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On the size of the cometary tail magnetic field

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Summary. Ershkovich has recently criticized earlier arguments by Mendis for substantial magnetic fields ($B \approx 100\gamma$) in the cometary plasma tail. It is shown that these criticisms are unjustified. Contrary to Ershkovich's claim, it is shown that the folding rate of the tail rays is indeed indicative of a substantial tail magnetic field. Much of the disagreement stems from Ershkovich's idealization of the plasma tail of the comet as a uniform plasma cylinder immersed in the solar wind and separated from it by a well-defined discontinuity surface across which pressure balance is maintained. The observations are indicative of a dynamical situation, not one of static equilibrium. The rapidly folding tail ray morphology is analogous to that of the terrestrial magnetotail, wherein the 'cometary tail magnetic field' is simply the vector sum of the interplanetary magnetic field and the fields associated with the current system in the plasma tail. Ershkovich's arguments for small magnetic fields based on his interpretation that the amplitude of the helical waves observed in comet tails is unconvincing since it is based on his highly oversimplified and inaccurate model. These helical waves are likely to be associated with field-aligned current discharges from the tail.

In the absence of direct measurements of the cometary plasma tail magnetic field to date, there has been a continuing discussion about its likely magnitude based on theoretical interpretations of the observed morphology and dynamics of plasma features in the cometary tail (e.g. Hyder, Brandt & Roosen 1974; Ershkovich 1976a, b, 1977; Ip & Mendis 1975, 1976a; Mendis 1977a, 1978; Brandt & Mendis 1979).

Following the early model of Alfvén (1957), Ip & Mendis (1976a) consider the plasma tail streamers to be magnetic flux tubes being swept up by the cometary ionosphere. Identifying the folding of these streamers on to the tail axis, with the drift of plasma in crossed electric and magnetic fields, these authors showed that the strength of the tail magnetic field $B_t(y)$ at a distance y from the tail axis, is given by

$$B_t(y) = f v_s B_s / v_D(y) \tag{1}$$

where v_s and B_s are, respectively, the velocity and magnetic field strength of the undisturbed solar wind, $v_D(y)$ is the 'folding-velocity' of the tail streamers and $f\ (\leqslant 1)$ is the fraction of the convectional electric field of the solar wind transmitted to the cometary plasma. Typically $v_D(y)$ decreases from around 100 km/s far from the axis, when the streamers are first observed to around 1 km/s by the time they merge on the tail axis. Taking $v_s \approx 300$ km/s and $B_s \approx 5\gamma$ the convectional electric field of the solar wind ≈ 1.5 mV/m. The dawn-to-dusk electric field in the terrestrial magnetotail is typically about 0.15 mV/m (McCoy *et al.* 1975), which gives $f \approx 0.1$ in the terrestrial case. Adopting the same value of f for the cometary case and taking $v_D \approx 1$ km/s, Mendis (1977a) obtained $B_t(0) \gtrsim 100\gamma$.

Recently, Ershkovich (1978) has rediscussed this problem. He considers the motion of a magnetic flux tube through the cometary coma (see his fig. 1). Then with the y and z axes taken parallel to the undisturbed interplanetary field $\vec{B}_s$ and the tail axis, respectively, he obtains the downstream drift speed of the flux tube in the electric field fE as,

$$dz/dt = fE/B_y = fv_sB_s/B_y \tag{2}$$

where B_y is the magnetic field in the coma. Taking $B_y \lesssim 50\gamma$ (the value near the subsolar stagnation point calculated from the hypersonic pressure balance) and $B_s \approx 5\gamma$ one gets $dz/dt \gtrsim 0.1\,fv_s$. Then if d is the downstream drift of the central part flux tube through the coma during the time its extensions into free flowing solar wind close on to the tail axis with a characteristic time T, $d = (dz/dt)\,T \gtrsim 0.1 fv_sT$. Taking $T = 14.6$ hr (Wurm & Mammano 1972) and $v_s \approx 400$ km/s one gets:

$$f \lesssim 5 \times 10^{-7} d \tag{3}$$

where d is in km. Ershkovich then cites the observation of Wurm & Mammano (1967) that streamers originate as close as 10^3 km from the nucleus and goes on to take $d = 2 \times 10^3$ km. With this value of d, $f \lesssim 10^{-3}$ and consequently from equation (1), $B_t(0) \sim 1\gamma$.

