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CURRENT COLLECTION BY A LONG WIRE IN NEAR-EARTH ORBIT*

P. J. Morrison and W. B. Thompson
Department of Physics
and P. R. Williamson**
Department of Applied Physics and Information Science
University of California
San Diego, La Jolla, California 92093
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ABSTRACT

Investigation of the current collected by a long wire in space has application to long antennas and the proposed space shuttle tethered subsatellite. Langmuir's result for current collection by a moving probe in a plasma is used to obtain expressions for the voltage and current as functions of position along a wire. Two cases are considered: firstly, one end of the wire is grounded to the plasma and secondly, the wire is allowed to assume a natural grounding point. Results are obtained as a function of the wire resistivity, length and diameter for various particle densities. Calculations for a 2mm diameter copper wire show that a current of 0.066 amperes of oxygen ions will be collected by a tether of 10 km in length.

I. INTRODUCTION

Several experiments proposed for the space shuttle will require long wires to be deployed from the space shuttle orbiter while in a near-earth orbit. Among these experiments are VLF antennas and tethered sub-satellites with wires extending on the order of 10 to 100 km from the vehicle. A significant potential difference between a long wire and the ambient plasma will develop because of the induced emf due to motion across the magnetic field lines of the earth. If the wire is bare, current will be collected from the plasma and flow in the wire.

In section II we describe our coordinate system, motivate the appropriate current-voltage relation for leakage current and obtain differential equations for the current and voltage as a function of position along a wire. The following two sections deal with the tethered subsatellite and a free floating wire respectively. Numerical results are obtained.

II. GENERAL DISCUSSION

We assume that a wire points radially outward[1] while in an eastward directed equatorial orbit with velocity v. The wire determines a coordinate axis with the point z=0 corresponding to the point of attachment to some spacecraft and z increasing away from the earth. For an equatorial orbit the geomagnetic filed, $\underline{B}$, is assumed to be constant in magnitude and direction. The vectors, $\underline{B}$, $\underline{z}$, and $\underline{v}$ determine an orthogonal coordinate system (Fig. 1).

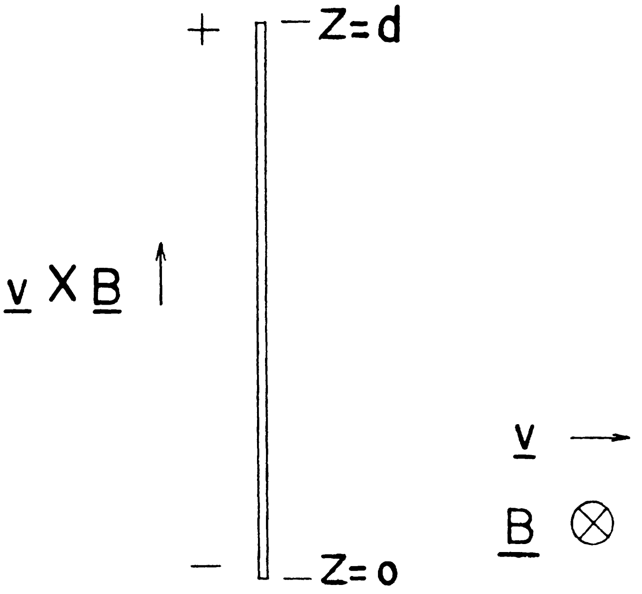

Figure 1. Coordinate system formed by $\underline{v}$, $\hat{z}$, and $\underline{B}$.

The motion of the wire in the magnetic field gives rise to a $(\underline{v}X\underline{B})\cdot\underline{z}$ induced emf in the wire. Since $\underline{v}X\underline{B}$ is directed in the positive $\underline{z}$ direction, the outer end obtains a positive charge; electrons are driven downward. It is apparent that the electric field due to charge separation in the wire, as seen from the plasma reference frame, has a component directed in the negative $\underline{z}$ direction. Hence, $dV/dz>0$ and $V(d)>V(0)$, where V is the wire potential relative to the ambient plasma potential and d is the outer end of the wire. This change in potential per unit length, dV/dz, is equal to the difference between the emf and the resistive loss per unit length. Since the emf must exceed the resistive loss,

$$\frac{dV}{dz} = \mathcal{E} - \Lambda i \tag{1}$$

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** Present address: Center for Research in Aeronomy, UMC 41, Utah State University, Logan, Utah 84322.

