Reciprocal Space Magnetic Field: Physical Implications

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Outline

Effective dynamics of Bloch electron

Motivation: For a large class of crystalline materials, the equations of motion of Bloch electrons are different:

$$\begin{aligned}\dot{\boldsymbol{x}} &= \frac{1}{\hbar}\frac{\partial \mathcal{E}_n(\boldsymbol{k})}{\partial \boldsymbol{k}} - \dot{\boldsymbol{k}} \times \boldsymbol{\Omega}(\boldsymbol{k}) \\ \hbar\dot{\boldsymbol{k}} &= -e\boldsymbol{E} - e\dot{\boldsymbol{x}} \times \boldsymbol{B}\end{aligned}$$

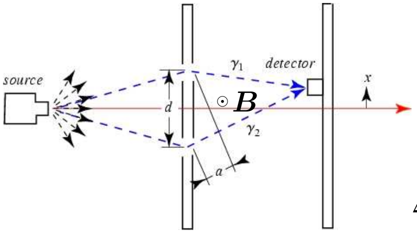
$\boldsymbol{\Omega}(\boldsymbol{k})$ – reciprocal space magnetic field. It presents in systems:

- breaking time-reversal symmetry: magnetic materials
- breaking spatial inversion symmetry: surfaces, interfaces, nanotubes $\cdots$

Sundaram and Niu, PRB **59**, 14915(1999); Marder, *Condensed Matter Physics*.

Physical consequences?

Aharonov-Bohm effect



$$\Delta\varphi_1 = \frac{e}{h}\int_{\gamma_1} \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{s}$$

$$\Delta\varphi_2 = \frac{e}{h}\int_{\gamma_2} \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{s}$$

$$\Delta\varphi = \Delta\varphi_2 - \Delta\varphi_1 = \frac{e}{h}\oint_{\gamma} \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{s}$$

$$\oint_{\gamma} \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{s} = \int_S (\boldsymbol{\nabla}\times\boldsymbol{A})\cdot \mathrm{d}S = \int_S \boldsymbol{B}\cdot \mathrm{d}S \equiv \phi_B$$

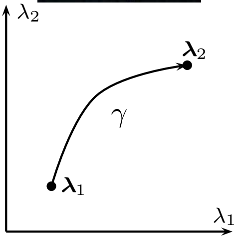
Berry phase

Considering a quantum system controlled by a set of parameters that are slowly varying with time:

$$H \equiv H[\boldsymbol{\lambda}(t)]$$

At t_1: $|\psi(t_1)\rangle = |\psi[\boldsymbol{\lambda}(t_1)]\rangle$
At t_2: $|\psi(t_2)\rangle \equiv \exp(i\varphi)\,|\psi[\boldsymbol{\lambda}(t_2)]\rangle$

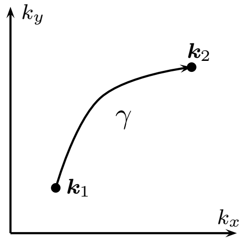
$$\varphi = -\frac{1}{\hbar}\int_{t_1}^{t_2} E[\boldsymbol{\lambda}(t)]\mathrm{d}t + \varphi_B$$



Berry Phase:

$$\varphi_B = i\int_\gamma \mathrm{d}\boldsymbol{\lambda}\cdot\langle\psi(\boldsymbol{\lambda})\,|\,\boldsymbol{\nabla}_{\boldsymbol{\lambda}}\psi(\boldsymbol{\lambda})\rangle$$

Berry phase and $\boldsymbol{k}$-space magnetic field



$$\psi(\boldsymbol{x}) = e^{i\boldsymbol{k}\cdot\boldsymbol{x}} u_{n\boldsymbol{k}}(\boldsymbol{x})$$

$$\hat{H}(\boldsymbol{k}) u_{n\boldsymbol{k}} = \epsilon_n(\boldsymbol{k}) u_{n\boldsymbol{k}}$$

$$\hat{H}(\boldsymbol{k}) = \frac{(-i\hbar\boldsymbol{\nabla} + \hbar\boldsymbol{k})^2}{2m} + V(\boldsymbol{x})$$