This value of d chosen by Ershkovich is without foundation. The correct value for d is the characteristic distance through which the interplanetary magnetic flux tube is slowed down. This slowing down is largely due to the loading of the solar wind plasma with cometary ions formed by photoionization and charge exchange (Biermann, Brosowski & Schmidt 1967) and the area of interaction can be estimated approximately by balancing the total mass flow of the solar wind across it to the mass production rate of cometary molecules:

$$n_sv_sm_s\pi d^2 \approx \dot{Q}m_c \tag{4}$$

where m_s and m_c are the average masses of the solar wind and cometary molecules and $\dot{Q}$ is the cometary production rate (in molecule/s). Typically $\dot{Q} \approx 10^{30}\,\mathrm{s}^{-1}$ at 1 AU for a medium bright comet (e.g. Bennett 1970 II). Taking $m_s \approx m_H$, $m_c \approx 20 m_H$, $n_s \approx 5\,\mathrm{cm}^{-3}$ and $v_s \approx 300$ km/s we get from equation (4), $d \approx 2 \times 10^6$ km. Detailed hydrodynamic calculations (e.g. Brosowski & Wegmann 1972; Wallis 1973) show that the distance of the outer collisionless shock is at a distance of about 3×10^5 km. That we do not see the flux tubes at that distance is simply due to the fact that they are then not sufficiently loaded with ions of cometary origin. The shock transition occurs when the mean molecular weight of the inflowing contaminated solar wind increases by a factor ≈ 33 per cent (Biermann *et al.* 1967), which corresponds to the addition of only 1–2 per cent cometary ions. In fact, in some major plasma comets like Morehouse (1908 III), 'parabolic' ion (CO^+) envelopes are first observed on the sunward side of the nucleus at a distance of about 3×10^5 km. They are thereafter observed to 'collapse' toward the nucleus (Eddington 1910).

Taking $d \approx 3 \times 10^5$ km we get from equation (2), $f \approx 0.15$. The actual value of f is probably somewhat lower than that obtained by this two-dimensional model. Consequently, the magnitude of the electric shielding factor $f \approx 0.1$ adopted earlier by Mendis (1977a) by analogy to the terrestrial magnetotail seems quite reasonable, which of course means that $B_t(0) \gtrsim 100\gamma$. Indeed it is difficult to understand why the value of f in the cometary case should be over two orders of magnitude smaller than its corresponding value in the terrestrial case as claimed by Ershkovich (1978).

A further comment on the observation of Wurm & Mammano (1967) regarding the 'point of origin' of the tail streamers is in order. Examination of their figure 3, which is a drawing, clearly indicates, from the lack of symmetry about the nucleus, that the lowest portions of the streamers on the sunward side are not a part of the magnetic flux tube to which they are connected. They seem more or less to radiate out from the nucleus and are, perhaps, similar to the diverging fans of radial rays clearly seen in the visual drawings of other bright comets (e.g. Comet Donati (1858 VI), Rahe *et al.* 1969), to connect like spokes on to the sunward envelope.