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Here $\mathcal{E} = (\underline{v} \times \underline{B}) \cdot \hat{z}$, $\hat{z}$ being a unit vector in the $\underline{z}$ direction, Λ is the resistance per unit length of the wire, and i is the current in the wire.

The current in eq. (1) includes leakage from the plasma. In order to ascertain the effect of this leakage current it is necessary to utilize the current-voltage relation for a long moving probe. The appropriate expression is obtained from Langmuir probe theory within the following approximations:

$$\frac{|eV|}{kT} \gg 1, \quad \frac{|eV|}{\frac{1}{2} m_+ v^2} \gg 1, \quad \frac{R_s}{R} \gg 1 \tag{2}$$

where e is the electron charge, k is Boltzmann's constant, T is the ambient plasma temperature, R is the wire radius, R_s is the characteristic length of the sheath, and m_+ is the mass of the positive ion present in the plasma (here O^+).

Large Potential Approximations. Here we consider the first and second inequalities of (2). For a near-earth orbit (~400 km), we assume $\mathcal{E}$ =0.2 volts/meter and $T = 10^3$ °K ($kT/e = 0.09$ volts). It is seen that, except for at most a small portion of the wire about a point where the potential is zero, $|eV|/kT \gg 1$. (See Figs. 2, 4 and 7.) For most of the wire the potential is quite large. A 10 km wire grounded at one end will attain, at the other end, a potential near 2 kv.

The current collected by a probe is composed of two components: one due to collection of accelerated charged particles and the other due to retarded charged particles of the opposite sign. If the potential is large the retarded current contribution will be small; we neglect it.

The second approximation $|eV|/\frac{1}{2}m_+v^2$ 1, compares the swept up kinetic energy of the positive ions to the wire's potential. For a wire speed of 7 km/sec and $m_+ = 16$ amu, $\frac{1}{2}m_+v^2/e \sim 4$ volts. Although this approximation is not as good as the previous one, it still holds for all but a small portion of the wire. Again, it breaks down near a point where V is zero. (The validity of this approximation also assures that $|eV|/\frac{1}{2}m_-v^2 \gg 1$.)

Large Sheath Approximation. Here, we consider the last inequality of (2). This approximation is valid if the Debye shielding length, λ_D is large compared to the wire radius. If we assume a density, n, of $2 \times 10^{11}\,m^{-3}$ for oxygen ions and electrons then $\lambda_D = 5 \times 10^{-3}$ m. A typical wire radius is about 10^{-3} m, so $\lambda_D \sim 5$ R. Actually, since the sheath radius increases with potential,[2] R_s may be many Debye lengths.

The appropriate current-voltage relation under these conditions is:[3,4]

$$J_p = \pm \frac{2\,ne^{3/2}}{\pi\sqrt{2m_\pm}} \sqrt{\mp V} \tag{3}$$

where J_P is the current drawn from the plasma through a unit element of surface area normal to the wire and $m_\pm$ is the mass of the accelerated particle. The upper sign corresponds to positive ion collection where the potential is negative; the lower coresponds to electron collection.

We now combine eqs. (1) and (3) to obtain an equation for the potential. Differentiating eq. (1) with respect to z, we obtain:

$$\frac{d^2V}{dz^2} = -\Lambda \frac{di}{dz}, \tag{4}$$

since the emf per unit length is assumed to be constant. Now, the change in current per unit length, di/dz, is due, only to the current leakage from the plasma. Hence,

$$\frac{di}{dz} = 2\pi R\, J_p\,. \tag{5}$$

Substitution of eq. (3) into eq. (5) and the result into eq. (4) yields the following nonlinear differential equation for the potential:

$$\frac{d^2V}{dz^2} = \mp \frac{4\Lambda R\, ne^{3/2}}{\sqrt{2m_\pm}} \sqrt{\mp V}\,. \tag{6}$$

If we assume that the density, n, is constant then this equation possesses an explicit solution in terms of elliptic integrals. We present this in Appendix A along with a useful expansion applicable for most cases of practical interest. In the next sections we apply this equation to the two examples and integrate numerically.

