ($> 10^{12}$ W, or 1 terawatt (TW)), which have come to be known as “table-top-terawatt” (“$T^3$”) lasers.[2] One of the major applications of such lasers has been the experimental realization of laser wakefield acceleration (LWFA),[3] a technique for compact, high-gradient charged particle acceleration proposed theoretically by Tajima and Dawson in 1979.[4] In LWFA, a single short, intense laser pulse propagates into a plasma of electron density $n_e$ that ranges typically from $\sim 10^{17} \text{ cm}^{-3}$ to $10^{19} \text{ cm}^{-3}$. When the focused intensity of the pulse approaches $I \sim 10^{18} \text{ W/cm}^2$, it expels a significant fraction $\delta n_e/n_e$ of plasma electrons from within its envelope by the ponderomotive force,[5] leaving in its “wake” an electron density wave of amplitude $\delta n_e/n_e$ oscillating at the plasma frequency $\omega_p$, and propagating at the group velocity $v_g \sim c$ of the driving pulse, analogous to the wake behind a boat. The internal space-charge fields of this wake reach values $10^8 < E_x < 10^{10} \text{ V/cm}$ that are $10^2$ to $10^4$ larger than the breakdown limit of conventional RF accelerators. They therefore provide a high-gradient accelerating structure for charged particles that are injected with energy above its capture threshold.

At lower plasma densities ($n_e \sim 10^{17} \text{ cm}^{-3}$), the natural plasma frequency $\omega_p = \sqrt{n_e e^2/(m_e e_0)} \sim 10^{13} \text{ s}^{-1}$ is close to the reciprocal pulse duration $\tau^{-1}$ available from $T^3$ lasers ($\tau \sim 10^{-13}$ s), creating a resonant excitation condition that enables somewhat less intense laser pulses to generate high amplitude wakes. This regime was therefore a natural focus of early LWFA experiments.[6, 7] Charged particle acceleration into such “resonant” wakefields, however, has required external injection of $\sim 1 \text{ MeV}$ electrons e.g. from a RF linac.[8] Moreover, acceleration has been limited to $< 3 \text{ MeV},[8]$ with low yields because the interaction length over which the accelerating field $E_x \sim 10^8 \text{ V/cm}$ acts is limited to the Rayleigh length $z_R < 3 \times 10^{-2} \text{ cm}$
of the laser focus in the absence of a guiding mechanism.

At higher plasma densities \(n_e \sim 10^{19} \text{cm}^{-3}\), \(\omega_p \gg \tau^{-1}\), creating a far off-resonant excitation condition that necessitates higher laser intensity to form a substantial wake. In this regime, usually called the “self-modulated” (SM) LWFA,[9] forward Raman,[10] self-modulation and relativistic self-focusing[11] instabilities dominate laser pulse propagation and wakefield formation. However, once the wakefield forms, its accelerating field \(E_x \sim 10^{10} \text{V/cm}\) is strong enough that it can capture and accelerate electrons from the hot thermal tail of the background plasma, thus circumventing the need for external injection. Moreover, self-focusing extends the interaction length to \(\sim 10^{-1} \text{ cm}\), and helps collimate the electron beam. As a result, several groups have observed a high yield (> 1nC/pulse)[12, 13]. There is a substantial number of electrons having energies up to 10 MeV and beyoned with the maximum energy extends to tens of MeV.[14] using an intense multi-TW laser pulse focused into a gas jet. While most SM-LWFA demonstrations have been essentially single-shot, repetition rates as high as 10 Hz have been achieved.[13]

Because of its efficient multi-MeV electron yield and simple set-up, the SM-LWFA has become the most promising candidate for near-term LWFA applications. Within the past two years, review articles[15] and conference proceedings papers[16] on laser acceleration have begun to envision a multitude of “low energy” applications — including materials science, structural biology, nuclear medicine, food sterilization, and transmutation of nuclear waste — with increasing frequency. In particular, applications to cancer radiation therapy have been envisioned for some time.[17] However, to our knowledge no quantitative analysis of these possible applications has been carried out. The purpose of this paper is to evaluate the feasibility of designing a medical accelerator for radiation treatment based on existing SM-LWFA performance, and to explore ways of improving this performance for medical applications within the bounds of existing laser technology.

In Sec. 2, we analyze the characteristics of electron beams produced by existing laser accelerators, based on recent SM-LWFA data,[13] in the context of generally accepted clinical requirements for radiation therapy.[18, 19] We find that, once energy windowing and beam delivery to the patient are taken into account, the dose rate falls short of the clinically useful threshold of 4 Gy/min by an order of magnitude or more. The primary limiting factor is the low repetition rate of the driving laser, necessitated in turn by the high intensity requirement of the SM-LWFA. In Sec. 3, we describe a modified scheme for exciting the wakefield in which the primary driving pulse is “seeded” by a weaker laser pulse that is shifted downward in frequency
by $\omega_p$[20]. The seed pulse lowers the intensity requirements of the primary pulse, thereby enabling high repetition rates. We present results of particle-in-cell simulations to evaluate the improvement that can be achieved.

In Sec. 4 we discuss the potential advantages that Raman seeding, and LWFA as a whole, may be able to bring one step closer to radiation therapy and other medical applications. Our conclusion is summed up in Sec. 5.

2 Suitability of existing laser accelerators for radiation therapy

Conventional radiotherapy utilizes linear accelerators that produce nearly mono-energetic electron beams and x-rays, thus enabling control of dose distribution. SM-LWFA, on the other hand, produces poly-energetic electron beams. The output spectrum matches the useful energy range (6-25 MeV) for clinical applications fairly well. However, direct use of the unfiltered output, while maximizing dose rate, would significantly compromise the quality of dose distribution. Thus energy windowing is essential for clinical use of the SM-LWFA.

Kainz et al. [18] demonstrated that for energies near the center of the clinical useful range (6-25 MeV), an energy-bin of width $\Delta E \sim 5$ MeV is required to retain adequate depth control. Purdy et al. [19] have stated that, to produce clinical beams with a dose rate of 4Gy/min, electron beam currents of 100 nA and 100 $\mu$A are required for 6-MeV electron and 6-MV x-ray beams respectively. Currents of 20 nA and 20 $\mu$A are required for 25-MeV electron and 25-MV x-ray beams respectively. These clinical requirements provide the basis for evaluating the suitability of SM-LWFA for radiation therapy.

