HAMILTONIAN STRUCTURE OF A COLLISIONLESS RECONNECTION MODEL VALID FOR HIGH AND LOW β PLASMAS

E. TASSI*

Dipartimento di Energetica, Politecnico di Torino,
Torino, 10129, Italy
**E-mail: emanuele.tassi@polito.it*

P. J. MORRISON

Department of Physics and Institute for Fusion Studies, University of Texas
Austin, Texas 78712, United States

D. GRASSO

Dipartimento di Energetica, Politecnico di Torino,
Torino, 10129, Italy

The noncanonical Hamiltonian formulation of a recently derived four-field model describing collisionless reconnection is presented. The corresponding Lie-Poisson bracket is shown to be a sum of a direct and semi-direct product forms and to possess four infinite independent families of Casimir invariants. Three out of four of these families are directly associated with the existence of Lagrangian invariants of the model. Two of the invariants generalize previously discovered invariants of a two-field model for reconnection in low-β plasmas. Finally a variational principle is given for deriving general equilibrium equations and an example of an equilibrium solution is described explicitely.

Keywords: magnetic reconnection, noncanonical Hamiltonian systems

1. Introduction

Magnetic reconnection (MR) is a phenomenon of great relevance for both laboratory and astrophysical plasmas. In fact MR is believed to play a key role in events such as solar flares, magnetospheric substorms, and sawtooth oscillations in tokamaks [1,2]. Through the process of MR magnetic energy can be converted into kinetic and thermal energy of a plasma, concomitant with an alteration of the topology of the magnetic field. Whereas early works on MR adopted a plasma description based on collisional resistive magnetohydrodynamics (MHD), in subsequent years a great effort has been devoted also to investigating reconnection occurring in plasmas, such as the high-temperature tokamak plasmas where collisions can be neglected. In such plasmas, finite electron inertia can be responsible for the violation of the frozen-in condition that allows MR to take place. In the context of collisionless reconnection mediated by electron inertia, a fruitful line of research originated with the derivation of a three-field model [3] valid for low-β plasmas, where β is the ratio between plasma and magnetic pressure. Two-dimensional two-field reduced versions of this model were intensively investigated in a number of works [4–6]. More recently a collisionless four-field model that is valid not only for plasmas with $\beta \ll 1$ was derived [7]. This model makes it possible to investigate MR for a wider range of values of β and also for length scales comparable with the ion skin depth. Because this model is free from dissipative terms, a natural and important question is whether or not it can be cast into noncanonical Hamiltonian form [8], as is the case for other dissipation-free plasma models, such as for instance the one mentioned above [3]. Apart from its formal elegance, an Hamiltonian formulation lends information on the dynamics described by the system without the need for solving directly the model equations. In particular, for the two-field reconnection model knowledge of the Casimir invariants, obtained through the Hamiltonian formulation, made it possible to give an explanation for the formation of the cross-shaped structures in the current density and vorticity fields observed in numerical simulations of collisionless reconnection [5]. Moreover, the Hamiltonian formalism can greatly simplify the search for exact stationary solutions of the system and for sufficient conditions for formal stability [9].

2. Model equations

The four-field model derived by Fitzpatrick and Porcelli [7] reads

$$\frac{\partial(\psi - d_e^2\nabla^2\psi)}{\partial t} + [\varphi, \psi - d_e^2\nabla^2\psi] - d_\beta[\psi, Z] = 0, \tag{1}$$

$$\frac{\partial Z}{\partial t} + [\varphi, Z] - c_\beta[v, \psi] - d_\beta[\nabla^2\psi, \psi] = 0, \tag{2}$$

$$\frac{\partial \nabla^2\varphi}{\partial t} + [\varphi, \nabla^2\varphi] + [\nabla^2\psi, \psi] = 0, \tag{3}$$

$$\frac{\partial v}{\partial t} + [\varphi, v] - c_\beta[Z, \psi] = 0. \tag{4}$$

Equation (1) is a reduced Ohm's law where the presence of finite electron inertia, which makes it possible for MR to take place, is indicated by the terms proportional to the electron skin depth d_e. Equations (2), (3) and (4) are obtained from the electron vorticity equation, the ion vorticity equation, and the parallel ion momentum equation, respectively.