This phenomenon has a natural explanation according to the plasma tail model of Ip & Mendis (1976a), wherein the cross-tail current caused by the folding of the tail rays, sporadically disrupts due to a current instability, flows along the field lines and eventually discharges partially into the cometary coma from the sunward side causing enhanced ionization there. It is straightforward to show, on the basis of the cometary atmosphere and ionosphere models developed by Mendis, Holzer & Axford (1972) and by Ip & Mendis (1976b) that the cross-conductivities become an appreciable fraction of the parallel conductivity only when $r \lesssim 5 \times 10^3$ km. Consequently, the tail-aligned current follows the field lines until $r \approx 5 \times 10^3$, at which point a faction (~ 1 per cent) is initially discharged into the inner coma. Since the plasma is unlikely to be inhomogeneous, the current will be channelled preferentially along filaments of higher conductivity. This, in turn, will cause increased local ionization and thereby increased conductivity along these paths. We suspect that the streamers which diverge radially from the inner coma and connect with the sunward envelope are produced this way.

Ershkovich (1978) further argues that the large cross-tail current in the neutral sheet predicted by Ip & Mendis (1976a) is too large, on the basis that the observed decrease in the drift velocity $v_D(y)(=fE/B_t(y))$ is due to a decrease in the electric field fE rather than due to an increase in the magnetic field $B_t(y)$. This is groundless since the maximum effect of the convectional electric field of the solar wind should be felt in the neutral sheet. Furthermore, since the cross-tail current is maintained by the drift of cometary plasma across the neutral sheet, a self-consistent model (Ip & Mendis 1976a) shows that tail current electrons and ions are accelerated to the Alfvén potential $\phi_A \approx B_t^2(0)/4\pi n_e$. Consequently, $fE \approx B_t^2(0)/4\pi n_e dt$ where dt is the breadth of the neutral sheet. This shows that fE cannot tend to zero without $B_t(0)$ tending to zero at the same time.

Ershkovich (1978) also makes the statement that 'the assumption that currents are field-aligned (Mendis 1977a) contradicts the fact that the force-free field in a closed system is stable against small perturbations (Woltjer 1958)'. This is not so. The point that was made by Mendis (1977a) was that double-solenoidal current system formed by the cometary cross-tail current (just as in the terrestrial magnetotail) is unstable, and that its sporadic disruption causes field-aligned currents to flow as is also believed to happen in the terrestrial case (Bostrom 1974). This is completely consistent with Woltjer's theorem in so far as the cometary tail can be considered a closed system. The force-free state in a closed system being the state of minimum magnetic energy, would indeed be the state that all cosmic plasmas, including comet tails, would strive to attain.

Much of the disagreement between Ershkovich (1978) and us stems from the completely different models chosen by us for the plasma tail of a comet. Ershkovich idealizes it to be a plasma cylinder immersed in the magnetized solar wind and separated from the solar wind by a well-defined tangential discontinuity across which there is static pressure equilibrium. He is then justified in talking about 'internal' (cometary) and 'external' (undisturbed solar wind) magnetic fields as well as pressures. However, even in such a case, it is inappropriate to use the pressure balance condition in the way he does, when field-aligned currents flow (Mendis 1977a). The Lorentz force $\mathbf{F}_L$ is given (with obvious notation) by

$$\mathbf{F}_L = \frac{1}{c}\,\mathbf{j} \times \mathbf{B} = \frac{1}{4\pi}\,(\mathbf{B} \cdot \boldsymbol{\nabla})\,\mathbf{B} - \nabla \left(\frac{B^2}{8\pi}\right) \tag{5}$$

when field-aligned current flow $\mathbf{j} \times \mathbf{B}$ for them is zero. One can no longer neglect the first term on the right side of equation (5). Indeed it is this first term that is associated with the observed pinching and filamentation of the current carrying cometary plasma. The field-aligned component could be of any arbitrary value and yet not produce a magnetic pressure opposing the solar wind pressure. We use a phenomenological model based on the 'folding umbrella' morphology of the tail streamers which we identify as magnetic flux tubes (a point of view, incidentally, that Ershkovich seems to agree with) closing on to the tail axis. In this situation the tail magnetic field is the vector sum of the interplanetary field and the field associated with the current system in the tail (the double solenoidal current system bisected by the tail current sheet and the field-aligned system), and the artificial division into 'external' and 'internal' is unjustified. All that one can meaningfully speak about is the 'cometary tail magnetic field' which is, of course, not intrinsic but rather is generated by the interaction of the magnetized solar wind and the cometary ionosphere. Strong observational evidence for this model comes from the as yet unpublished photographs of the plasma tail of Comet Kobayashi-Berger-Milon (1975h) which show the outer ends of the tail rays bending away from the tail axis, as depicted in fig. 3 of Mendis (1978; Moore 1978, private communication).