Berry phase:

$$\varphi_B = i \int_\gamma \left\langle u_{n\boldsymbol{k}} \left| \frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} \right.\right\rangle \cdot d\boldsymbol{k}$$

A-B phase in real space:

$$\varphi_{AB} = \frac{e}{\hbar} \int_\gamma \boldsymbol{A}(\boldsymbol{r}) \cdot d\boldsymbol{s}$$

$\boldsymbol{k}$-space magnetic field: $\boldsymbol{\mathcal{A}}_{\boldsymbol{k}}(\boldsymbol{k}) = i \left\langle u_{n\boldsymbol{k}} \left| \frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} \right.\right\rangle$

$$\boldsymbol{\Omega}(\boldsymbol{k}) = \boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_{\boldsymbol{k}}(\boldsymbol{k}) = i \left\langle \frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} \right| \times \left| \frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} \right\rangle$$

Symmetry consideration

$$\boldsymbol{\Omega}(\boldsymbol{k}) = \boldsymbol{\nabla}_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_{\boldsymbol{k}}(\boldsymbol{k}) = i \left\langle \frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} \right| \times \left| \frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} \right\rangle$$

With time-reversal symmetry:

$$\boldsymbol{\Omega}(\boldsymbol{k}) = -\boldsymbol{\Omega}(-\boldsymbol{k})$$

With spatial inversion symmetry:

$$\boldsymbol{\Omega}(\boldsymbol{k}) = \boldsymbol{\Omega}(-\boldsymbol{k})$$

With both symmetry:

$$\boldsymbol{\Omega}(\boldsymbol{k}) = 0$$

Implication

Theory of everything:

$$\hat{H} = \sum_i \frac{[\hat{\boldsymbol{p}}_i + e\boldsymbol{A}(\boldsymbol{r}_i)]^2}{2m_e} + \sum_\mu \frac{[\hat{\boldsymbol{P}}_\mu - Z_\mu e\boldsymbol{A}(\boldsymbol{R}_\mu)]^2}{2M_\mu} - \sum_{i\mu} \frac{Z_\mu e^2}{|\boldsymbol{r}_i - \boldsymbol{R}_\mu|} + \sum_{i<j} \frac{e^2}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}$$

More realistic approach – Effective Hamiltonian:

$$\hat{H} = \sum_i \frac{[\hat{\boldsymbol{p}}_i + e\boldsymbol{A}(\boldsymbol{r}_i)]^2}{2m_e^*} + \sum_{i<j} V_{ee}(\boldsymbol{r}_i - \boldsymbol{r}_j) + \hat{H}_{e-ph} + \hat{H}_{ph}$$

Examples: BCS theory; Quantum Hall effect ...

With the k-space magnetic field:

?

Directions for exploration

- **Applications** – the presence of an extra reciprocal field provides new freedom for designing novel devices.
- **Fundamentals** – concept of phase space; effective quantum mechanics
- **Physical effects** – Luttinger's theorem; orbital magnetization; magnetoresistance; superconductivity...

Outline

1. Introduction

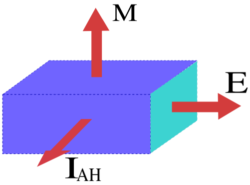
2. Physical implications of the reciprocal space magnetic field
   - Design novel transport devices
   - Inhomogeneous phase space
   - Effective quantum mechanics

3. Conclusion

Anomalous Hall effect

Hall effect in ferromagnetic metal:

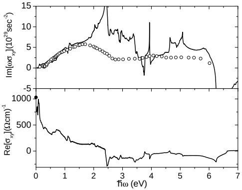
$$\rho_{xy} = R_0 B + (4\pi M) R_s$$



Intrinsic contribution:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \frac{1}{\hbar}\frac{\partial \mathcal{E}_n(\boldsymbol{k})}{\partial \boldsymbol{k}} - \dot{\boldsymbol{k}} \times \boldsymbol{\Omega}(\boldsymbol{k}) \\ \hbar \dot{\boldsymbol{k}} &= -e\boldsymbol{E} \end{aligned}$$

$$\begin{aligned} I_{AH} &\equiv -e\langle \dot{\boldsymbol{x}} \rangle \\ &= -\frac{e^2}{\hbar}\boldsymbol{E} \times \int_F \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^d}\boldsymbol{\Omega}(\boldsymbol{k}) \end{aligned}$$



Karplus, Luttinger (1954); Jungwirth, Niu, MacDonald (2002); Fang et al. (2003); Yao et al. (2004).

Anomalous Hall insulator

$$\sigma_{AH} = -\frac{e^2}{\hbar}\int_F \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^d}\boldsymbol{\Omega}(\boldsymbol{k})$$

For a fully occupied 2D band (band insulator):

$$\sigma_{AH} = -\frac{e^2}{\hbar}\int_{BZ} \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^2}\Omega_z(\boldsymbol{k})$$

$$\int_{BZ} \mathrm{d}^2\boldsymbol{k}\Omega_z(\boldsymbol{k}) = 2\pi C_n$$

Quantized anomalous Hall effect:

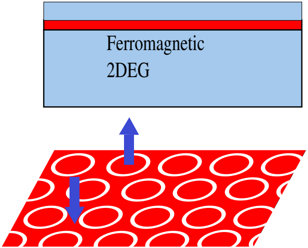
$$\sigma_{AH} = -\frac{e^2}{h}C_n$$

Protected by the band gap, extraordinarily robust!

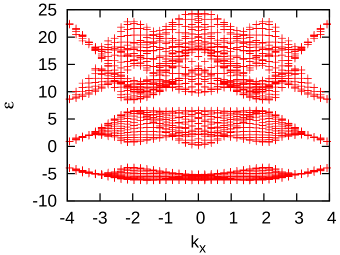
Design an anomalous Hall insulator

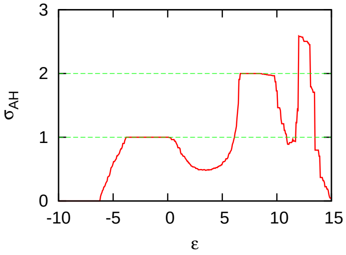
Objective: A 2D ferromagnetic band insulator

- A ferromagnetic 2DEG confined in an asymmetric quantum well
- Patterned with non-magnetic rings arranged as a triangular lattice
- Electron density tunable by the gate voltage

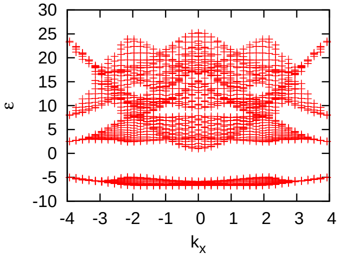


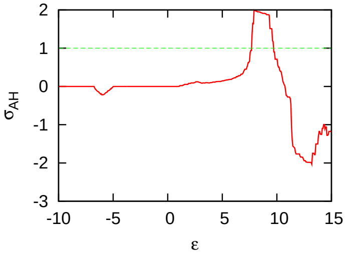
Parameters: Strength of Rashba spin-orbit coupling; Exchange interaction between electron and magnet; Lattice constant; ring radius





Reversed magnetization





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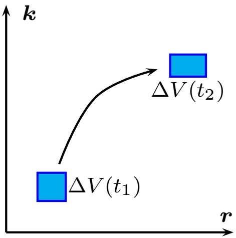
Breakdown of Liouville's theorem

Liouville's theorem: Ensemble density in phase space is conserved.