Considering a Cartesian coordinate system (x, y, z) and taking z as an ignorable coordinate, the fields ψ, Z, φ and v are related to the magnetic field $\mathbf{B}$ and to the ion velocity field $\mathbf{v}$ by the relations $\mathbf{B} = \nabla\psi\times\hat{\mathbf{z}}+(B^{(0)}+c_\beta Z)\hat{\mathbf{z}}$ and $\mathbf{v} = -\nabla\varphi \times \hat{\mathbf{z}} + v\hat{\mathbf{z}}$, respectively. Here $B^{(0)}$ is the constant guide field, whereas $c_\beta = \sqrt{\beta/(1+\beta)}$ and $d_\beta = d_i c_\beta$ with d_i indicating the ion skin depth. The ions are assumed to be cold, but electron pressure perturbations are taken into account and are given by $p = P^{(0)} + B^{(0)}p_1 + p_2$, with $P^{(0)}$ a constant background pressure, p_1 coupled to the magnetic field via the relation $p_1 \simeq -c_\beta Z$, and p_2, which at the lowest order is decoupled from the system. Notice that in this context the parameter β is defined as $\beta = (5/3)P^{(0)}/{B^{(0)}}^2$. In the above formulation all the quantities are expressed in a dimensionless form according to the following normalization: $\nabla = a\nabla$, $t = v_A t/a$, $\mathbf{B} = \mathbf{B}/B_p$, where a is a typical scale length of the problem, B_p is a reference value for the poloidal magnetic field, and v_A is the Alfvén speed based on B_p and on the constant ion density. Finally, we specify that $[f, g] = \nabla f\times\nabla g\cdot\hat{\mathbf{z}}$, for generic fields f and g. Notice that in the limit of perfectly conducting plasma (i.e. $d_e = 0$) the above model is equivalent to the Hamiltonian model derived in [10] when field line curvature is neglected.

3. Hamiltonian formulation

Dissipation-free fluid models for plasmas admit a noncanonical Hamiltonian formulation [8]. In short this means that it is possible to reformulate an n-field model as

$$\frac{\partial\zeta_i}{\partial t} = \{\zeta_i, H\}, \qquad i = 1, \cdots, n \tag{5}$$

where ζ_i are suitable field variables, H is the Hamiltonian functional, and $\{,\}$ is the Poisson bracket consisting of an antisymmetric bilinear form satisfying the Jacobi identity.

One way to derive a noncanonical Hamiltonian formulation is to proceed by first searching for a conserved functional that is a natural candidate for the Hamiltonian of the model. If one considers for instance a squared domain

$\mathcal{D}$ in the xy plane with doubly periodic boundary conditions, the four-field model (1)–(4) admits the following constant of motion:

$$H = \frac{1}{2}\int_{\mathcal{D}} d^2x(d_e^2 J^2 + |\nabla\varphi|^2 + v^2 + |\nabla\psi|^2 + Z^2) \tag{6}$$

with $J = -\nabla^2\psi$ indicating the parallel current density. The quantity H represents the total energy of the system. The first term refers to the kinetic energy due to the relative motion of the electrons with respect to ions along the z direction. The second and the third terms account for the kinetic ion energy, whereas the last two terms refer to the magnetic energy. Adopting $\psi_e = \psi - d_e^2\nabla^2\psi$, $U = \nabla^2\varphi$, Z, and v as field variables and (6) as Hamiltonian, it is possible to show that the model can indeed be cast in a noncanonical Hamiltonian form with Poisson bracket, of Lie-Poisson type, defined as