Ershkovich (1978) further refers to the surface current which is responsible for the difference $B_e - B_i$ where B_e and B_i are, respectively, the 'external' and 'internal' magnetic fields. Clearly he is using the analogy of the terrestrial magnetotail where the cross-tail current does indeed close along a thin boundary sheet on to the solar wind due to the fact that the plasma density (and, consequently, the electrical conductivity) within the distant terrestrial magnetotail is significantly lower than that of the solar wind ($n_e \approx 0.1\,\mathrm{cm}^{-3}$ in the terrestrial plasma sheet, while $n_e \approx 0.01\,\mathrm{cm}^{-3}$ in the high latitude lobes; Akasofu (1977). In the case of the comet, however, the situation is quite different, with the plasma density in the tail being significantly greater than that of the solar wind ($n_e \approx 10$–$100\,\mathrm{cm}^{-3}$ in the comet's plasma tail, see, e.g. Mendis & Ip (1977). Consequently, the cross-tail current closes through the entire volume of the cometary plasma tail lobes with a density given by

$$j_x = \frac{c}{4\pi}\,\frac{dB_t(y)}{dy}. \tag{6}$$

Ershkovich (1978) now concedes that the cometary magnetic field evaluation based on his dispersion relation (equation (4)) will only be effective when reliable observations which could discriminate between plasma bulk motions and wave motions in comet tails become available. Such observations are unfortunately not available at present (see, e.g. Brandt & Mendis 1979). He, however, claims that a second method he has derived, namely the comparison of the amplitude of non-linear helical waves (see equation (4) of Ershkovich 1978) with observation, is essentially free of this restriction, and that this indicates that the cometary magnetic field is $\lesssim$ the interplanetary magnetic field. There is a circularity in this argument since it presupposes that the observed helical waves in comet tails are indeed excited via the Kelvin–Helmholtz instability at the sharp tangential discontinuity surface between the cometary tail plasma and the solar wind that the author envisages and, as we have pointed out earlier, the observational evidence does not support such a model. Even if it did a cometary magnetic field $\gtrsim 50\,\gamma$ would inhibit the growth of this instability (Mendis & Ip 1977), so would a sufficiently thick boundary layer. According to Smith & von Goeler (1968), who considered the stability of an inhomogeneous, low β plasma (typically $\beta \lesssim 0.1$ for a comet tail) with a uniform magnetic field, subject to a velocity shear taking into account both ion finite Larmor radius effects as well as Landau damping of the waves, the condition for the excitation of the Kelvin–Helmholtz instability is roughly given by

$$\frac{\partial v_0}{\partial x} \gg \frac{\Omega_{\rm ci} a_{\rm i}}{L} \tag{7}$$