$$\Delta V = \Delta \boldsymbol{r} \Delta \boldsymbol{k}$$

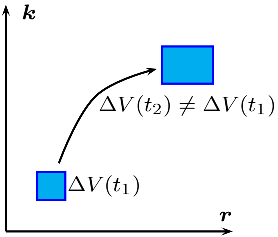
$$\frac{1}{\Delta V}\frac{\mathrm{d}\Delta V(t)}{\mathrm{d}t} = \boldsymbol{\nabla}_{\boldsymbol{x}} \cdot \dot{\boldsymbol{x}} + \boldsymbol{\nabla}_{\boldsymbol{k}} \cdot \dot{\boldsymbol{k}} = 0$$



With k-space magnetic field:

$$\frac{1}{\Delta V}\frac{\mathrm{d}\Delta V(t)}{\mathrm{d}t} \neq 0$$

$$\Delta V(t) = \frac{\Delta V(0)}{1 + (e/\hbar)\boldsymbol{B} \cdot \boldsymbol{\Omega}(\boldsymbol{k})}$$



Non-uniform phase space

Phase volume of a quantum state:

$$(2\pi)^d \rightarrow \frac{(2\pi)^d}{1+(e/\hbar)\boldsymbol{B}\cdot\boldsymbol{\Omega}(\boldsymbol{k})}$$

Statistical Physics – Phase space measure:

$$D(\boldsymbol{r},\boldsymbol{k}) = \frac{1}{(2\pi)^d} \rightarrow D(\boldsymbol{r},\boldsymbol{k}) = \frac{1}{(2\pi)^d}\left(1+\frac{e}{\hbar}\boldsymbol{B}\cdot\boldsymbol{\Omega}(\boldsymbol{k})\right)$$

Physical quantity:

$$\langle\hat{O}\rangle = \int \mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{k} D(\boldsymbol{r},\boldsymbol{k})\mathcal{O}(\boldsymbol{r},\boldsymbol{k})f(\boldsymbol{r},\boldsymbol{k})$$

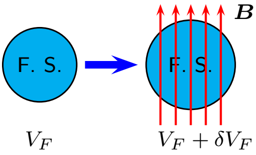
$f(\boldsymbol{r},\boldsymbol{k})$ — Distribution function

Physical effects

Fermi sea volume:

$$n_e = \int_{V_F + \delta V_F} \frac{d\boldsymbol{k}}{(2\pi)^d} \left(1 + \frac{e}{\hbar} \boldsymbol{B} \cdot \boldsymbol{\Omega}\right)$$

$$\delta V_F = -\frac{e}{\hbar} \int_{V_F} \boldsymbol{B} \cdot \boldsymbol{\Omega}$$

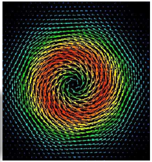


Luttinger's theorem breaks down!

Orbital magnetization:

$$E(\boldsymbol{B}) = \int_{V_F + \delta V_F} \frac{d\boldsymbol{k}}{(2\pi)^d} \left(1 + \frac{e}{\hbar} \boldsymbol{B} \cdot \boldsymbol{\Omega}\right) [\epsilon_0(\boldsymbol{k}) - \boldsymbol{B} \cdot \boldsymbol{m}(\boldsymbol{k})]$$

$$\boldsymbol{M} = -\frac{\partial E}{\partial \boldsymbol{B}} = \frac{e}{2\hbar} \int_{V_F} \frac{d\boldsymbol{k}}{(2\pi)^d} i \left\langle \frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} \right| \times \left[2\mu - \epsilon(\boldsymbol{k}) - \hat{H}\right] \left| \frac{\partial u_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} \right\rangle$$