$$\begin{aligned}
\{F,G\} = \int d^2x\, &(U[F_U, G_U] + \psi_e([F_{\psi_e}, G_U] \\
&+[F_U, G_{\psi_e}] - d_\beta([F_Z, G_{\psi_e}] + [F_{\psi_e}, G_Z]) + c_\beta([F_v, G_Z] + [F_Z, G_v])) \\
&+Z([F_Z, G_U] + [F_U, G_Z] - d_\beta {d_e}^2[F_{\psi_e}, G_{\psi_e}] + c_\beta {d_e}^2([F_v, G_{\psi_e}] + [F_{\psi_e}, G_v]) \\
&-\alpha[F_Z, G_Z] - c_\beta\gamma[F_v, G_v]) + v([F_v, G_U] + [F_U, G_v] \\
&+c_\beta {d_e}^2([F_Z, G_{\psi_e}] + [F_{\psi_e}, G_Z]) - c_\beta\gamma([F_v, G_Z] + [F_Z, G_v]))\big)\,,
\end{aligned} \tag{7}$$

where $\alpha = d_\beta + c_\beta {d_e}^2/d_i$, $\gamma = {d_e}^2/d_i$, and subscripts indicate functional differentiation.

4. Casimir invariants

Lie-Poisson brackets for noncanonical Hamiltonian systems are characterized by the presence of Casimir invariants. A Casimir invariant is a functional that annihilates the Lie-Poisson bracket when paired with any other functional, i.e. a Casimir C satisfies

$$\{F, C\} = 0, \tag{8}$$

for every functional F. Thus Casimir invariants constraints the nonlinear dynamics generated by the Poisson bracket for any choice of Hamiltonian. For the derivation of the Casimirs of the four-field model we can proceed in the following way. First, multiplying Eq. (4) times d_i and adding it to Eq. (1) yields

$$\frac{\partial D}{\partial t} + [\varphi, D] = 0, \tag{9}$$

where $D = \psi_e + d_i v$. Equation (9) indicates that the field D is a Lagrangian invariant that remains constant along the contour lines of φ. The presence of this Lagrangian invariant also suggests that using D as one of the variables will simplify the Lie-Poisson bracket. Indeed, upon replacing ψ_e with D as field variable, Eq. (8) for the four-field model reads

$$\begin{aligned}\{F, C\} = \int d^2x\, (&F_U[C_U, U] + F_D[C_U, D] + F_U[C_D, D] \\ &+ c_\beta F_v[C_Z, D] + c_\beta F_Z[C_v, D] + F_Z[C_U, Z] + F_U[C_Z, Z] \\ &- \alpha F_Z[C_Z, Z] - c_\beta \gamma F_v[C_v, Z] + F_v[C_U, v] + F_U[C_v, v] \\ &- \alpha F_v[C_Z, v] - \alpha F_Z[C_v, v]) = 0.\end{aligned} \tag{10}$$

After integrating by parts, collecting the terms multiplying the same functional derivatives of F, and using the arbitrariness of F one obtains the following system of equations for C:

$$[C_U, D] = 0, \tag{11}$$

$$[C_U, U] + [C_D, D] + [C_Z, Z] + [C_v, v] = 0, \tag{12}$$

$$-c_\beta[C_v, D] - [C_U, Z] + \alpha([C_Z, Z] + [C_v, v]) = 0, \tag{13}$$

$$c_\beta[C_Z, D] - c_\beta \gamma [C_v, Z] + [C_U, v] - \alpha[C_Z, v] = 0. \tag{14}$$

A functional integration of (11) yields that C can be of the form

$$C(U, D, Z, v) = \int d^2x (U\mathcal{F}(D) + g(D, Z, v)), \tag{15}$$

where $\mathcal{F}$ and g represent arbitrary functions of their arguments. Equation (12) is automatically satisfied for any choice of C with an integrand that depends only upon the field variables and not their spatial derivatives, and therefore imposes no constraints. Using (12) and substituting (15) into (13) yields

$$\left(-c_\beta \frac{\partial^2 g}{\partial v^2} - \alpha \frac{\partial^2 g}{\partial v \partial D}\right)[v, D] - \left(c_\beta \frac{\partial^2 g}{\partial v \partial Z} + \mathcal{F}'(D) + \alpha \frac{\partial^2 g}{\partial D \partial Z}\right)[Z, D] = 0, \tag{16}$$