where $a_{\rm i}$ is the ion Larmor radius, $\Omega_{\rm ci}$ is the ion cyclotron frequency and L is the scale length of the equilibrium density variations. While the value of all these quantities are uncertain, taking $T_{\rm i} \approx 10^4$ K, $L \approx 1000$ km (the radius of a tail ray) and $v_0 \approx 300$ km/s, we obtain the condition for the Kelvin–Helmholtz instability that the thickness of the 'boundary layer' is $d \ll 10^4$ km. There are also other problems with Ershkovich's analysis here. First, the Mach numbers of the flow in both the solar wind and the comet are $\gg 1$. The assumption of incompressibility in such a case is unjustified (Landau & Lifshitz 1959). Furthermore, it is not at all apparent from the analysis that the non-linear state of the Kelvin–Helmholtz instability is a stationary helical structure, convected down the tail without distortion – it is merely an assumption. We believe that these helical waves are excited by the classical kink instability due to axial currents, as proposed by Hyder *et al.* (1974), which are caused by the sporadic disruption of the cross-tail current (Ip & Mendis 1976a). The instability is excited when the axial current is sufficiently large, that the average energy density in the azimuthal field (B_ϕ) is greater than twice the average energy density in the axial field (B_z) (see Alfvén & Fälthammar (1963), for discussion, as well as experimental evidence). This condition is approximately equivalent to $B_\phi > \alpha B_z$, where α is a factor of order unity. Ershkovich's (1978) statement that in the 'undisturbed case' (i.e. when no axial currents are flowing), $B_z \gg B_\phi$, while being a truism, is irrelevant to the actual situation which is characterized by large axial currents. Consequently, his equation (6), which is based on this assumption, and the inferences drawn therefore are invalid.

There are a few more general remarks that we wish to make. First, if there is indeed a discontinuity surface separating the distant cometary tail plasma from the solar wind, it is not at all apparent that it should be a tangential discontinuity. It is more likely to be a contact discontinuity (see Spreiter, Summers & Alksne 1966). This is because any magnetic or velocity discontinuity that may exist on the forward part of the induced cometary magnetosphere (which may well be bounded by a tangential discontinuity surface (Mendis 1977b)) has ample time to diffuse, even with very slight departures from perfectly conducting or inviscid flow.

Secondly, whatever may be the nature of this assumed discontinuity, its proper stability analysis should include not only the effects of compressibility and the finite thickness of the boundary layer (since models with singular surfaces generally tend to give large growth rates), but also take into account Landau damping and finite Larmor radius effects (the ion Larmor radius being typically $\approx 10^2$ km). In this case the very applicability of ideal MHD analysis is questionable and quantitative estimates obtained are very suspect.

Finally, the use of growth rates or phase velocity information, obtained from a dispersion relation, in order to interpret large amplitude disturbances such as those in comet tails (as in Ershkovich *et al.* 1972) is categorically incorrect. Perturbation theory, and dispersion relations derived therefrom, only describe tendencies of an equilibrium configuration; they should not be applied to finite amplitude structures. Consequently, the use of such techniques in order to estimate magnetic fields is unwarranted.

While Ershkovich (1978) seems to be aware of some of these difficulties which are addressed in his more recent papers (see, e.g. Ershkovich & Heller (1977) and references to his earlier papers contained therein), so far a proper analysis, taking all or even most of the aforementioned points into account has not been done. We recognize this is not a trivial matter. However, such an analysis is required before we can have any confidence in the predictions of these mathematical models.

On the other hand, the simple phenomenological model that has been developed (Ip & Mendis 1976a; Mendis 1978) is based on the observed folding of cometary tail rays into the cometary plasma tail axis. If we are granted that these rays delineate the magnetic field lines (as Ershkovich (1978) himself does), it is difficult to avoid the conclusion that there is significant enhancement of magnetic field over the ambient solar wind value, near the tail axis. Clearly one sees the density of rays increase toward the tail axis. Furthermore, the plasma tail is very far from a static equilibrium (as assumed by Ershkovich) with a total ray folding time of 10–15 hr and perceptible changes of structure noticed over 2–3 hr.

These arguments too, however, are circumstantial, and the final solution to this important question of the size of cometary magnetic fields may have to await direct *in situ* measurements during a future comet tail fly-by mission as is being presently discussed in relation to the 1986 apparition of Comet P/Halley. If the tail magnetic field happens to be even larger than what we presently believe it to be (i.e. $\gtrsim 1000\,\gamma$ rather than $\approx 100\,\gamma$) it may be barely possible to measure it from the Zeeman splitting of sharp molecular emission lines at radio wavelengths (Mendis & Ip 1977).