We take the recent data of Leemans et al. [13] as an example. Their electron beams were produced by a laser pulse with a wavelength $\lambda = 0.8\mu$m, and a spotsize $w \sim 6\mu$m. Its peak power is up to 10 TW at a repetition rate 10 Hz, at a peak intensity $I = 7.3 \times 10^{18}$W/cm$^2$. The average momentum distribution of output electrons per pulse is fitted to the expression $f(p) = Aexp(-p/p_0)$ with $A = 4.8 \times 10^9/(\text{MeV}/c)$ and $p_0 = 3.3$ MeV/c. Total charge produced per pulse is given by $F(0, \infty) = eA\rho_0 \approx 2.5$ nC, where $F(p_1,p_2) = \int_{p_1}^{p_2} f(p)dp$.

The dose rate is defined as the product of the “effective fluence” the “mass stopping power for water”. Let $\eta$ be the fraction of the selected
electrons that reaches the patient. The dose rate is given by

\[
\mathcal{D} = \left( \frac{\eta I}{eA} \right) \cdot \frac{1}{\rho} \frac{dE}{dx}. \tag{1}
\]

Using for water phantom \( \rho = 1g/cc, \) \( dE/dx \sim 2MeV/cm, \) and the cross section \( A = \pi r^2 \) with the standard fiducial cross section of \( 25cm \times 25cm, \) \( r = (25/\sqrt{2}) \) cm,

\[
\mathcal{D}(Gy/min) = 0.122 \times \eta I(nA). \tag{2}
\]

The current of the selected electrons is \( I = \epsilon f \Delta N, \) with \( \Delta N \) being the number of electrons within the selected energy bin and \( f \) the pulse repetition rate.

As an example, we estimate achievable dose rate for \( E_{cent} = 9 \) MeV and \( \Delta E = 5 \) MeV. The fitted algebraic form gives 9.4% electrons produced in the bin. This corresponds to 0.24 nC per pulse at the rate of 10 Hz, giving 2.4 nA beam current. Using eq.(2) the corresponding dose rate is \( \sim 0.11 Gy/min, \) which is a factor 40 smaller than the minimum required dose rate for clinical application. As \( E_{cent} \) increases, the dose rate decreases further.

The existing SM-LWFA could still be useful for preliminary “patient site” test treatments with reduced beam cross section. For example, one could run tests at using a cross section of \( 5cm \times 5cm, \) the typical size of a tumor, thereby yielding a dose rate of 2.5 Gy/min. Nevertheless, it is important to explore means to increase the dose rate substantially in working toward a clinically viable dosage. One possibility is to increase the total yield of electrons per pulse. However, current SM-LWFAs operate close to the saturation, or beam-loading limit, in which total electron energy produced is comparable to plasma wave energy. [21] Thus yield can not be substantially increased without increasing laser pulse energy, which would require a corresponding reduction in pulse repetition rate, resulting in no net improvement in dose rate. A second possibility is to concentrate the output in a narrower energy bin. However, control of the energy spectrum is extremely difficult in view of the short wavelength of the plasma waves in the SM-LWFA. The greatest opportunity for improvement, therefore, lies in increasing the repetition rate \( f. \) Since repetition rate of solid state laser systems is limited by heat removal, the pulse energy must be reduced proportionately. Thus a method must be found to produce a comparable electron yield with lower pulse energy.
3 Raman-seeding: Improving laser accelerators for radiation therapy applications

We now simulate the effect on LWFA of a weak Raman seed pulse, shifted in frequency from the main laser pulse by approximately $\omega_p$, using the particles-in-cell (PIC) method [22]. In PIC simulations, the plasma system is modeled using finite-size macro-particles, each representing a large number of electrons. Their finite size suppresses unphysically high collision rates that would occur with point particles, while allowing a coarse-grain description of plasma dynamics. Space is divided into cells comparable in size to the macro-particles, and Maxwell’s equations and the Lorentz force describe laser-plasma interactions.[23]

To model Raman seeding qualitatively, we use 1D PIC simulations, in which assuming $y$-polarization, the main ($E_y$) and seed ($E'_y$) pulses propagate in the x-direction, but do not depend on transverse spatial coordinates:

$$E_y = E_0 f(x - x_c, \sigma_x, k),$$
where $f(\Delta x, \sigma, k) = \exp \left[ -\frac{\Delta x^2}{2\sigma^2} \right] \sin kx, \quad (3)$$

$$E'_y = E'_0 f(x - x'_c, \sigma'_x, k'),$$
with $\omega' = \omega - \omega_p$, and $k' \approx k - k_p. \quad (4)$

Here $x_c, \sigma_x$ and $k = 2\pi/\lambda$ are, respectively, the center location, the width and the wave number of the pulse. $k_p$ and $\omega_p$ satisfy the dispersion relation $\omega^2 = \omega_p^2 + (ck_p)^2$. 1D simulations are useful to document how the main LWFA mechanisms — Raman forward scattering, longitudinal wave breaking, particle trapping — depend qualitatively on experimental parameters such as main and seed pulse intensity, at low cost in computer time. The results are most reliable for large spot size ($w >> \lambda_p$, or plasma wavelength) and early time in wakefield growth. 2D simulations, which include laser intensity variation along one transverse spatial dimension, are needed for smaller spot sizes, and to account for purely 2D phenomena — relativistic self-focusing, transverse self-modulation, transverse wave-breaking etc. — that play important roles in practical LWFA. 2D simulation allows us to estimate laser pulse energy needed for energetic electron production quantitatively. For discussion of the transition from 1D to 2D phenomena see Ref.[24]. For our related 1D and 2D investigations see also Refs.[25, 26, 27].

