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Editor

Resistive Dynamics of Magnetic Islands with Curvature and Pressure

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1. RESISTIVE DYNAMICS OF MAGNETIC ISLANDS

Here we analytically derive the resistive nonlinear dynamics of thin magnetic islands including the effects of pressure and curvature. This combines and extends the nonlinear island calculation of Rutherford[1], who neglected interchange effects, and the linear stability results of Glasser, Greene, and Johnson.[2]

A principal result is that there is a critical island width Δx_c. Islands wider than Δx_c are dominated by Δ', measuring the magnetic free energy, while narrower islands are dominated by pressure and curvature in the island vicinity. Δx_c is given by

$$\Delta x_c^s \Delta_*' \sim k_2(E+F) \quad , \tag{1}$$

where E, F, H, and $D_I = E+F+H$ are standard measures of magnetic curvature[2], $s = \sqrt{1-4D_I}$, and Δ_*' is the finite-β generalization[2] of Δ'. k_2 is roughly 6.3.

Note that Δx_c is <u>independent of resistivity</u>.

To obtain this result, a Grad-Shafranov equation is derived for thin islands which describes the resonant magnetic field in the island vicinity. Interestingly, the curvature enters this nonlinear equation through an expression proportional to the Mercier ideal interchange criterion.

An expansion in the tokamak inverse aspect ratio ε is used for simplicity, using either $\beta \sim \varepsilon^2$ or $\beta \sim \varepsilon$. Only for high β, $\beta \sim \varepsilon$, can Δx_c be a significant fraction of the minor radius. For destabilizing curvature and stabilizing Δ', Equation (1) gives the saturated island width for a single resistive interchange.

Nonlinear islands grow relatively slowly, so following Rutherford we neglect inertia. Therefore, our starting equations are

$$\underset{\sim}{j}\times\underset{\sim}{B} = \underset{\sim}{\nabla}p \ , \tag{2}$$

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{j}_{\parallel} = \underset{\sim}{B}\cdot\underset{\sim}{\nabla}p\times\underset{\sim}{\nabla}(-1/B^2) \ , \tag{3}$$

and Ohm's law $E_{\parallel} = \eta j_{\parallel}$.

The island grows on an equilibrium field $\underset{\sim}{B}_0 = \underset{\sim}{\nabla}\chi\times\underset{\sim}{\nabla}(q(\chi)\vartheta - \zeta)$. We focus attention on the region near a rational surface with $\dot{\ell} = m/n = q_0$. We define a periodic angle coordinate $\alpha = \vartheta - \zeta/q_0$. All quantities f have resonant and nonresonant parts. Since these parts generally behave differently, we define an averaging operator to select out the resonant part, $\bar{f}(\chi,\alpha) = \oint d\zeta\ f(\chi,\alpha,\zeta)/\oint d\zeta$; the nonresonant part is $\tilde{f} = f-\bar{f}$. We also define

$$[A,B] = \underset{\sim}{\nabla}\zeta\cdot(\underset{\sim}{\nabla}A\times\underset{\sim}{\nabla}B) \ . \tag{4}$$

The total average magnetic flux, ψ_h, through a helical ribbon $\alpha = \text{const}$ plays the role of a flux function for the island. ψ_h is obtained from the current I by Ampere's Law. I is found by considering the average $(\bar{I})$ and nonresonant $(\tilde{I})$ parts of Equations (2) and (3). To requisite order $\bar{p}$ is a function of ψ_h. $\bar{I}$ is given by

$$[\psi_h, \bar{I}] = [\tilde{p}, \bar{h}] + \overline{[\tilde{p}, \tilde{h}]} - \overline{[\tilde{\psi}, \tilde{I}]} \ , \tag{6}$$

where h gives the effect of curvature, $h = -1/B_0^2 + 2P_0/B_0^4$. Without pressure and curvature, $\bar{I}$ is a function of ψ_h. $\bar{h}$ gives the effect of average normal curvature, and the last two terms give the effect of geodesic curvature and Pfirsch-Schluter currents.

Explicit expressions for the last two terms are obtained using the nonresonant parts of Equations (2), (3), and (5). The calculation is lengthy but similar to linear theory, since much of the magnetic nonlinearity is negligible for the nonresonant component.

The results for I are substituted into Equation (5), and equilibrium quantities are Taylor-expanded about the rational surface $\chi = \chi_0$. We thus obtain

$$\frac{\partial^2 \psi_h}{\partial \chi^2} = J_*(\psi_h) + (\chi - \chi_0)(G_1 + G_2) \frac{\partial p(\psi_h)}{\partial \psi_h} \ , \tag{8}$$

where J_* is an arbitrary function, where G_1 and G_2 are (to relevant order in ε) E+F and H divided by $(\partial p/\partial \chi_0)\left[q_0/(\partial q/\partial \chi_0)\right]^2$.

The function $J_*(\psi_n)$ is found similarly to Rutherford's analysis; it is driven by an inductive electric field. The pressure gradient is determined in sources deep in the plasma interior, and is determined in the island region by an assumed diffusion process, but with $p(\psi_h)$.

The resulting evolution equation for A(0,t) is best expressed in terms of the island width $\Delta\chi = 4 \quad q_0^2 A/(\partial q_0/\partial \chi)$,

$$\frac{k_1}{\eta}\frac{\partial\Delta\chi}{\partial t} = \Delta_*'\Delta\chi^{s'} + k_2\frac{(E+F)}{\Delta\chi} , \qquad (10)$$

where $s' = -1 + \sqrt{1-4D_I}$, $k_1 \simeq 3$, $k_2 \simeq 6.3$. For $\beta=0$, this agrees with Rutherford's result (the island width grows linearly with time). The driving term from pressure dominates for $\Delta\chi < \Delta\chi_c$ given by Equation (1).

[1] RUTHERFORD, P.H., Nonlinear growth of the tearing mode, Phys. Fluids 16 (1973) 1903.

[2] GLASSER, A.H., GREENE, J.M., JOHNSON, J.L., Resistive instabilities in general toroidal plasma configurations, Phys. Fluids 18 (1975) 875.