III. TETHERED SUBSATELLITE

A space shuttle tethered subsatellite system called the Tethered Balloon Current Generator[6] has been proposed and a moderately sized system consists of a 30 meter diameter conducting balloon attached to the space shuttle via a 10 kilometer wire. The tethered balloon is gravity-gradient stabilized in a position radially outward as the shuttle describes an eastward orbit. The geometry is the same as in Fig. 1, where the point d corresponds to the balloon end.

Since the balloon has a large surface area, the conductive coupling with the plasma will be large; hence, we assume that the balloon is at the same potential as the plasma. This is of course not rigorously correct, since the potential at this point, will depend upon the current, via the current-voltage characteristics of the balloon; but it does not vary much over the range of current of interest. (If the balloon is at zero potential, then a current in excess of 1 amp will be collected on its surface due to electron and ion impingent thermal fluxes. If an equal number of electrons and ions are collected, i.e. i=0, then the balloon will attain a potential on the order of

-0.5 volts.) Therefore, $V(d)=0$ (the point d corresponds to z_0 as used in Appendix A).

Since dV/dz must be greater than zero, all points of the wire have negative potential and ions are collected. This implies that the upper signs in eqs. (A. 14) and (A. 15) are appropriate. Knowledge of the current at the point d, $i(d)$, or $dV(d)/dz$ will complete the solution. Realistically, the current is controlled from the shuttle, therefore, the more appropriate conditon is $i(0)$, the value of the current at $z=0$. We use the boundary conditon $i(0)=0$.

Figures 2 and 3 show results for the case of zero resistivity ($\rho=0$), where eq. (6) possesses a linear solution and di/dz varies as $(d-z)^{1/2}$. In Fig. 2, we plot ϕ, the negative of V, as a function of z. Figure 3 is a plot of di/dz vs. z for two values of the density. It is seen that di/dz scales with Rn. The maximum current $i(d)$ is 0.066 amps for $n=10^{12}m^{-3}$; this is due entirely to leakage from the plasma.

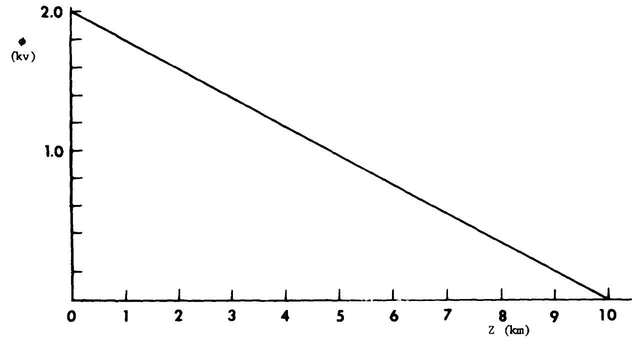

Figure 2. Plot of ϕ (negative of the potential) as a function of z for: the resistivity $\rho=0$, $n=2X10^{11}m^{-3}$, and $R=10^{-3}m$.

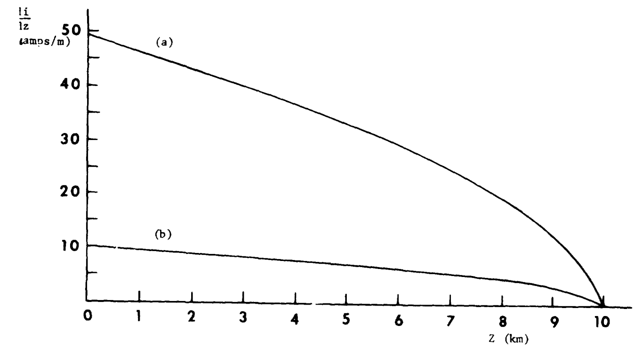

Figure 3. Plot of di/dz as a function of z for: (1) $\rho=0$, $R=10^{-3}m$, $n=10^{12}m^{-3}$, $i(d)=0.066$ amps and (b) $\rho=0$, $R=10^{-3}m$, $n=2X10^{11}m^{-3}$, $i(d)=0.033$ amps.