3.1 1D PIC simulation of Raman-seeded LWFA

Our 1D simulations use $2^{11}$ cells, with about 10 PIC particles per cell. For our purposes, forward Raman scattering can be described very well as a
4-wave instability—i.e., a recursive interaction among the incident wave (at frequency $\omega_0$), the plasma wave (at $\omega_p$), the Stokes wave (at $\omega_- = \omega_0 - \omega_p$) and the anti-Stokes wave (at $\omega_+ = \omega_0 + \omega_p$). There is also a 3-wave instability, from recursive interaction among the first three waves, that becomes more pronounced at large scattering angle. Decker et al.\cite{10} discuss the competition among the various types of Raman instabilities in detail. To compare our 1D simulation output with the analytic theory of Raman instability growth,\cite{10} we approximate the instability growth as a pure 4-wave instability at early time.

We denote the seed-strength parameter by: $b = (a_{\text{seed}}/a_0)^2$. Here $a_0$ is a convenient laser field strength parameter defined by $a_0 \equiv E_0/(m\omega c) = e/(m\omega c)^{1/2}$. This parametrization is convenient because typically $a_0 > 1$ is required to generate a substantial wakefield with a single laser pulse. Including the seed contribution, the expression of Decker et al.\cite{10} for asymptotic wakefield growth from 4-wave instability at the tail of the pulse becomes,

$$\delta E_x \propto a_{\text{noise}} \frac{\exp(\zeta)}{\sqrt{2\pi \zeta}}, \quad (5)$$

where $\zeta = G \sqrt{(t - t_1)t_1 \omega_p^2}$ and $G = a_0/\sqrt{2\gamma_p}$. Here $t_1 = \text{pulselength}/c$ and $\gamma_p \equiv \omega/\omega_p \approx \lambda_p/\lambda$ is the ratio of the main pulse frequency to the plasma frequency, a convenient way of parametrizing the electron density $n_e$ in the plasma. $a_{\text{noise}}$ is the noise amplitude. When the seed contribution dominates, $a_{\text{seed}}$ replaces $a_{\text{noise}}$.

3.2 Dependence of LWFA on Raman seed intensity

Fig. 1 is a 1D PIC simulation output for which the main pulse has intensity $a_0 = 0.5$ (corresponding to $I = 5.3 \times 10^{17} W/cm^2$) and wavelength $\lambda = 0.8\mu$. Electron density in the plasma is $n_e = 4.85 \times 10^{19} cm^{-3}$ (i.e. $\gamma_p = 6$). Fig.1a shows the wakefield amplitude as a function of the square root of the normalized strength of the seed $\sqrt{b} \equiv \sqrt{I_{\text{seed}}/I_0} = a_{\text{seed}}/a_0$, at $t=100$, where the time variable $t$, unless otherwise specified, will always be given in units of $\omega_p^{-1}$. Even for $\sqrt{b} = 0$, there is some noise contribution to the wakefield,\cite{10} because of the finite pulse length. For the present laser-plasma system, $\sqrt{b} = 0$ defines the background noise level. As $a_{\text{seed}}$ increases, the seed contribution begins to supplant background noise. Above a threshold $a_{\text{seed}}$, wakefield amplitude then rises linearly with $a_{\text{seed}}$ (see eq(5)) until other effects take over. Fig 1a suggests that at $t=100$, this linear region begins at around $\sqrt{b} \sim 0.04$ and extends to at least 0.1. Fig. 1b shows how
the maximum wake field varies as a function of time. The points connected by the thick-lines indicate the time behavior of the maximum wakefield for the seed strength parameters at: $\sqrt{b} = 0., 0.01, 0.05$, and 0.1. As the strength of the seed increases, the corresponding curve of connected points occur earlier and earlier in time. The shape of the growth curves as predicted by eq.(5) for the cases $\sqrt{b} = 0, 0.05$ and 0.1 are shown by the light-curves, for comparison. The normalizations of the $\sqrt{b} = 0$ and $\sqrt{b} = 0.05$ cases are arbitrary chosen. Based on the normalization of the $\sqrt{b} = 0.05$ case, the curve of the $\sqrt{b} = 0.1$ case is predicted by eq.(5). In the small $t(< 100)$ region, the prediction agrees with the PIC results reasonably well.

From eq.(5) one sees, when $b << 1$, the G-factors for all cases illustrated are similar. Nevertheless, the parameter-$b$ controls the threshold in $t$, which in turn controls the overall growth pattern in the small $t$-region. Fig.1b also shows that the exponential rise predicted by eq.(5) applies only over a relatively small time interval. Subsequently other competing effects dampen the exponential growth. The main conclusion to be drawn from Fig. 1 is that a relatively weak seed pulse ($0.04 < b < 0.1$) can substantially accelerate wakefield growth. It turns out as $b$ further increases, the detuning effect sets in. Here the larger is the $b$ value, the more precise experimental control is needed to prepare the shifted seed frequency. For our investigation below we choose to work with $b = 0.1$.

3.3 Dependence of Raman-seeded LWFA on main pulse intensity

Raman seeding enables hot electrons to be generated efficiently for main pulse intensities even lower than the value $a_0 = 0.5$ used in the example of Fig. 1. To illustrate this, we illustrate in Fig. 2, temporal evolution of the laser plasma system for $a_0 = 0.35$ and $a_{seed} = 0.01$, at time $t=100, 300$ and $650$. On the left it shows that as time increases, the laser pulse becomes more and more modulated. On the right, it shows the corresponding wakefield excitation, it first increases and then decreases. The point $t=300$ is near the maximum of the excitation. To illustrate this, Fig. 3 compares unseeded (open circles, dashed curves) and Raman-seeded (filled squares, solid curves) LWFA for $a_0 = 0.35$ and $a_{seed} = 0.1a_0$. Fig. 3a shows the growth of the normalized wakefield potential $\phi = eE_x/(m\omega c)$ with time, and Fig. 3b the maximum electron momentum $p_x^{\text{max}}$ vs. time, where for relativistic electrons $p_x/cm \approx KE/mc^2$ in the relativistic domain. For this simulation, the main pulse has wavelength $\lambda = 1\mu$ and pulse length $\sim 2\sigma_x = 100fs$, the seed pulse has pulse length $\sigma'_x = \sigma_x$ and position $x'_c = x_c + 0.5\sigma_x$, and the
plasma has density \( n_e = 3.3 \times 10^{19} \text{cm}^{-3} \) (i.e. \( \gamma_p = 5.8 \)). For the unseeded case, \( \phi \) never exceeds 0.1, whereas for the seeded case \( \phi \) reaches a maximum \( \phi_0 \approx 0.6 \) at \( t = 300\omega_p^{-1} \), sufficient to trap electrons\(^{[28]}\). As expected, this maximum occurs somewhat later than the corresponding maximum at \( a_0 = 0.5 \) (see Fig. 1b). The maximum kinetic energy is related to \( \phi_0 \) by\(^{[3]}\)