Confirmed by: Thonhauser, Ceresoli, Vanderbilt, Resta, cond-mat/0505518

More physical effects

Density of states at Fermi surface and specific heat:

$$\rho(\mu, \boldsymbol{B}) = \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^d} \left(1 + \frac{e}{\hbar}\boldsymbol{B} \cdot \boldsymbol{\Omega}\right) \delta\left(\epsilon(\boldsymbol{k}) - \boldsymbol{B} \cdot \boldsymbol{m}(\boldsymbol{k}) - \mu\right)$$

$$C_e = \frac{\pi^2}{3} k_B^2 T \rho(\mu, \boldsymbol{B})$$

Magnetoresistivity in linear B:

$$\sigma_{xx} = e^2 \rho(\mu, \boldsymbol{B}) v_F^2(\mu, \boldsymbol{B}) \tau(\mu, \boldsymbol{B})$$

$$v_F(\mu, \boldsymbol{B}) = \frac{v_F^0(\mu) - \boldsymbol{B} \cdot [\partial \boldsymbol{m}(k_F)/\partial k_F]}{1 + (e/\hbar)\boldsymbol{B} \cdot \boldsymbol{\Omega}}$$

$$\frac{1}{\tau(\mu, \boldsymbol{B})} = \int_\mu \frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^d} \left(1 + \frac{e}{\hbar}\boldsymbol{B} \cdot \boldsymbol{\Omega}\right) W_{\boldsymbol{k}\boldsymbol{k}'} \left(1 - \frac{\boldsymbol{k} \cdot \boldsymbol{k}'}{k^2}\right)$$

Outline

Effective quantum mechanics

Conventional:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\mathcal{E}_n\left(-i\boldsymbol{\nabla} + \frac{e}{\hbar}\boldsymbol{A}\right) - e\phi(\boldsymbol{r})\right]\psi$$

With k-space magnetic field:

$$\hat{H} = \mathcal{E}_n(\hat{\boldsymbol{k}}) - e\phi(\hat{\boldsymbol{r}})$$

$$[\hat{x}_\mu, \hat{x}_\nu] = i\frac{\epsilon_{\mu\nu\gamma}\Omega_\gamma}{1 + (e/\hbar)\boldsymbol{B}\cdot\boldsymbol{\Omega}}$$

$$[\hat{k}_\mu, \hat{k}_\nu] = -i\frac{(e/\hbar)\epsilon_{\mu\nu\gamma}B_\gamma}{1 + (e/\hbar)\boldsymbol{B}\cdot\boldsymbol{\Omega}}$$

$$[\hat{x}_\mu, \hat{k}_\nu] = i\frac{\delta_{\mu\nu} + (e/\hbar)B_\mu\Omega_\nu}{1 + (e/\hbar)\boldsymbol{B}\cdot\boldsymbol{\Omega}}$$

$\boldsymbol{\Omega} = 0$ – Pierles substitution:

$$\hat{\boldsymbol{k}} \to -i\frac{\partial}{\partial\boldsymbol{r}} + \frac{e}{\hbar}\boldsymbol{A}(\boldsymbol{r})$$

$\boldsymbol{B} = 0$:

$$\hat{\boldsymbol{r}} \to i\frac{\partial}{\partial\boldsymbol{k}} + \boldsymbol{\mathcal{A}}_{\boldsymbol{k}}(\boldsymbol{k})$$

General case: Quantum mechanics in non-commutative geometry

Many-body Hamiltonian

Theory of everything:

$$\hat{H} = \sum_i \frac{[\hat{\boldsymbol{p}}_i + e\boldsymbol{A}(\boldsymbol{r}_i)]^2}{2m_e} + \sum_\mu \frac{[\hat{\boldsymbol{P}}_\mu - Z_\mu e\boldsymbol{A}(\boldsymbol{R}_\mu)]^2}{2M_\mu} - \sum_{i\mu} \frac{Z_\mu e^2}{|\boldsymbol{r}_i - \boldsymbol{R}_\mu|} + \sum_{i<j} \frac{e^2}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}$$

More realistic approach:

$$\hat{H} = \sum_i \frac{[\hat{\boldsymbol{p}}_i + e\boldsymbol{A}(\boldsymbol{r}_i)]^2}{2m_e^*} + \sum_{i<j} V_{ee}(\boldsymbol{r}_i - \boldsymbol{r}_j) + \hat{H}_{e-ph} + \hat{H}_{ph}$$