where $'$ indicates derivative with respect to the argument of the function. In the latter expression the coefficients multiplying the brackets '[,]' must vanish independently. This leads to the relation

$$c_\beta \frac{\partial g}{\partial v} + \alpha \frac{\partial g}{\partial D} = Z\mathcal{F}'(D) + K(D), \tag{17}$$

with K an arbitrary function of D. Analogously, (14), (12), and (15) yield

$$\left(-c_\beta \frac{\partial^2 g}{\partial Z \partial v} + \mathcal{F}'(D) - \alpha \frac{\partial^2 g}{\partial Z \partial D} \right) [D, v] - \left(c_\beta \frac{\partial^2 g}{\partial Z^2} + c_\beta \gamma \frac{\partial^2 g}{\partial v \partial D} \right) [D, Z] \\ - \left(c_\beta \gamma \frac{\partial^2 g}{\partial v^2} - \alpha \frac{\partial^2 g}{\partial Z^2} \right) [v, Z] = 0, \tag{18}$$

which leads to

$$c_\beta \frac{\partial g}{\partial v} + \alpha \frac{\partial g}{\partial D} = -ZW(v, D) + Y(v, D) \tag{19}$$

with W and Y arbitrary functions. A comparison of (17) with (19) leads to

$$W(v, D) = -\mathcal{F}'(D), \qquad Y(v, D) = K(D). \tag{20}$$

If one chooses g such that $g_v = 0$, then, upon integration of (17) with respect to D, one obtains

$$g(D, Z) = \frac{Z}{\alpha} \mathcal{F}(D) + \mathcal{H}(D) \tag{21}$$

where $\mathcal{H}' = K(D)$. This allows us to identify the two independent infinite families of Casimirs given by the following:

$$C_1 = \int d^2x \left(U + \frac{Z}{\alpha} \right) \mathcal{F}(D), \tag{22}$$

$$C_2 = \int d^2x \mathcal{H}(D). \tag{23}$$

If one sets $\mathcal{F}(D) = 0$ and $\mathcal{H}(D) = 0$, then the general solution for g becomes

$$g = g_+ \left(\chi - \sqrt{\frac{\gamma\alpha}{c_\beta}} Z \right) + g_- \left(\chi + \sqrt{\frac{\gamma\alpha}{c_\beta}} Z \right), \tag{24}$$

where $\chi = D - (\alpha/c_\beta)v$ and $g_\pm$ are arbitrary functions of their arguments. Therefore two additional independent infinite families of Casimirs are given by

$$C_3 = \int d^2x g_+ \left(D - \frac{\alpha}{c_\beta} v - \sqrt{\frac{\gamma\alpha}{c_\beta}} Z \right) = \int d^2x g_+(T_+), \tag{25}$$

$$C_4 = \int d^2x g_- \left(D - \frac{\alpha}{c_\beta} v + \sqrt{\frac{\gamma\alpha}{c_\beta}} Z \right) = \int d^2x g_-(T_-). \tag{26}$$

Knowledge of the functional dependence of the Casimirs suggests a simplification of the Lie-Poisson bracket will occur if the Poisson bracket is written in terms of the new coordinates

$$D = D, \tag{27}$$

$$\omega = U + \frac{Z}{\alpha}, \tag{28}$$

$$T_+ = \psi_e - \gamma v - \sqrt{\frac{\gamma\alpha}{c_\beta}} Z, \tag{29}$$

$$T_- = \psi_e - \gamma v + \sqrt{\frac{\gamma\alpha}{c_\beta}} Z. \tag{30}$$

Indeed, in the new coordinates the Lie-Poisson bracket reads

$$\begin{aligned} \{F, G\} = \int d^2x (\omega[F_\omega, G_\omega] + D([F_D, G_\omega] + [F_\omega, G_D]) \\ + T_-[F_{T_-}, G_{T_-}] + T_+[F_{T_+}, G_{T_+}]). \end{aligned} \tag{31}$$