\[
KE_{\text{max}} = 4\gamma_p^2\phi_0mc^2 = 4 (5.8)^2 \times 0.6 \times 0.5 \approx 40 \text{ MeV},
\]

consistent with the simulated \( p_x^{\text{max}} \) in Fig. 3b. The peak in \( KE_{\text{max}} \) occurs about \( 200\omega_p^{-1} \) later than the peak in \( \phi \) because of the time required for acceleration. We can estimate this delay as one dephasing time \( t_{\text{dephase}} = 2\pi\gamma_p^2/\omega_p \approx 221 \), consistent with the simulated delay.

In order to plot the efficiency of Raman-seeded LWFA systematically as a function of \( a_0 \), we must take into account the time scales illustrated in Figures 1b and 3. We must choose an interaction time (and thus length) large enough to include the main wakefield peak (up to \( \sim 400\omega_p^{-1} \) plus the dephasing time \( \sim 200\omega_p^{-1} \) at all \( a_0 \) values being compared. In other words \( t_{\text{final}} \sim 600\omega_p^{-1} \) or interaction length \( L_{\text{int}} \sim 600 \times 2\pi/5.8 \sim 0.65 \text{ mm} \). Using this criterion, Figure 4 shows how (a) \( \phi_{\text{max}} \), (b) \( KE_{\text{max}} \), (c) and (d) the percent electron production scale with \( a_0 \) for seeded SM-LWFA, while maintaining seed amplitude \( a_{\text{seed}} = 0.1a_0 \). The threshold for electron trapping and acceleration to MeV energies is near \( a_0 \sim 0.28 \). \( KE_{\text{max}} \) rises steadily up to \( a_0 \approx 0.35 \), then saturates, despite continuing increase of \( \phi \), indicating deviation from the expectation from a linear theory. Here the maximum kinetic energy of electrons is about 40 MeV and the corresponding production of hot electrons (\( KE > 1 \text{ MeV} \)) is about 0.15%.

The 1D simulations indicate, therefore, that \( a_0 \sim 0.35 \) is about the lowest main pulse field strength for which Raman-seeded LWFA produces energetic electrons efficiently. This field strength is indeed substantially smaller than typical field strengths (\( 1 < a_0 < 2 \)) required in single-pulse SM-LWFA experiments, suggesting that intensities (\( I \propto a_0^2 \)) ten to one hundred smaller than conventional SM-LWFA experiments may be feasible. Moreover, the electron kinetic energy spectrum in Figure 5, extracted from 1D simulations of RS-LWFA for \( a_0 = 0.35 \) at \( t = 600\omega_p^{-1} \), shows a plateau of electron number extending out to about 40 MeV, following a steep drop from low energies up to \( \sim 2 \text{ MeV} \). The range from \( 1 < KE < 40 \text{ MeV} \) is indeed useful for radiation therapy.
3.4 Electron yield and main pulse energy for Raman-seeded LWFA

The critical remaining questions are: 1) How many electrons are generated within this useful range? and 2) What laser pulse energy is required to generate them? From 1D simulations, we are limited to approximate answers to these questions. To answer the first question approximately, we transformed the spectrum $dN_e/d\gamma$ of PIC particles, expressed in terms of the Lorentz factor $\gamma$, to a kinetic energy distribution $dN_e/dE$ using the expression

$$
\frac{dN_e}{dE} = \frac{f}{m_ec^2} \left[ \frac{dN}{d\gamma} \right],
$$

where $f = N_e/N_T$ is the ratio of the total number of electrons to the total number of PIC particles within the co-moving window. $N_e = n_eAL$ in turn depends on plasma density $n_e = 3.3 \times 10^{19}/\text{cm}^{-3}$, the length $L = 184\mu$m of the co-moving frame window and the effective interaction area $A$. To generate the $dN/dE$ values shown in the vertical scale of Figure 5, we took $A = \pi r_0^2 = 1000\mu^2$, equivalent to a disc of radius $r_0 = 18\mu$. The spot size $w$ may be determined by the relationship $A = \pi w^2/2$, which leads to $w_0 = \sqrt{2}r_0 \approx 25\mu$m. This is the smallest area for which we have found that 1D simulations accurately represent RS-LWFA physics up to $t = 600\omega_p^{-1}$, based on independent 2D simulations [27]. However, it is not necessarily the smallest $A$ providing medically useful electron yield. Moreover, it exceeds spot areas typically used in conventional SM-LWFA experiments. Nevertheless, within this conservative assumption, the hot electrons ($KE > 1\text{MeV}$) in Figure 5 constitute a fraction $f_{\text{hot}} = N_{\text{out}}/N_T = 0.0015$ of the total plasma electrons. This corresponds to a number of hot electrons $N_e^{\text{out}}$ and charge $Q_e^{\text{out}}$ per pulse of

$$
N_e^{\text{out}} = \frac{N_{\text{out}}}{N_T}n_eAL \approx 6 \times 10^{12}, \text{ and } Q_e^{\text{out}} = eN_e^{\text{out}} \approx 1.5\text{nC},
$$

and requires pulse energy $U_{\text{pulse}} = A\tau_{\text{pulse}}I_0 \approx 170\text{ mJ}$ for the assumed pulse duration $\tau_{\text{pulse}} = 100\text{ fs}$. The hot electron yield per pulse matches conventional SM-LWFA output. (Using the lower cutoff of $KE > 1\text{MeV}$, the data of Leemans et al. discussed in Sec. 2 leads to the hot electron yield of 1.7 nC.) Moreover $U_{\text{pulse}}$ is several times smaller than typical SM-LWFA pulses, despite the artificially large $A$ assumed here. Such pulses could be generated at repetition rates as high as 50 Hz with a typical $\sim 10\text{ W}$ average power laser system, a modest improvement over current SM-LWFA As. To see whether further improvement — i.e. similar $Q_e^{\text{out}}$ and $dN/dE$ with smaller
$U_{\text{pulse}}$ by decreasing $A$ — is feasible, we must turn to 2D simulations of RS-LWFA. The 2D simulation is more versatile. In this context our 1D simulation results presented up to this point may be regarded as the 1D regime results based on 2D simulation.