With the k-space magnetic field:

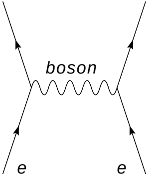
$$\hat{H} = \sum_i \frac{\hbar^2 \hat{\boldsymbol{k}}_i^2}{2m_e^*} + \sum_{i<j} V_{ee}(\hat{\boldsymbol{r}}_i - \hat{\boldsymbol{r}}_j) + \hat{H}_{e-ph}(\hat{\boldsymbol{r}}) + \hat{H}_{ph}$$

$$[\hat{\boldsymbol{r}}_i^\mu, \hat{\boldsymbol{r}}_i^\nu] \neq 0$$

Superconductivity – Boson exchange mechanism

Theory of superconductivity:

- Electrons develop attractive interaction through exchanging bosons (collective excitation).
- Cooper pairs

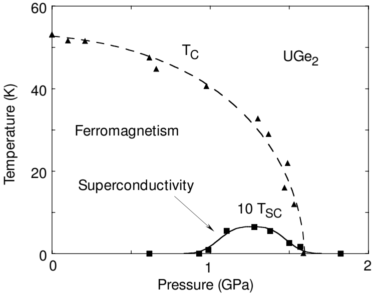


Bosonic excitations responsible for the superconductivity:

- Phonon – conventional BCS superconductivity
- Charge density wave – Kohn and Luttinger, PRL, 1965
- Spin fluctuation – ^{3}He; heavy-fermion superconductors; high-T_c superconductors? ...

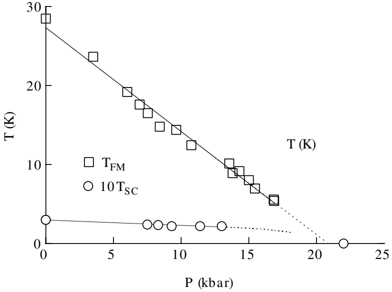
Ferromagnetic superconductors

UGe_2



Sexena et al., Nature, 2000

$ZrZn_2$

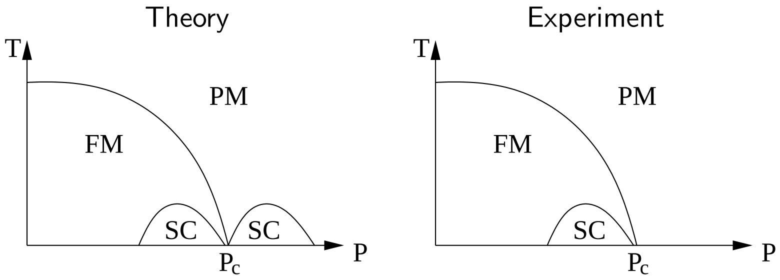


Pfleiderer et al., Nature, 2001

Prevailing picture: Attractive e-e interaction induced by the enhanced spin fluctuation near a quantum critical point.

Spin fluctuation?

Puzzle: The superconducting state is only observed in the ferromagnetic phase, not in the paramagnetic phase.



Alternative theory?

Effective quantum mechanics

Quantum mechanics in non-commutative geometry:

$$
\begin{aligned}
[\hat{x}_\mu, \hat{x}_\nu] &= i\frac{\epsilon_{\mu\nu\gamma}\Omega_\gamma}{1+(e/\hbar)\boldsymbol{B}\cdot\boldsymbol{\Omega}} \rightarrow i\epsilon_{\mu\nu\gamma}\Omega_\gamma \\
[\hat{k}_\mu, \hat{k}_\nu] &= -i\frac{(e/\hbar)\epsilon_{\mu\nu\gamma}B_\gamma}{1+(e/\hbar)\boldsymbol{B}\cdot\boldsymbol{\Omega}} \rightarrow 0 \\
[\hat{x}_\mu, \hat{k}_\nu] &= i\frac{\delta_{\mu\nu}+(e/\hbar)B_\mu\Omega_\nu}{1+(e/\hbar)\boldsymbol{B}\cdot\boldsymbol{\Omega}} \rightarrow i\delta_{\mu\nu}
\end{aligned}
$$

