3. Field due to a disk.

One can think of a disk as being made up of concentric rings.

\[ \Delta E_{\text{disk}} = \frac{k \Delta Q_{\text{ring}}}{r^3} \]

where \( \Delta Q_{\text{ring}} \) is the charge of a ring with radius \( r \) and thickness \( \Delta r \).

\[ E_{\text{disk}} = \int \left[ k \left( \frac{Q}{4\pi \varepsilon_0} \right) \right] 2\pi r \Delta r \times \frac{z}{r^3} \]

\[ = \left( \frac{k Q}{4\pi \varepsilon_0} \right) \int_0^R r \frac{dr}{r^3} \]

Evaluate \( I_2 \):

\[ I = \int_0^R r \frac{dr}{r^3} \]

where \( p = \sqrt{r^2 + z^2} \)

With \( z \) being fixed, \( p^2 = r^2 + z^2 \).

Taking \( \frac{d}{dr} \) on both sides leads to \( \frac{d}{dr} \text{RHS} = 2p \frac{dp}{dr} \).

And \( \frac{d}{dr} \text{RHS} = \frac{d}{dr} [r^2 + z^2] = 2r \frac{dr}{dr} = 2r \).

\[ 2r \frac{dp}{dr} = 2r \] or \( r \, dp = p \, dr \)

\[ I = \int_0^R p \frac{dp}{r^3} = \int_0^R \frac{dp}{p^2} = -\frac{1}{p} \bigg|_0^R = -\frac{1}{R} - \frac{1}{\infty} = -\frac{1}{R} \]