Response to "Comment on 'Dielectric energy versus plasma energy, and Hamiltonian action-angle variables for the Vlasov equation' " [Phys. Fluids B 4, 3038 (1992)]

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In Ref. 1 Best correctly suggested that the usual dielectric or wave energy expression is deficient because of its inability to describe the effect of resonant particles. For this reason reference to Best's paper in our paper (Ref. 2) would have been appropriate. However, there are substantial differences between these two works—most notably, the energy density expression he presents is not (when integrated) equal to the energy of a linear perturbation in Vlasov theory. The correct (exact) expression is given by either Eq. (42) or Eq. (98) of Ref. 2. We comment on this and other discrepancies.

In Ref. 1 and in the preceding Comment it is argued that the energy of a wave packet depends upon second-order quantities via initial conditions [cf. Eq. (8.4) of Ref. 1]. The calculation of this work proceeds by expanding about a Maxwellian equilibrium distribution function. Since the energy of a perturbation is a second-order quantity, an energy expression to this order, upon expansion, will contain both first- and second-order quantities. In Ref. 1 an attempt is made to reexpress the second-order quantities in terms of first-order quantities. To this end a series of tedious calculations, that spans in addition to Ref. 1 at least two other papers (Refs. 3 and 4), are made and it is observed that the energy contains a second-order contribution that depends upon the initial conditions. However, in the course of these calculations several approximations are made. For example, in Eq. (5.8) of Ref. 1 certain second-order quantities are neglected. [Later, in Eq. (8.4), it is observed that the neglected part contributes a constant value to the energy, but there it is erroneously stated that this contribution is not essential.] In Eqs. (6.6) and (6.7) a narrow spectrum has been assumed and terms of the order of the square of the group velocity divided by the phase velocity have been neglected. In our papers (Refs. 2 and 5–7) we have made no approximations and have observed that the energy can be expressed in terms of first-order quantities alone. Our calculation is simple and concise [see Ref. 5, Eqs. (19)–(21) and Ref. 2, Eqs. (37)–(42)]. Moreover, our result is to be expected since the linearized theory by itself forms a Hamiltonian system for any initial condition; hence the energy is necessarily a bilinear expression in the first-order quantities—just as it is for simple oscillator systems with a finite number of degrees of freedom.

Contrary to the claim in the Comment, there is nothing ambiguous about the fact that the energy constant should depend on the initial conditions for first-order quantities. The energy of a simple oscillator clearly depends upon the initial momentum and linear displacement from equilibrium. Although the initial conditions for first-order quantities can be chosen arbitrarily, the initial second-order perturbation of the distribution function that appears in the energy cannot simply be set to zero. This is because, as we have shown, this second-order quantity is determined by the first-order quantities. Thus the expression that Best claims to be the energy of a Landau damped wave is incomplete.

In Best's Comment he has asserted that the condition we called dynamical accessibility is responsible for the fact that the energy is expressible in terms of first-order quantities. This is not the case. Dynamical accessibility, which is merely the statement that perturbations are caused by forces that are generated by a one-particle Hamiltonian, e.g., electromagnetic forces, is not at issue here, since Best's calculations are confined to Maxwellian equilibria where the constraint plays no role. The energy in this case reduces to the early result of Refs. 8 and 9. We have also treated the case where the constraint is not imposed, as can easily be seen from the simple derivation of the general energy expression in Ref. 2. We note that the Maxwellian energy expression is the simplest case of the general energy expression we derived for arbitrary Maxwell–Vlasov equilibria and arbitrary electromagnetic perturbations (Refs. 2 and 5–7).

In the context of Best's Eq. (8.1) it is stated that an additional "constant" piece of the total energy is "set to zero" with the argument that it "has no physical relevance." In reality relation (8.1) is a direct consequence of Eqs. (6.8) and (6.10) and the constant piece need not be set to zero in an *ad hoc* manner. Thus Best has obtained the strange result that the total energy perturbation should vanish for all perturbations. This strange result is due to the approximations made and the terms omitted.