In Figs. 4 and 5 we plot the deviation from the $\rho=0$ case. Observe that the deviation scales with $\rho n/R$. For a reasonable resistivity, $\rho=1.5X10^{-8}\Omega$-m, (Cu), wire radius $R=10^{-3}$m and $n=2X10^{11}m^{-3}$, the deviation is quite small. In the figures we plot cases where the deviation is larger. For plots (a) and (c) we used the extreme resistivity, $\rho=10^{-6}\Omega$-m (nichrome) to demonstrate this finite resistance effect.

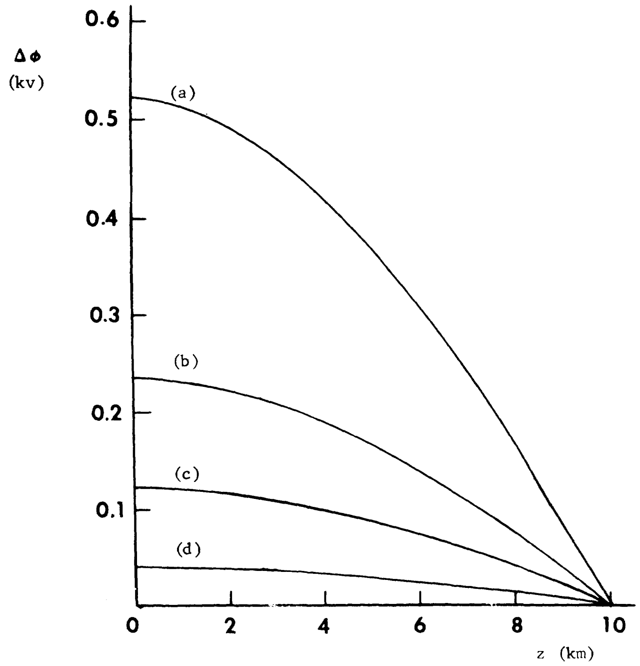

Figure 4. Plot of $\Delta\phi$, the deviation from the $\rho=0$ case, as a function of z for: (a) $\rho=10^{-6}\Omega$-m, $R=10^{-3}m$, $n=10^{12}m^{-3}$, $i(d)=0.27$ amps, (b) $\rho=10^{-7}\Omega$-m, $R=2.5X10^{-4}m$, $n=10^{12}m^{-3}$, $i(d)=0.076$ amps, (c) $\rho=10^{-6}\Omega$-m, $R=10^{-3}m$, $n=2X10^{11}$ m^{-3}, $i(d)=$ 0.063 amps, and (d) $\rho=1.5X10^{-4}\Omega$-m, $R=2.5X10^{-4}m$, $n=10^{12}m^{-3}$, $i(d)=0.081$ amp.

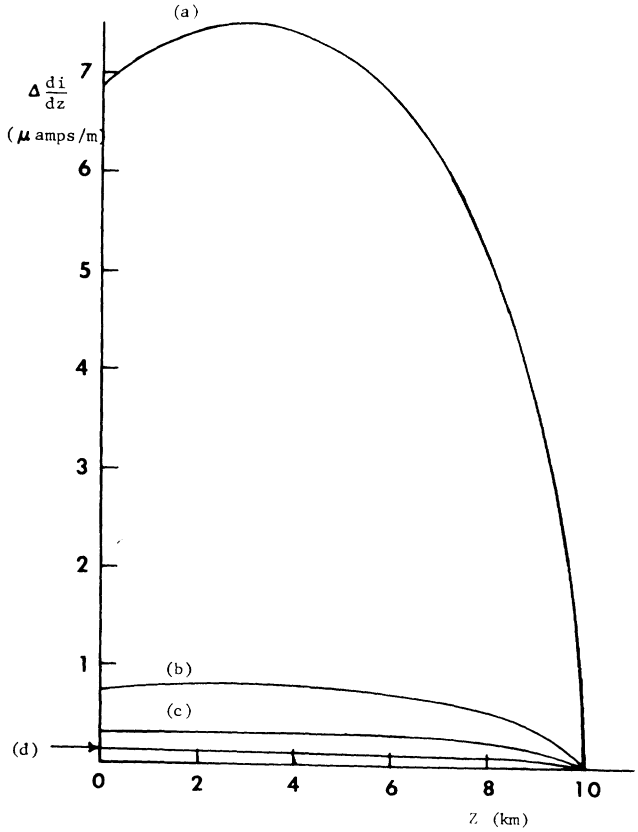

Figure 5. Plot of $\Delta di/dz$, the deviation from the $\rho=0$ case, as a function of z. (a), (b), etc. correspond to the same parameters as in Figure 4.