This form reveals the algebraic structure of the Lie-Poisson bracket, which can be identified as a sum of direct product and semi-direct product forms [9,11]. Making use of the coordinates suggested by the form of the Casimirs, the model equations can be rewritten in the compact form

$$\frac{\partial D}{\partial t} = -[\varphi, D], \tag{32}$$

$$\frac{\partial \omega}{\partial t} = -[\varphi, \omega] + \frac{1}{{d_e}^2 + {d_i}^2}[D, \psi], \tag{33}$$

$$\frac{\partial T_+}{\partial t} = -\left[\varphi + \frac{d_\beta}{d_e} \sqrt{1 + \frac{d_e^2}{d_i^2}} \psi, T_+ \right], \tag{34}$$

$$\frac{\partial T_-}{\partial t} = -\left[\varphi - \frac{d_\beta}{d_e} \sqrt{1 + \frac{d_e^2}{d_i^2}} \psi, T_- \right] \tag{35}$$

with $\omega = U + Z/\alpha$ a "generalized" vorticity. This formulation displays the existence of the three Lagrangian invariants D, T_+ and T_- associated with the families of Casimirs C_2, C_3 and C_4, respectively. The existence of such invariants implies that the values of D, T_+ and T_- remain constant on the contour lines of φ, $\varphi + \frac{d_\beta}{d_e}\sqrt{1+\frac{d_e^2}{d_i^2}}\psi$, and $\varphi - \frac{d_\beta}{d_e}\sqrt{1+\frac{d_e^2}{d_i^2}}\psi$, respectively. It implies also that the area enclosed by the contour lines of the Lagrangian invariants remains constant. Notice also that T_+ and T_- in the limit $\beta \to 0$ and $d_i \to \infty$ tend to the Lagrangian invariants $G_\pm = \psi - d_e^2\nabla^2\psi \pm d_e\rho_s U$ of the two-field model derived in [3]. The family C_1 is of a different nature and one of the constraints imposed by it is that the total value of ω within an area enclosed by a contour line of D remains constant.

5. Equilibria

The knowledge of the Casimir invariants makes it possible to construct a variational principle [9] that can greatly simplify the search for exact equilibrium solutions of the system. Indeed setting to zero the first variation of the free energy functional $F = H + C_1 + C_2 + C_3 + C_4$ yields

$$-d_e^2\nabla^2\psi + \psi = D - d_i v(\psi, D) \tag{36}$$

$$\frac{D-\psi}{d_i^2+d_e^2} + \mathcal{F}'(D)\nabla^2\mathcal{F}(D) + \frac{d_i\,\mathcal{F}'(D)}{c_\beta(d_i^2+d_e^2)}Z(\psi, D) + \mathcal{H}'(D) = 0, \tag{37}$$

with $v(\psi, D)$ and $Z(\psi, D)$ given by

$$\begin{aligned} v = \frac{d_i}{d_i^2+d_e^2}\Bigg[D - \frac{1}{2}\Bigg(h_+^{-1}\left(\frac{c_\beta\sqrt{d_i^2+d_e^2}}{2d_e}\psi + \frac{\mathcal{F}(D)}{2}\right) \\ -h_-^{-1}\left(\frac{c_\beta\sqrt{d_i^2+d_e^2}}{2d_e}\psi - \frac{\mathcal{F}(D)}{2}\right)\Bigg)\Bigg] \end{aligned} \tag{38}$$

$$\begin{aligned} Z = -\frac{d_i}{d_e}\frac{1}{\sqrt{d_i^2+d_e^2}}\Bigg[\frac{1}{2}\Bigg(h_+^{-1}\left(\frac{c_\beta\sqrt{d_i^2+d_e^2}}{2d_e}\psi + \frac{\mathcal{F}(D)}{2}\right) \\ +h_-^{-1}\left(\frac{c_\beta\sqrt{d_i^2+d_e^2}}{2d_e}\psi - \frac{\mathcal{F}(D)}{2}\right)\Bigg)\Bigg] \end{aligned} \tag{39}$$

where h_+ and h_- are arbitrary invertible functions of their arguments. At equilibrium, the relations