This is distinct from the expression of Eq. (42) or Eq. (98) of Ref. 2 for the energy of longitudinal perturbations, which are clearly nonzero and non-negative for Maxwellian equilibria. (Best's statement that we "found in section VIIC and Appendix C that the total energy can take...negative values,..." is simply untrue for Maxwellian equilibria.) The counterpart of Eqs. (42) and (98) for transverse

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perturbations were given in Refs. 5 and 6 and can also be written as (Ref. 7) follows:

$$\mathscr{E}=\frac{V}{32}\sum_{k}\int v_{\parallel}\,\frac{|\epsilon_T-c^2/v_{\parallel}^2|}{\mathrm{Im}\,\epsilon_T}\,|\mathbf{E}|^2 dv_{\parallel}\,, \tag{1}$$

with obvious definitions of symbols. Note that the above energies are not just "constants," but expressions involving equilibrium and first-order quantities.

Contrary to Best's claim the numerical value of our energy expression certainly has meaning. In general energy is only defined up to a constant. However, the energy here is the *difference* between the energy of the perturbed state and the energy of the equilibrium state. It is well known that the sign of the energy of linear perturbations has practical meaning. In particular, negative energy modes are seen to be destabilized in traveling wave tubes and, in general, in beam plasma systems (see, e.g., Ref. 10 and references therein). In the theory of Hamiltonian dynamical systems the energy of a linearly stable mode has an invariant signature that is a basis for distinguishing normal forms (see, e.g., Ref. 11). We would also like to point out that there is an intimate similarity between Penrose's criterion and the criterion for the existence of negative-energy waves. The total energy expressions we have derived, contrary to energy density, are quite useful for describing stability in Vlasov plasmas. (See, e.g., in addition to our Refs. 2 and 5–7, Refs. 8, 9, and 12.)

Contrary to Best's claim, the integrand of Eq. (42) in our paper is the exact energy density. In the derivation no terms have been omitted because of space integration.

In the Comment and in Ref. 1 it is argued that for small Landau damping one can distinguish two time scales for energy propagation of a wave packet. In particular, it is argued that a portion of the energy, which has a density given by the dielectric energy expression, propagates at a group velocity while another portion precurses this and travels at a phase velocity. However, upon examination of the exact energy density, as given, for example, by the integrands of the space integrals in Eq. (44) of Ref. 2, one observes that in general the energy has contributions moving at a spectrum of velocities, including the group velocity. The last term of this expression propagates at the group velocity as it Landau damps while the second and third terms, which involve time integrals over the electric field, propagate at no particular velocity. Best's result arises because important terms have been neglected. It is perplexing to us why it is claimed in Ref. 1 and in the Comment that energy density is important in contradistinction to the total energy. The total energy sheds light on transport, as we have described above and, most importantly, only the total energy is a constant of the motion.

Contrary to Best's claims in the Comment our derivation of the general energy expression in Ref. 2 does not make use of Van Kampen type modes and does not require products of generalized functions. The transformation involving the perturbed distribution function that is described in Sec. IX of Ref. 2 is related, but not equivalent, to Van Kampen's calculation because it makes no assumption about time dependence. This transformation is used for the purpose of diagonalizing the energy expression, which has already been obtained by other means. The diagonalization procedure is akin to the procedure for transforming elementary linear oscillator systems into normal coordinates. The mathematics employed is of a character similar to that used to prove Parsevals theorem in Fourier analysis.

In addition to the claim in Ref. 1 and in the Comment that perturbations of a homogeneous Maxwellian Vlasov plasma do not necessarily change the energy, it is claimed that perturbations change the entropy. This latter claim is also incorrect in general. When perturbations are caused by, e.g., electromagnetic fields the constraint of dynamical accessibility is fulfilled and, as shown in Refs. 2, 5, and 6, integrals over any function of the distribution function are conserved. If mechanisms exist that produce perturbations that violate this constraint, it would seem appropriate to incorporate them into the dynamics. Such a dynamics is outside of Vlasov theory, which of course, conserves entropy.

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