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Dynamically accessible perturbations and magnetohydrodynamic stability
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In Ref. 1, the stability conditions of magnetized plasma flows were obtained by exploiting the Hamiltonian structure of the magnetohydrodynamics (MHD) equations. Three kinds of energy principles were considered: an energy principle in Lagrangian variables, an energy-Casimir principle for symmetric equilibria in an Eulerian variable noncanonical formulation of MHD, and, finally, an energy principle based on dynamically accessible variations, i.e., on variations that explicitly preserve invariants of the system. General criteria for stability were obtained, along with comparisons between the three different approaches.

In particular, it was shown that the stability of helically-symmetric equilibria can be assessed by considering the second variation of the energy-Casimir functional

\[ \delta^2 \tilde{\mathcal{V}}[Z] = \int_V \left[ \rho \left( \frac{\partial v}{\partial \rho} \right)^2 + 2 \left( v - \frac{1}{k} \mathbf{h} \right) \cdot \left( \frac{\partial v}{\partial \mathbf{B}} \right) + \left( \rho U_{\rho \rho} + 2U_{\rho} \right) \left( \frac{\partial v}{\partial \rho} \right)^2 \right. \]

\[ - \mathcal{F} \cdot \left( \frac{\partial \mathbf{B}}{\partial \mathbf{B}} \right) - 2 \left( \mathcal{F}' \mathbf{B} + \frac{1}{k} \rho \mathcal{G} \mathbf{h} \right) \cdot \left( \frac{\partial \mathbf{B}}{\partial \mathbf{B}} \right) \]

\[ + 2 \left( \rho U_{\rho \rho} S' + U_{\rho} S' - J' - \frac{1}{k} \rho v_0 \mathcal{G} \right) \left( \frac{\partial \mathbf{B}}{\partial \mathbf{B}} \right) \cdot \left( \frac{\partial \mathbf{B}}{\partial \mathbf{B}} \right) \]

\[ + \left( \rho T S'' + \rho U_{\rho} S'' - \rho J'' - kB_0 \mathcal{H} - \left( k^2 |\sin 2x| \mathcal{H}' - \frac{1}{k} \rho v_0 \mathcal{G}'' \right) \cdot \mathbf{B} \cdot \mathcal{F}' \right) \left( \frac{\partial \mathbf{B}}{\partial \mathbf{B}} \right) \]

\[ \left. \left( \frac{\partial \mathbf{B}}{\partial \mathbf{B}} \right) \cdot \left( \frac{\partial \mathbf{B}}{\partial \mathbf{B}} \right) \right] \right] d^3 r. \]  

(2)

Although the above expression is given correctly in Ref. 1, when reformulating it in terms of the physical equilibrium quantities, an algebraic error was made that affects Eq. (74) of Ref. 1 and modifies the form of the following expressions in Sec. III. In the present erratum, we provide the correct form of these equations. In addition, a few notational changes are introduced to make the present text more self-contained.

By grouping as in Ref. 1 the variation of the plasma equilibrium quantities \( \delta \rho, \delta v, \delta \mathbf{B}, \delta \psi \) into convenient linear combinations \( \delta \mathbf{S}, \delta \mathbf{Q}, \delta \mathbf{R}_\perp, \delta \mathbf{R}_\parallel, \) and \( \delta \psi \), but keeping the components of \( \delta \mathbf{R} \) along the symmetry direction \( \mathbf{h} \) and perpendicular to it separate, after some algebra, we obtain the following from Eq. (1):

\[ \delta^2 \tilde{\mathcal{V}}[Z] = \int_V \left[ a_1 |\delta \mathbf{S}|^2 + a_2 (\delta \mathbf{Q})^2 + a_3 (\delta \mathbf{R}_\parallel)^2 + a_4 |\delta \mathbf{R}_\perp|^2 \right. \]

\[ + a_5 (\delta \psi)^2 \right] d^3 r, \]  

(3)