3.5 2D PIC simulation

Our 2D PIC simulations are based on the Vorpal code[29]. The simulation box had dimensions $L_x = 200\mu m$ and $L_y = 50\mu m$ with corresponding gridlines $2000 \times 300$, and there were 5 PIC particles per cell. At $t = 0$ the laser pulse started propagating toward the simulation box from a starting point outside. Upon its arrival at the center of the window, the pulse-moving frame took over, so the pulse stayed at the center of the window for the rest of the simulation.

Most parameters used are comparable to those used for 1D PIC simulation, i.e. the main pulse has wavelength $\lambda = 0.8\mu m$, pulse length $\tau_{\text{pulse}} = 100fs$, the seed strength $b = 0.01$ and $\gamma_p \sim 6$ as before. For the laser intensity, from general considerations one expects the $a_0$ for 2D should be larger than that of 1D. Here we will present the case with intensity $a_0 = 0.5$ ($I_0 = 5.3 \times 10^{17}W/cm^2$) and refer the reader to ref.[27] for more details. This $a_0 = 0.5$ case has a special significance. Here the diffraction effect, which is present in 2D simulation, is approximately canceled by the relativistic self-focusing effect, since here the relevant ratio $P/P_c = (\pi a_0^2/(2\gamma_p \lambda))^2 \approx 1.0$, where $P_c$ is the critical density[3].

With the parameters chosen, the plasma electron density is $n = 4.85 \times 10^{19}/cm^3$. The energy of the pulse is $U_{\text{pulse}} = (\pi \omega^2/2)\tau_{\text{pulse}} I \sim 30$ mJ, where the pulse intensity $I = 1.35 \times 10^8 a_0^2/\lambda(\mu)$ $\sim 5.3 \times 10^{17}W/m$ was used. The Rayleigh length $L_R = \pi \omega^2/\lambda \sim 141\mu m$ and “Rayleigh time”: $t_R = L_R/c = 184/\omega_p$.

Figure 6 presents the 2D PIC simulation results after the pulse has traveled 1 Rayleigh length ($t \sim 200\omega_p^{-1}$). Fig. 6a shows, there is already a significant pulse modulation. Fig. 6b shows the corresponding wakefield at the same time. Notice the wave structure with a maximum amplitude at around $\phi_0 = 0.6$ near $x \sim 100\mu m$, which is close to the center of the pulse. Then as $x$ increases, there is a “wave breaking” which begins at around $x = 80\mu m$. This is to be compared to the 1D case, where the corresponding wakefield maintains a relatively large amplitude over the entire left-region (see Fig. 2). Fig 6c shows the longitudinal momentum distribution of the accelerated electrons. Notice that the maximum momentum reached here is $p_x/mc \sim 35$ or $p_x \sim 17MeV/c$, which begins at the end portion of
the wakefield wave-structure.

For the present small-\(w\) 2D simulation, 2D effect becomes more and more noticeable as time progresses. At \(t = t_R\), it turned out there is an spread in the \(y\)-direction is \(\sqrt{<y^2>} - <y^2> \sim 0.7w\) and spread in angle: by \(\sqrt{<\theta^2>} - <\theta^2> \sim 11.6^\circ\). At \(t = 2t_R\), their values are \(\sim 1.9w\), and \(\sim 10.9^\circ\) respectively.

The conversion from 2D simulation quantities to the charge of the produced hot electrons (\(KE > 1\text{MeV}\)) is done by

\[
Q_{\text{out}}^e = \frac{N_{\text{out}}^e}{N_T} (L_x L_y w) ne,
\]

At \(t = 1t_R\), \(N_{\text{out}}^e = 3977\), \(N_T = 3 \times 10^6\) and \(w = 6\mu m\), yielding \(Q = 0.62\text{nC}\) per pulse. And at \(t = 2t_R\), \(Q = 0.58\text{nC}\). So a same order of magnitude of the charge of hot electrons as that for the 1D regime can be produced with much less pulse energy (\(\sim 30mJ\) vs \(\sim 170mJ\)).

The kinetic energy distributions of hot electrons at \(t = t_R\) and \(t = 2t_R\) are shown in Fig. 7a and Fig. 7b, each in a semilog plot. It is instructive to compare them with the energy spectrum of the 1D case shown in Fig. 5. First consider the maximum energy. We recall within 1D approximation, the maximum electron energy is given by \(E_{\text{max}} = 4\gamma_p^2 mc^2 \phi_0 \sim 40\ \text{MeV}\), for \(\phi_0 \sim 0.6\) (see Sec. 3.4). For the present small spot-size case only in half of the 1D-acceleration range where the beam is being accelerated and at the same time being focused.[3] So in the “small spotsize, \(w\)” approximation, \(E_{\text{max}}^w \sim 2\gamma_p^2 mc^2 \phi_0\).

Next consider \(dN/dE\) distributions for the small-\(w\) 2D case and for the 1D case. Here the difference is striking. For the small-\(w\) 2D case, \(dN/dE\) falls-off approximately exponentially (see Fig. 7a, 7b), while for 1D-case, Fig. 5 shows that large number of electrons is present in high energy portion of the spectrum. The rapid fall-offs in Fig. 7 may be correlated to, among other things, the occurrence of wave-breaking (see Fig. 6b). Once the wave breaks, no strong wakefield will be available to accelerate electrons to high energies. Also a closer examination on the transverse structure of the pulse at \(t = t_R\) and at \(2t_R\) in 2D simulation output reveals that as time increases, the pulse tends to spread out in the transverse direction and it breaks up into small pulse-lets. In other words through interaction with the plasma medium, the pulse undergoes a “filamentation” process which further reduces the strength of the pulse in producing wakefield, and in turn in the capability in generating energetic electrons.