IV. FREE FLOATING WIRE

We now consider an isolated, free floating wire which is appropriate for VLF antennas and small tethered subsatellites when no current is emitted from either the shuttle or the subsatellite. In this case the ends of the wire assume an unspecified floating potential relative to the ambient plasma and no current flows through either end of the wire. The wire assumes a natural grounding point, c, such that above this point (increasing z) the potential is positive and below, negative (Fig. 6) (dV/dz remaining positive). Hence, we use $i(0)=0$, $i(d)=0$, and $V(c)=0$.

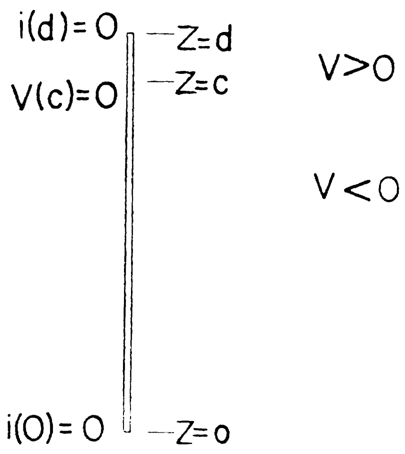

Figure 6. Schematic depiction of floating wire.

The point, c, at which the potential is zero will be assumed to correspond to zero plasma leakage current density. We further assume that in the region $c<z\leqslant d$, where $V>0$, only electron collection takes place; the retarded current is neglected as discussed previously. Similarly, when $0\leqslant z<c$, where $V<0$, electron retarded current is neglected and the leakage here is due only to positive ion collection.

The above assumptions reduce the problem to the solution of two differential equations in two regions with matching conditions on the voltage and current at the point c. In Appendix B we determine the point c.

$$c = \frac{d}{1+\left(\frac{m_-}{m_+}\right)^{1/3}} \tag{7}$$

When $m_-=m_e$ and $m_+=16$ amu, $c\sim0.97$ d.

The solution when $c<z\leqslant d$ corresponds to eqs. (A. 14) and (A.15) with the lower sign where $z_0=c$. When $0 \leqslant z<c$ the upper sign is appropriate.

As an example we consider a 100 km wire with radius $2.5\text{X}10^{-4}$m in a region where the particle density is $2\text{X}10^{11}\,m^{-3}$. Plots of ϕ(-V) versus z are shown in Fig. 7 (upper), where the resistivity, $\rho=1.5\text{X}10^{-8}\,\Omega$-m(Cu) is plotted along with the zero resistivity. The lower portion of Fig. 7 shows -di/dz versus z. The maximum current occurs at the point c; the magnitude of the leakage current density or di/dz equaling zero at this point.