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References

Akasofu, S-I., 1977. *Physics of Magnetospheric Substorms,* p. 15, D. Reidel. Dordrecht, Holland.
Alfvén, H., 1957. *Tellus,* 9, 92.
Alfvén, H. & Fälthammar, C-G., 1963. *Cosmical Electrodynamics,* p. 120, Clarendon Press, Oxford.
Biermann, L., Brosowski, B. & Schmidt, H. U., 1967. *Sol. Phys.,* **1**, 254.
Bostrom, R., 1974. *Magnetospheric Physics,* p. 45, ed. McCormac, B. M., D. Reidel, Dordrecht, Holland.
Brandt, J. C. & Mendis, D. A., 1979. *Solar System Physics – a Twentieth Anniversary Review,* eds Kennel, C. F., Lanzerotti, L. T. & Parker, E. N., North-Holland, in press.
Brosowski, B. & Wegmann, R., 1972. *Max-Planck Institute Publ. MPI/PAE Astr. 46.*
Eddington, A., 1910. *Mon. Not. R. astr. Soc.,* **70**, 442.
Ershkovich, A. I., 1976a. *Planet. Space Sci.,* **24**, 287.
Ershikovich, A. I., 1976b. *Mon. Not. R. astr. Soc.,* **177**, 649.
Ershkovich, A. I., 1977. Proceedings of the L. D. deFeiter Memorial Symposium, eds Shea, M. A., Smart, D. F. & Wu, S. T., *Astrophys. Space Sci. Lib.,* **71**, p. 305, D. Reidel, Dordrecht, Holland.
Ershkovich, A. I., 1978. *Mon. Not. R. astr. Soc.,* **184**, 952.
Ershkovich, A. I. & Heller, A. B., 1977. *Astrophys. Space Sci.,* **48**, 365.

Ershkovich, A. I., Nusinov, A. A. & Chernikov, A. A., 1972. *Planet. Space Sci.,* **20,** 1235.

Hyder, C. L., Brandt, J. C. & Roosen, R. G., 1974. *Icarus,* **23,** 601.

Ip, W-H. & Mendis, D. A., 1975. *Icarus,* **26,** 457.

Ip, W-H. & Mendis, D. A., 1976a. *Icarus,* **29,** 147.

Ip, W-H. & Mendis, D. A., 1976b. *Icarus,* **28,** 389.

Landau, L. D. & Lifshitz, E. M., 1959. *Fluid Mechanics,* p. 24, Pergamon Press, Oxford.

McCoy, J. E., Lin, R. P., McGuire, R. E., Chase, L. M. & Anderson, K. A., 1975. *J. geophys. Res.,* **80,** 3217.

Mendis, D. A., 1977a. *Mon. Not. R. astr.Soc.,* **179,** 5P.

Mendis, D. A., 1977b. *The Study of Travelling Interplanetary Phenomena,* p. 291, eds Shea, M. A., Smart, D. F. & Wu, S. T., D. Reidel, Dordrecht, Holland.

Mendis, D. A., 1978. *Moon Planets,* **18,** 361.

Mendis, D. A., Holzer, T. E. & Axford, W. I., 1972. *Astrophys. Space Sci.,* **15,** 313.

Mendis, D. A. & Ip, W-H., 1977. *Space Sci. Rev.,* **20,** 145.

Rahe, J., Donn, B. & Wurm, K., 1969. *Atlas of Cometary Forms,* p. 10, NASA SP-198.

Smith, C. G. & von Goeler, S., 1968. *Phys. Fluids,* **11,** 2665.

Spreiter, J. R., Summers, A. L. & Alksne, A. Y., 1966. *Planet. Space Sci.,* **14,** 223.

Wallis, M. K., 1973. *Planet. Space Sci.,* **21,** 1647.

Woltjer, L., 1958. *Proc. Nat. Acad. Sci.,* **44,** 489.

Wurm, K. & Mammano, A., 1967. *Icarus,* **6,** 281.

Wurm, K. & Mammano, A., 1972. *Astrophys. Space Sci.,* **18,** 273.