To sum up the above example leads to the general conclusion that, by focusing to spot sizes below the 1D limit (\(w_0 < 25\mu m\)), RS-LWFA can
produce medically-relevant electron output up to the mid-range energy of medical application, say near 15 MeV with much lower pulse energy (30 mJ vs. 170 mJ) than in the 1D regime. From 1D simulation work, we expect that adequate electrons in the 15-25 MeV region can be produced for sufficiently large spot-size. To properly answer the question how low can \( a_0 \) be to generate the full energy spectrum for medical application requires a careful search in the parameter space, where the spot-size along with the plasma- and laser- parameters are allowed to vary. We will defer this numerical effort to future work.

4 Discussion

The analysis of the previous two sections shows that a laser- energy-threshold-lowering scheme such as Raman-seeding would be a critical enabling factor for LWFA to satisfy generally-accepted clinical requirements for radiation therapy applications: dose rate > 4 Gy/min, beam size 25 \( \times \) 25 cm, electron energy range 5-25 MeV and energy width \( \Delta E \sim 5 \) MeV. In comparing demonstrated SM-LWFAs[13] to proposed RS- LWFA, we used average power \( P_{\text{avg}} = U_{\text{pulse}} f \approx 10 \) W as a standard capability of existing solid-state terawatt laser systems, limited primarily by heat removal from the gain medium. This figure should be considered in a wider context. TW-class systems providing \( P_{\text{avg}} = 20 \) W (20 mJ, 1 kHz) already exist.[30] Moreover, development of higher average power solid-state laser systems is a forefront area of current laser technology research.[31] Besides medical therapy, other applications driving this development include laser-generated x-ray sources for photolithography, laser ranging and atmospheric sensing. Key enabling technologies include diode arrays for more efficient pumping of the laser gain medium, active correction of thermal wavefront distortions and beam multiplexing strategies. Average powers well over 100 W have been demonstrated in prototype solid-state laser systems with ns or longer laser pulses. It is reasonable to predict that short-pulse systems suitable for LWFA with \( P_{\text{avg}} > 100 \) W will some day be available. In this larger context, our canonical \( P_{\text{avg}} = 10 \) W is a conservative estimate of the capability of TW laser systems.

Even if near-future improvements in laser technology are taken into account, however, conventional SM-LWFAs are likely to fall short of clinical dose requirements. A comparison between experimental results of Leemans et al. discussed in Section 2 and the 2D simulation are shown in Table II. For \( E = 9 \pm 2.5 \) MeV electrons, the former gives \( \sim 0.1 \) Gy/min using \( P_{\text{avg}} = 5W \)
— (i.e. 0.5 J at 10 Hz) scale to $\sim 0.2$ Gy/min at $P_{\text{avg}} = 10W$ (0.5 J, 20 Hz), and $\sim 2$ Gy/min at $P_{\text{avg}} = 100W$ (0.5 J, 200 Hz). On the other hand, our 2D PIC simulations show that the proposed RS-LWFA driven by a $P_{\text{avg}} = 10$ W system (.03 J, 330 Hz) could produce $\sim 2.4$ Gy/min for the 9 MeV energy bin, which scales to 24 Gy/min for $P_{\text{avg}} = 100$ W (.03 J, 3.3 KHz). Higher repetition rate systems will also help to enable clinical dose rates for high energy bins (e.g. $E \sim 20 \pm 2.5$ MeV). In our simulations, 20 MeV electrons are produced with approximately one order of magnitude lower efficiency than 10 MeV electrons (see Fig. 7). However, this distribution is neither definitive nor typical. The energy spectrum is sensitive to details of the driving pulse temporal shape and spot-size or more generally the focus profile and to the density distribution of the plasma. For example, we show elsewhere[27] that implementation of RS-LWFA in a preformed plasma channel can facilitate the energization of higher energy electrons. Further 2D PIC simulations of both longitudinal and transverse electron phase-space distribution that take these factors into account, as well as experiments, will be needed to optimize the distribution for medical applications. For now, our estimated dose rates near mid-range energies indicate that RS-LWFA-based radiation therapy applications are feasible.

Demonstration of feasibility provides, of course, only a necessary condition to motivate further research into LWFA-based radiation therapy applications. To provide a sufficient condition, potential advantages of medical LWFA over existing technology should also be articulated. Compactness is the chief advantage of high-gradient accelerators over conventional technology. The SM-LWFA and RS-LWFA accelerate electrons to several tens of MeV in only $\sim 1$mm path length, as opposed to several meters in conventional rf linacs. Associated with compactness is the potential for lower cost and greater mobility of the clinical apparatus, which might make LWFA-based systems attractive for smaller medical facilities. However, for a complete evaluation of space requirements, the laser delivery and vacuum systems surrounding the interaction region must also be taken into account. To this end, a schematic diagram of a medical LWFA together with the two laser pulse compressor units is illustrated in Fig. 8. Some of the ingredients of the accelerator have been considered earlier[17, 25, 32]. The main pulse from the lower compressor is joined by the Raman seed pulse generated using

a known experimental technique[33] from the upper compressor. The combined pulse is transported by reflecting mirrors and enters a vacuum chamber through a window made of $MgF_2$ or $LiF$. The pulse is then reflected and focused to the interaction region at the central gas cell at at-
mospheric pressure. After the interaction region, energetic electrons emerge from the chamber through the downstream aperture. The remainder of the laser pulse is redirected by a concave mirror to the dump. The length of the chamber is estimated to from 30 cm down to about 5 cm. We refer the reader to the Appendix for more details on the accelerator chambers.

The interaction of the driving laser pulse with the MgF$_2$ or LiF brings out an important secondary advantage of RS-LWFA over SM-LWFA. The SM-LWFA, because it requires such an intense pulse, could not use the window on the vacuum chamber, nor any transit in air between last compressor grating and interaction region, because of nonlinear optical ("B-integral") distortion in the window and air. Consequently the compressor must be attached to the accelerator vacuum chamber, making the clinical part of the apparatus much bulkier. RS-LWFA, on the other hand, can use substantially weaker pulses, which can go through air and high-band gap window. Consequently the whole laser system can be separated from the smaller accelerator arm, making the clinical part of the apparatus smaller and more mobile. This is a significant secondary argument for threshold-lowering schemes such as RS-LWFA.