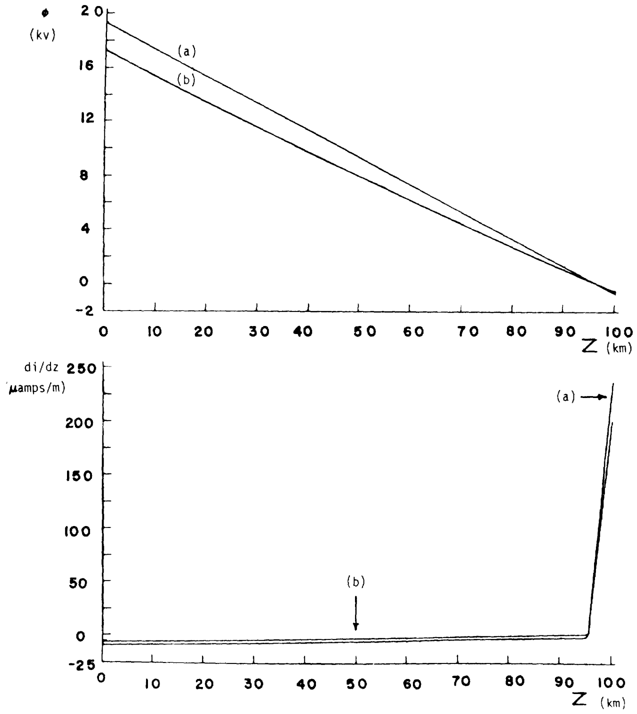

Figure 7. (upper) Plot of ϕ (negative of the potential) as a function of z for the floating wire. (lower) Plot of -di/dz versus z. (a) $\rho=0$, $n=2X10^{11}m^{-3}$, $R=2.5X10^{-4}m$, (b) $\rho=1.5X10^{-8}\,\Omega$-m, $R=2.5X10^{-4}m$, $n=2X10^{11}m^{-3}$. The maximum current occurs at point c; $i(c)=0.5$ amps.

V. CONCLUSION

We have described the current-voltage properties of an orbiting long wire of finite resistivity and diameter. Two applications have been considered and deviation from the zero resistivity case was seen to be small for practical values of the parameters involved. The point of ground for a floating wire was determined.

ACKNOWLEDGEMENTS

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APPENDIX A

Here we obtain the solution to eq. (6) which is representable in terms of elliptic integrals. Also, since for most cases of practical interest the deviation from the ρ=0 case is small, we develop expansions for V and i.

Integrating eq. (6) from $z=z_0$ to z with $V(z_0)=0$ yields:

$$\left(\frac{dV}{dz}\right)^2 = \left(\frac{dV}{dz}\right)^2_{z=z_o} + \frac{16ne^{3/2}}{3\sqrt{2m_\pm}}(\mp V(z))^{3/2} \quad \text{(A.1)}$$

Using eq. (1) we define,

$$\left(\frac{dV}{dz}\right)_{z=z_o} = \mathcal{E} - \Lambda i(z_o) \equiv K \quad \text{(A.2)}$$

and substitution into (A.1) yields:

$$\frac{dV}{dz} = \sqrt{\frac{16ne^{3/2}\Lambda R}{3\sqrt{2m_\pm}}(\mp V(z))^{3/2} + K^2} \quad \text{(A.3)}$$

Substituting the dimensionless variables

$$\psi_\pm = \mp\left(\frac{16ne^{3/2}\Lambda R}{3\sqrt{2m_\pm}\,K^2}\right)^{2/3} V \; ;$$

$$\xi_\pm = K\left(\frac{16ne^{3/2}\Lambda R}{3\sqrt{2m_\pm}\,K^2}\right)^{2/3} z \quad \text{(A.4)}$$

into (A.3) leads to the integral,

$$\int_0^{\psi_\pm} \frac{d\psi_\pm}{\sqrt{\psi_\pm^{3/2}+1}} = \mp(\xi_\pm - \xi_{o\pm}) \quad . \quad \text{(A.5)}$$

Substituting $\psi_\pm=x^2$ ($\psi_\pm$ is nonnegative) implies,

$$\int_0^{\sqrt{\psi_\pm}} \frac{2x\,dx}{\sqrt{x^3+1}} = \left\{ \frac{2\sqrt{\psi_\pm^{3/2}+1}}{\sqrt{\psi_\pm}+1+\sqrt{3}} - \frac{2}{\sqrt{3}+1} \right.$$

$$+\left(\frac{\sqrt{3}-1}{3^{1/4}}\right)\left(F(\varphi,k) - F(\beta,k)\right) \quad \text{(A.6)}$$

$$\left. - 2\cdot 3^{1/4}\left(E(\varphi,k) - E(\beta,k)\right)\right\} , \quad (6)$$

where

$$\cos\varphi = \frac{\sqrt{\psi_\pm}+1-\sqrt{3}}{\sqrt{\psi_\pm}+1+\sqrt{3}}, \quad \cos\beta = \frac{1-\sqrt{3}}{1+\sqrt{3}},$$

and $\quad k^2 = \sin^2\frac{5\pi}{12} = \frac{2+\sqrt{3}}{4}$.

F and E are elliptic integrals of the first and second kinds, respectively.