Zero B field limit:

$$\hat{x}_\mu \rightarrow i\frac{\partial}{\partial \boldsymbol{k}_\mu} + \mathcal{A}_\mu(\boldsymbol{k})$$

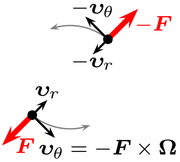
Electron-electron interaction:

$$V_{ee}(\boldsymbol{x}_i - \boldsymbol{x}_j) \rightarrow V_{ee}(\hat{\boldsymbol{x}}_i - \hat{\boldsymbol{x}}_j)$$

Electron-electron interaction – Classical picture

Scattering between two electrons:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \frac{1}{\hbar}\frac{\partial \mathcal{E}_n(\boldsymbol{k})}{\partial \boldsymbol{k}} - \boldsymbol{F}_{ee} \times \boldsymbol{\Omega}(\boldsymbol{k}) \\ \hbar\dot{\boldsymbol{k}} &= \boldsymbol{F}_{ee}(\boldsymbol{r} - \boldsymbol{r}') \end{aligned}$$



"Screw" scattering – transient pairing of electrons with finite angular momentum

Superconductivity with unconventional pairing?

Quantum approach – Second quantization

$$V_{ee}(\boldsymbol{x}_i - \boldsymbol{x}_j) \rightarrow v(\hat{\boldsymbol{x}}_i - \hat{\boldsymbol{x}}_j)$$

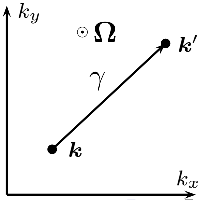
$$\hat{x}_\mu \rightarrow i\frac{\partial}{\partial \boldsymbol{k}_\mu} + \mathcal{A}_\mu(\boldsymbol{k})$$

$$V = \frac{1}{2\mathcal{V}} \sum_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{K}} u(\boldsymbol{k}, \boldsymbol{k}'; \boldsymbol{K}) c^\dagger_{\frac{\boldsymbol{K}}{2}+\boldsymbol{k}'} c^\dagger_{\frac{\boldsymbol{K}}{2}-\boldsymbol{k}'} c_{\frac{\boldsymbol{K}}{2}-\boldsymbol{k}} c_{\frac{\boldsymbol{K}}{2}+\boldsymbol{k}},$$

$$u(\boldsymbol{k}, \boldsymbol{k}'; \boldsymbol{K}) = v(\boldsymbol{k}' - \boldsymbol{k}) e^{i\gamma(\frac{\boldsymbol{K}}{2}+\boldsymbol{k}', \frac{\boldsymbol{K}}{2}+\boldsymbol{k}) + i\gamma(\frac{\boldsymbol{K}}{2}-\boldsymbol{k}', \frac{\boldsymbol{K}}{2}-\boldsymbol{k})}$$

Aharanov-Bohm phase under the $\boldsymbol{k}$-space magnetic field

$$\gamma(\boldsymbol{k}, \boldsymbol{k}') = \int_{\boldsymbol{k}}^{\boldsymbol{k}'} \boldsymbol{\mathcal{A}}(\boldsymbol{k}) \cdot \mathrm{d}\boldsymbol{k}$$



Application to a simple ferromagnetic metal

Model: An isotropic two-dimensional ferromagnetic metal with a constant topological field $\Omega_z(\boldsymbol{k}) = \Omega_0$

Bare electron-electron interaction:

$$v(\boldsymbol{r}) = V_0 \frac{\exp[-\kappa_{TF}(\sqrt{r^2+a^2}-a)]}{\sqrt{(r/a)^2+1}}$$