The issue of converting the electron beam from the LWFA into a clinically useful beam incident on the patient remains to be addressed. This requires beam energy selection, broadening, collimation, and monitoring. The LWFA is a more difficult issue than that of a conventional linac because the former beam is polyenergetic and has a slightly greater angular divergence. In linear accelerators, the beam is typically redirected and energy selected using a 270$^\circ$ achromatic magnet [34]. The beam is broadened using a dual scattering foil system, or occasionally a magnetic scanning system[34, 19]. Raischel et al [35] have investigated the use of an achromatic magnet system for transporting and broadening a LWFA beam, although the study was restricted to 5-cm diameter beams, too small for standard therapy application but adequate for dosimetric study. The treatment of the broadening and flattening of the beam leading to an acceptable beam at the patient for standard therapy application will be discussed in the companion paper[18]. A narrow beam could be used directly for rastering of the beam as well as for spotting narrow regions of tumor if detectable.

5 Conclusion

Existing Laser Wakefield Accelerators (LWFAs) produce well-collimated beams of electrons with a range of energies (1-25 MeV). This has the potential for
radiation therapy applications, thus motivating recent suggestions that LW- 
FAs might provide a compact, low-cost alternative to conventional medical 
linacs. Using 1D and 2D particle-in-cell simulations, we have quantitatively 
evaluated the feasibility of such proposals. Our results show that existing 
LWFAs require such energetic laser pulses (∼ 1J) that the dose rate falls well 
short of clinical levels once realistic laser repetition rates, electron energy 
windowing and beam delivery to patient are taken into account. On the 
other hand, we show that Raman-seeded (RS) LWFA can sufficiently reduce 
the threshold laser pulse energy required for producing multi-MeV electrons 
that clinical dose rates become feasible using existing or near-future laser 
and beam delivery technology. The results motivate further theoretical and 
experimental research into RS-LWFA as a potential medical accelerator.

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NSF-FOCUS005739. While we were completing this work, we learned that 
John M. Dawson had passed away. We would like to dedicate this paper 
to his memory. This work is also dedicated to Lynda Chiu, who is cur- 
rently fighting our common enemy, cancer. We also thank our colleagues 
Kenneth R. Hogstrom and Kristofer K. Kainz at The University of Texas 
M.D. Anderson Cancer Center for their invaluable input, especially their 
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code and invaluable communications throughout our use of the code.

Appendix: The accelerator chamber

Vacuum region and gas-cell region: In order to reach intensities su-
fficient for LWFA, laser pulses from a CPA system must be focused to a 
spot-size in the range 5µ < w < 20µ into an atmospheric density gas. A 
reflective focusing optics, such as an off-axis-parabolic (OAP) mirror, is pref-
ered to prevent distortion of the intense pulse by dispersion, self-focusing 
or self-phase modulation on passing through a refractive lens. Ionization-
induced refraction[36] would prevent the laser pulse from focusing effective-
ly if such a dense gas filled the entire focusing region. Consequently, the OAP 
must be housed inside a vacuum chamber that maintains background pres-
sure < 1 Torr. The high density gas is restricted to a region ∼ 1mm long at 
the laser focus using either a supersonic gas nozzle or a flowing gas cell with 
sub-mm entrance and exit orifices for the laser pulse. The latter method al-
ows a greater range and spatial uniformity of gas density[37, 38, 26]. With 
either method, an efficient (600cfm ∼ 280 liters/sec) mechanical pump is
required to maintain the pressure difference between the gas-cell or jet and the vacuum region.

**Chamber length:** We estimate the chamber length here. To maintain the coherence of laser pulse, the angle subtended by the mirror at the interaction region must be less than the diffraction angle, i.e. \( D/f \leq \theta_{\text{diffraction}} = \lambda/(2r_0) \), where \( D \) and \( f \) are the mirror diameter the laser pulse. This implies that

\[
f \geq f_{\text{min}} \approx \frac{2r_0}{\lambda} D.
\]

Assume the damage energy threshold of the mirror to be (with a conservative estimate) \( \Delta U_{\text{damage}}/\Delta A = 1 \text{ J/cm}^2 \).

- **1D regime:** From Table I \( U_{\text{pulse}} = 170 \text{ mJ} \). To avoid damage, this leads to \( U_{\text{pulse}} \leq \Delta U_{\text{damage}} = (1\text{ J/cm}^2) \cdot \frac{\pi D^2}{4} \), or \( D_{\text{min}} \approx 0.46 \text{ cm} \), which together with Table I and eq.(9) leads to the maximum value of \( f \approx 16.6 \text{ cm} \). Allowing for a comparable length interval before and after the interaction, leads to a chamber length of the order of \( 2f \approx 30 \text{ cm} \).

- **small-\( w \) case:** Notice the 2D simulation result gains a factor: \( (r_0D)_{2D}/(r_0D)_{1D} = \frac{(r_0\sqrt{U_{\text{pulse}}})_{2D}}{(r_0\sqrt{U_{\text{pulse}}})_{1D}} = (6/18) \times (30/170)^{1/2} \approx 1/7 \). The corresponding length will be \( \sim 5 \text{ cm} \). Here among other things the thickness of the gas-cell and the dimension of the mirror may be nonnegligible in determining the chamber length.