where
\[ a_1 = \frac{1}{\rho}, \quad a_2 = \frac{\rho}{c_s^2 - M^2 c_a^2}, \]
\[ a_3 = \frac{4\pi (c_s^2 - M^2 c_a^2)}{c_s^2 - M^2 (c_s^2 + c_a^2) + \frac{M^2}{4\pi \rho} B^2}, \]
\[ a_4 = \frac{4\pi}{(1 - M^2)} \frac{c_s^2 - M^2 (c_s^2 + c_a^2) + \frac{M^2}{4\pi \rho} B^2}{c_s^2 - M^2 (c_s^2 + c_a^2)} \]
\[ a_5 = -\gamma - a_2 \left| \frac{\delta S}{\delta \psi} \right|^2 \frac{\delta Q}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta R_h}{\delta \psi} \left| \frac{\delta \psi}{\delta \psi} \right|^2 \]
\[ \left( \frac{\delta \psi}{\delta \psi} \right)^2 \left( \frac{\delta \psi}{\delta \psi} \right)^2 \]
where, in the last equation, \( M_2 \) yields
\[ M_2 = \frac{1}{\rho} \frac{\rho}{c_s^2 - M^2 c_a^2}, \]
\[ \frac{4\pi (c_s^2 - M^2 c_a^2)}{c_s^2 - M^2 (c_s^2 + c_a^2) + \frac{M^2}{4\pi \rho} B^2} \]
\[ \frac{4\pi}{(1 - M^2)} \frac{c_s^2 - M^2 (c_s^2 + c_a^2) + \frac{M^2}{4\pi \rho} B^2}{c_s^2 - M^2 (c_s^2 + c_a^2)} \]
\[ \frac{M^2}{4\pi \rho} \frac{B^2}{c_s^2 - M^2 (c_s^2 + c_a^2)} \]
\[ \frac{2\frac{\delta S}{\delta \psi} \frac{\delta Q}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta R_h}{\delta \psi} \left| \frac{\delta \psi}{\delta \psi} \right|^2 \left( \frac{\delta \psi}{\delta \psi} \right)^2 \}
\[ M := \frac{1}{k^2} \frac{G' G^* + J' - (\rho U_{\rho s} + U_s) S - \frac{F F'}{\rho} B^2}{k^2} \]
\[ N := k H^* + \frac{1}{k} F G' + \frac{1}{k} F G' \]
\[ \text{while for } \delta R \text{, we obtain} \]
\[ \delta R = \frac{1}{a_4} \frac{\delta B}{\delta \psi} + \frac{M^2}{a_4 \sqrt{4\pi \rho}} \left( \frac{c_s^2 - M^2 (c_s^2 + c_a^2)}{4\pi \rho} \right) \]
\[ - \frac{2}{\rho} \frac{\delta S}{\delta \psi} \frac{\delta Q}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta R_h}{\delta \psi} \left| \frac{\delta \psi}{\delta \psi} \right|^2 \]
\[ \left( \frac{\delta \psi}{\delta \psi} \right)^2 \left( \frac{\delta \psi}{\delta \psi} \right)^2 \]
and we have defined the following functions of the equilibrium configuration:
\[ N := k H^* + \frac{1}{k} F G' + \frac{1}{k} F G' \]
Note that Eq. (3) differs from Eq. (74) of Ref. 1 insofar as the term proportional to \( \delta R \) in Eq. (74) is now split into two terms with different coefficients (\( a_3 \) and \( a_4 \)) that take the place of, and correct, the coefficient \( a_3 \) of Eq. (74) in Ref. 1.
Setting the variations \( \delta S, \delta Q, \) and \( \delta R_h \) to zero (as follows from the minimization of \( \delta^2 \delta^2[Z] \), provided \( a_1, a_2, \) and \( a_3 \) are positive) yields
\[ \delta^2 \delta^2[Z] = \int_V \left[ \frac{1}{a_4} \frac{\delta \psi}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta \psi}{\delta \psi} \frac{\delta \psi}{\delta \psi} \right] d^3 r. \]
\[ b_1 = k^2 \frac{1 - M^2}{4\pi} \frac{c_s^2 - M^2(c_s^2 + c_t^2)}{c_s^2 - M^2(c_s^2 + c_t^2) + \frac{M^4}{4\pi\rho} B_\perp^2}, \]

\[ b_2 = -Y - a_1 \left( \delta S \right)_{\hat{z}_s}^2 - a_2 \left( \delta Q \right)_{\hat{z}_s}^2 - a_3 \left( \delta R_{\delta} \right)_{\hat{z}_s}^2 + \nabla \cdot \left( \frac{\delta R_{\perp}}{\delta \psi} \times k\mathbf{h} \right), \]

\[ b_3 = k^2 \frac{1 - M^2}{4\pi} \frac{\frac{M^4}{4\pi\rho} B_\perp^2}{c_s^2 - M^2(c_s^2 + c_t^2) + \frac{M^4}{4\pi\rho} B_\perp^2}. \]

We note that

\[ k^2 \frac{1 - M^2}{4\pi} = b_1 + b_3, \]

which can be convenient if we rewrite Eq. (19) as

\[ \delta^2 \tilde{\gamma}[\mathcal{Z}] = \int_{\mathcal{V}} \left[ b_1 |\nabla \perp \delta \psi|^2 + b_2 (\delta \psi)^2 \right. \]

\[ + (b_1 + b_3) |\nabla \delta \psi \times e_\psi|^2]d^3r, \]

where $\nabla \perp \delta \psi$ denotes the component of the gradient of the variation $\delta \psi$ perpendicular to the equilibrium flux function $\psi$.

The Euler-Lagrange equation associated with the extrema of Eq. (19) is

\[ \nabla \cdot \left[ b_1 I + b_3 (I - e_\psi e_\psi) \right] : \nabla \delta \psi - b_2 \delta \psi = 0, \]

where $I$ is the identity tensor and $(I - e_\psi e_\psi)$ is the projector on the tangent plane to the $\psi$-surfaces. Equation (26) should take the place of Eq. (81) in Ref. 1—it represents the generalized form of the Newcomb equation$^2$ for MHD symmetric equilibria with flow.

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