Berry phase:

$$\gamma(\boldsymbol{k},\boldsymbol{k}') = \frac{1}{2}\Omega_0 k k' \sin\theta$$

Effective e-e interaction:

$$u(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{K}=0) = v(|\boldsymbol{k}-\boldsymbol{k}'|)\exp\left[i\Omega_0 k k' \sin\theta\right] = \sum_m u_m(k,k')e^{im\theta}$$

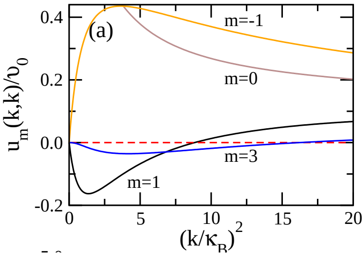
Effective electron-electron interaction

For channel m: $\kappa_u \equiv \kappa_{TF}/\sqrt{1+\kappa_{TF}a}$, $\phi_\Omega \equiv \Omega_0 \kappa_u^2$

$$u_m(k,k') \propto \begin{cases} I_m\left(\frac{kk'}{\kappa_u^2}\sqrt{1-\phi_\Omega^2}\right), & |\phi_\Omega| \le 1 \\ (-1)^m J_m\left(\frac{kk'}{\kappa_u^2}\mathrm{sgn}(\phi_\Omega)\sqrt{\phi_\Omega^2-1}\right), & |\phi_\Omega| > 1 \end{cases}$$

Attractive interaction:

- $|\Omega_0|\kappa_u^2 > 1$
- m is odd
- The angular momentum is parallel to the $\boldsymbol{\Omega}$-field.



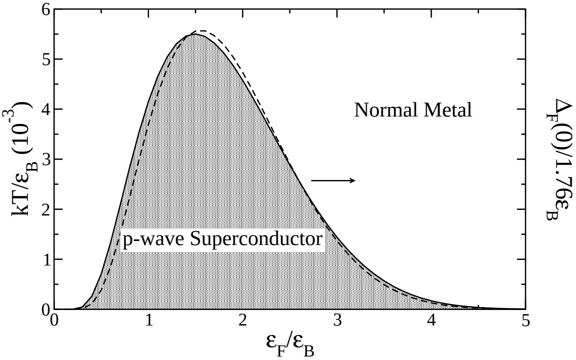
More realistic model:

$$\Omega(k) = \frac{\Omega_0 \kappa_B^2}{(k^2+\kappa_B^2)^2}$$

Superconductivity

$$\Delta(\boldsymbol{k}) = -\frac{1}{2}\sum_{\boldsymbol{k}'} u(\boldsymbol{k},\boldsymbol{k}')\frac{\tanh\frac{\beta\mathcal{E}(\boldsymbol{k}')}{2}}{\mathcal{E}(\boldsymbol{k}')}\Delta(\boldsymbol{k}')$$

Pairing is determined by the most attractive m-channel.



$$\Delta(\boldsymbol{k}) \sim k_x + ik_y$$

$$\frac{\Delta_F(0)}{kT_c} \approx 1.76$$

$$\Delta_F(0) \sim (\epsilon_F + \epsilon_B) \times \exp\left[-\frac{1}{\rho_F u(\epsilon_F,\epsilon_F)}\right]$$

Conclusion

Reciprocal space magnetic field:

- **Design novel transport devices** – anomalous Hall insulator
- **Inhomogeneous phase space** – Breakdown of Luttinger theorem; Fermi-surface related phenomena; orbital magnetization...
- **Quantum mechanics in non-commutative geometry** – Unconventional superconductivity in ferromagnetic metals...

Collaborators: Q. Niu and D. Xiao (UT-Austin)
Publication: Phys. Rev. Lett. **95**, 137204 (2005); More is coming

Conclusion

Reciprocal space magnetic field:

- **Design novel transport devices** – anomalous Hall insulator
- **Inhomogeneous phase space** – Breakdown of Luttinger theorem; Fermi-surface related phenomena; orbital magnetization...
- **Quantum mechanics in non-commutative geometry** – Unconventional superconductivity in ferromagnetic metals...

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Thank You For Your Attention!