Now defining $\epsilon_\pm = 4\Lambda R n e^{3/2}/(\sqrt{2m_\pm})$, eq. (6) becomes

$$\mp\frac{d^2V}{dz^2} = \epsilon_\pm\sqrt{\mp V} \quad \text{(A.7)}$$

Expaning $\mp$V in the smallness of $\epsilon^\pm$,

$$\mp V = \sum_\ell \pm V_\ell \quad , \quad \text{(A.8)}$$

where $V_{\ell+1}/(V_\ell) \sim O(\epsilon)$. Substituting (A.5) into (A.8) and equating the first two orders in $\epsilon^\pm$ to zero,

$$\frac{d^2(\mp V_o)}{dz^2} = 0 \quad \text{(A.9)}$$

$$\frac{d^2(\mp V_1)}{dz^2} = \epsilon_\pm(\mp V_o)^{1/2} \quad . \quad \text{(A.10)}$$

We solve these subject to the boundary conditions,

$$V_\ell(z_o) = 0, \quad \left.\frac{dV_o}{dz}\right)_{z=z_o} = K,$$

$$\left.\frac{dV_\ell}{dz}\right)_{z=z_o} = 0, \; \ell > 0.$$

The solution to (A.9) is

$$(\mp V_o) = K(z - z_o) \quad . \tag{A.11}$$

Substituting (A.11) into (A.10) and integrating,

$$\frac{dV_1}{dz} = \frac{2\epsilon K^{1/2}}{3} (\mp(z - z_o))^{3/2} \tag{A.12}$$

and integrating a second time,

$$\mp V_1(z) = \frac{4\epsilon_{\pm} K^{1/2}}{15} (\mp(z - z_o))^{5/2} \tag{A.13}$$

Therefore to order $\epsilon^{\pm}$,

$$\mp V(z) = K(z - z_o)$$

$$+ \frac{16 \Lambda R n e^{3/2} K^{1/2}}{15\sqrt{2m_{\pm}}} (\mp(z - z_o))^{5/2} \tag{A.14}$$

The corresponding equation for the current is obtained by differentiating (A.14) then substituting the result into eq. (1),

$$i(z) = i(z_o) - \frac{8 n e^{3/2} R K^{1/2}}{3\sqrt{2m_{\pm}}} (\mp(z - z_o))^{3/2} \quad . \tag{A.15}$$

APPENDIX B

Here we determine the point c. When $0 \leqslant z < c$, using (A.5),

$$\int_0^{\psi_+} \frac{d\psi_+'}{\sqrt{\psi_+'^{3/2} + 1}} = \xi_{c+} - \xi_+ \tag{B.1}$$

and when $c < z \leqslant d$,

$$\int_0^{\psi_-} \frac{d\psi_-'}{\sqrt{\psi_-'^{3/2} + 1}} = \xi_- - \xi_{c-} \quad . \tag{B.2}$$

Using (A.4), $\psi_+ = \psi_-(m_-/m_+)^{1/3}$ and $\xi_+ = \xi_-(m_-/m_+)^{1/3}$. Evaluating (B.1) at z=0 and (B.2) at z=d, then equating $\xi_{c+} = \xi_{c-}(m_-/m_+)^{1/3}$ implies,

$$\int_0^{\psi_+(0)} \frac{d\psi_+'}{\sqrt{\psi_+'^{3/2} + 1}}$$

$$= -\left(\frac{m_-}{m_+}\right)^{1/3} \int_0^{\psi_-(d)} \frac{d\psi_-'}{\sqrt{\psi_-'^{3/2} + 1}}$$

$$+ \xi_{d-}\left(\frac{m_-}{m_+}\right)^{1/3} \tag{B.3}$$

Now since we have assumed i(0)=0 and i(d)=0, using (A.3) evaluated at c and d, together with (1) and (A.4), we obtain,

$$\psi_+(0) = \psi_-(d) \tag{B.4}$$

Hence from (B.3) and (B.2),

$$-\int_0^{\psi_-(d)} \frac{d\psi}{\sqrt{\psi^{3/2} + 1}} = \frac{\xi_{d+}}{1 + \left(\frac{m_-}{m_+}\right)^{1/3}} = \xi_{c+} \tag{B.5}$$

Whereupon,

$$c = \frac{d}{1 + \left(\frac{m_-}{m_+}\right)^{1/3}} \tag{B.6}$$

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