**References**


Table I. 1D Simulation: Parameters and Results

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<th>Assigned parameters</th>
<th>1D PIC simulation</th>
<th>With assumed spot-size</th>
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<tbody>
<tr>
<td>Plasma number density</td>
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</tr>
<tr>
<td>Wavelength</td>
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<td>Pulse Length</td>
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<tr>
<td>Pulse-energy</td>
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<th>Deduced parameters</th>
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<tbody>
<tr>
<td>Pulse intensity(Sec. 3.3)</td>
<td>$a_0 = 0.35$</td>
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<tr>
<td>Seed strength $a_{seed}/a_0$ (Sec. 3.2)</td>
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<tr>
<td>Interaction length (Sec. 3.3)</td>
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<tr>
<td>spot-size $w$: (Sec. 3.4)</td>
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<td>Rayleigh length ($L_R$)</td>
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<table>
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<td>Peak wakefield: seeded / unseeded</td>
<td>$\sim 6$</td>
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<tr>
<td>Maximum electron energy</td>
<td>$\sim 40$ MeV</td>
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<tr>
<td>Number of electrons</td>
<td>$\sim 1.5$ nC per pulse</td>
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Table II. Exp-data (Unseeded) vs. Raman seeded simulation

<table>
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<tr>
<th>Cases</th>
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<th>$E &gt; 1$ MeV</th>
<th>$9 \pm 2.5$ MeV</th>
<th>Rep-rate</th>
<th>Clinic rate, $\sim 9$ MeV</th>
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<tr>
<td>Leemans</td>
<td>500 mJ</td>
<td>1.7 nC/pls</td>
<td>0.11 Gy/min</td>
<td>20 Hz</td>
<td>$\sim 0.23$ Gy/min</td>
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<tr>
<td>2D PIC</td>
<td>30 mJ</td>
<td>0.62 nC/pls</td>
<td>0.073 Gy/min</td>
<td>330 Hz</td>
<td>$\sim 2.4$ Gy/min</td>
</tr>
</tbody>
</table>

Figure Captions

Figure 1. Growth of the wakefield amplitude due to Raman instability. (a) The maximum wakefield at $t=100$, as a function of the normalized seed strength: $\sqrt{b} = a_{seed}/a_0$. Notice the presence of the threshold behavior. For $a_{seed}/a_0 < 0.04$, the wakefield stays essentially constant. Beyond this point, there is an increase in the wakefield and quickly turn to an approximate linear rise. (b) Raman instability growth as a function of time at $a_0=0.5$ for various seed parameter b. The PIC simulation results are shown as heavy curves with points. They correspond to $\sqrt{b} = 0.0, 0.01, 0.05$ and 0.1. Light
curves indicate the growth behavior of 4-wave instability as predicted by eq. (5). The normalizations at $\sqrt{b} = 0$ and 0.05 are adjusted to fit the data. Based on the $\sqrt{b} = 0.05$ normalization, $\sqrt{b} = 0.1$ curve is predicted by eq.(5), which in the small t (< 100) region, is in a reasonable agreement with the corresponding PIC behavior.

Figure 2. Temporal evolution of the laser pulse and the wakefield at $t = 100$, 300 and 650. The left column shows that growth of the pulse modulation process where the pulse is sliced into may pulselets. Right column illustrates the increase and the decrease of the wakefield excitation. The time $t=300$ is near the maximum of the excitations.

Figure 3. Temporal evolution of two maxima.

- Maximum of the normalized $x$-component electric field (wakefield) versus time for the seeded case (solid curve) and the unseeded case (dashed curve). For the seeded case the peak value is $\sim 0.61$ which is at $t \sim 300$. For the unseeded case the peak value is $\sim 0.11$, at $t \sim 100$.

- Maximum of the normalized $x$-component momentum ($\eta_x$) versus time for the seeded case (solid curve) and the unseeded case (dashed curve). For the seeded case the peak value is $\sim 77$, at $t \sim 600$. For the unseeded case the maximum is bounded by $\eta_x \sim 0.55$ in the t-range: 100 to 300. (c) Same as (b), except the plot is in a linear scale, which gives clearer display on the time dependence of the maximum momentum for the seeded case, especially in 300-600 region.

Figure 4. Laser intensity (or $a_0$) dependence of the maximum of wakefield, the maximum of kinetic energy and the corresponding percentage production for the seeded case.

- (a) Maximum of wakefield vs $a_0$. For each $a_0$ point, the corresponding maximum is determined by a time curve. For example for $a_0=0.35$, see Fig. 4a. Dashed curve is the contribution of one resonating pulse.

- (b) Maximum of kinetic energy vs. $a_0$. For each $a_0$ point, the corresponding maximum is determined by a time curve. For example for $a_0=0.35$, see Fig. 4b.

- (c) Percentage of energetic electrons ($KE \geq 1MeV$) produced vs. $a_0$. At each $a_0$, the production percentage is taken at the time where the corresponding kinetic energy is maximum. Notice that the ordinate is in a linear scale and the horizontal line is at 0.15%.
• (d) is same as (c), except that the ordinate is in a log-scale.

Figure 5. The optimized electron energy spectrum for $a_0 = 0.35$ at $t = 600$. For other simulation parameters, see Table 1.

Figure 6. 2D PIC simulation results at $a_0 = 0.5$, when the pulse has traversed about one Rayleigh length. (a) The modulated pulse and at $t = 200$, (b) The wakefield, (c) The $p_x/cm$ vs $x$ plot. This is an important show case, which illustrates that electron energy spectrum comparable to that provided by present source at 500mJ per pulse can be generated by a seeded laser pulse with an energy as low as 30 mJ per pulse. See Sec. 3.5 for details.

Figure 7. The electron kinetic energy (in MeV) spectrum for the Raman seeding case based on 2D PIC simulation. Parameters: $a_0 = 0.5$, the spot-size $w = 6 \mu m$, $\gamma_p = 6$, laser pulse energy: 30 mJ, with interaction lengths: (a) 1 Rayleigh length, and (b) 2 Rayleigh length. The straight lines included are respectively $\exp(-E/3.3)$ and $\exp(-E/4.5)$ to guide the eye.

Figure 8. A schematic diagram of the compact laser accelerator together with the two supporting laser-pulse compressors. The pump pulse from the lower compressor is joined by the Raman seed pulse from the upper compressor to form a seeded pulse. The combined pulse is transported by mirrors and enters the accelerator chamber through a window made out of MgF$_2$ or LaF. It is then reflected by an off-axis-parabolic mirror to be focused at the center of the gas cell. The intense pulse creates a plasma medium and generates the wakefield waves, which trap and accelerate electrons in the plasma. The electron beam exits to the right.
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