$$T_\pm = \pm h_\pm^{-1}\left(\frac{c_\beta\sqrt{d_i^2+d_e^2}}{2d_e}\psi \pm \frac{\mathcal{F}(D)}{2}\right) \tag{40}$$

also hold. Given the freedom in choosing the forms for $\mathcal{F}$, $\mathcal{H}$, h_+, and h_- it emerges that deriving exact solutions using the above variational principle

is considerably easier than solving the original system (1)–(4) with the time derivatives set to zero. Once the choice for the free functions $\mathcal{F}(D)$, $\mathcal{H}(D)$, $h_+^{-1}\left(\frac{c_\beta\sqrt{d_i^2+d_e^2}}{2d_e}\psi+\frac{\mathcal{F}(D)}{2}\right)$, and $h_-^{-1}\left(\frac{c_\beta\sqrt{d_i^2+d_e^2}}{2d_e}\psi-\frac{\mathcal{F}(D)}{2}\right)$ (or equivalently $g_+(T_+)$ and $g_-(T_-)$) is made, the problem amounts to solving the system (36)–(37) for ψ and D. The corresponding equilibrium solutions for v and Z are then simply obtained from (38)–(39).
A possible choice for the free functions is the following:

$$h_+(T_+)=\lambda T_+, \qquad h_-(T_-)=-\lambda T_-, \qquad \mathcal{F}(D)=D, \qquad \mathcal{H}'(D)=\tilde{d}D \tag{41}$$

with constants λ and $\tilde{d}$. For this choice one has

$$v=\frac{d_i}{d_i^2+d_e^2}\left(D-\frac{c_\beta\sqrt{d_i^2+d_e^2}}{2d_e\lambda}\psi\right), \qquad Z=-\frac{d_i}{2\lambda d_e}\frac{D}{\sqrt{d_i^2+d_e^2}}, \tag{42}$$

with D and ψ solutions of

$$\nabla^2\psi=a(\tilde{\lambda})\psi-bD, \tag{43}$$

$$\nabla^2 D=b\psi+d(\tilde{d})D \tag{44}$$

where $b=1/(d_i^2+d_e^2)$ and $a(\lambda)$ and $d(\tilde{d})$ are arbitrary constants. A particularly simple example of solution corresponds to $D=C\psi$, with $C=-\frac{1}{2b}\left(d-a\pm\sqrt{d^2-2ad+a^2-4b^2}\right)$, and ψ a solution of

$$\nabla^2\psi=(a(\lambda)-bC)\psi. \tag{45}$$

Considering a circular domain of unit radius and adopting polar coordinates (r,θ), the flux function admits the following solution

$$\psi(r,\theta)=C_1 J_1(\sqrt{bC-a(\lambda)}r)\cos\theta, \tag{46}$$

with dipolar structure. The corresponding equilibrium solutions for the fields D, v, and φ will be simply linear functions of ψ. Notice that the boundary conditions in this case imply that the choice of the arbitrary constants d and a must be such that $J_1(\sqrt{bC-a(\lambda)})=0$.

6. Conclusions

The four-field model derived in [7] has been shown to admit a noncanonical Hamiltonian formulation. The corresponding Lie-Poisson bracket is characterized by four independent infinite families of Casimir invariants. The families associated with the invariants $T_\pm$ generalize the families related to $G_\pm$ of the low-β two-field model derived in [3]. A natural question that is under investigation is whether the invariants $T_\pm$ play a role analogous to the one played by $G_\pm$ in the two-field limit in determining the alignment of current density and vorticity along the separatrices of the magnetic field during the nonlinear evolution of the system [5]. The problem of accessibility to a saturated state is also under investigation, in order to extend to this model the analysis carried out in [6]. By means of a variational principle the problem of finding exact equilibrium solutions has been reduced to the problem of solving a system of coupled partial differential equations possessing two arbitrary functions of D. Choosing the arbitrary functions to be linear functions of their arguments, the problem becomes linear and was shown to admit, in a specific case, solutions with dipolar structures.

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