9I 3 δ^2F: A Generalized Energy Principle for Determining Linear and Nonlinear Stability.*
P. J. MORRISON,** *Institute for Fusion Studies, The University of Texas at Austin.*

A generalization of the ideal MHD energy principle, δW, is presented. The generalization is applicable to the equilibria of all of the basic nondissipative plasma models. Thus, for example, one can treat fluid equilibria with flow, models with finite Larmour radius effects, and kinetic theories. The δW energy principle arises because the perturbed Hamiltonian for static MHD equilibria has kinetic and potential energy terms of standard form, in which case (Liapunov) stability is determined by the potential alone. More generally the Hamiltonian structure of plasma models in Eulerian variables is noncanonical[1] and the Hamiltonian is not of standard form. Nevertheless, there is a generalization of the Hamiltonian, a generalized free energy (F), that has equilibria as stationary points and for which definiteness of the second variation, δ^2F, is sufficient for stability[2]. This definiteness of δ^2F is a more dependable criterion for practical stability than conventional linear spectral stability. Indeed, sometimes spectral theory is highly misleading because nonlinear instability for arbitrarily small perturbations can arise. This can occur when δ^2F is indefinite, yet spectral stability theory indicates stability. Physically, δ^2F - not the second variation of the "usual" energy-is the appropriate perturbed energy. Identifying the appropriate energy yields a new, more general, definition of a negative energy mode: indefiniteness of δ^2F and concomitant spectral stability. One can use δ^2F in much the same spirit as δW; i.e. insert trial functions and then vary parameters to search for indefiniteness. A further test is used to distinguish linear instability from negative energy modes. Note that δ^2F is applicable even if the dielectric functional is intractable or not even defined. Many examples are available. In particular the δ^2F velocity thresholds for fluid and kinetic streaming instabilities are lower than those of conventional linear theory. Also MHD equilibria with flow and FLR effects[3] have been treated.

*Supported by the U.S. Department of Energy under Contract #DE FG05-80ET-53088.
**In collaboration with M. Kotschenreuther.

[1]P.J. Morrison and J.M. Greene, Phys. Rev. Lett. **45**, 790 (1980).
[2]D.D. Holm, J.E. Marsden; T. Ratiu and A. Weinstein, Phys. Rep. **123**, 3 (1985); R.D. Hazeltine, D.D. Holm, J.E. Marsden and P.J. Morrison, Proc. ICPP Lausanne **2**, 204 (1984); P.J. Morrison and S. Eliezer, Phys. Rev. A **33**, 4205 (1986).
[3]C. T. Hsu, *et al.*, see abstract this meeting.

9I 4 Dynamics of Resonant Magnetic Perturbations in Toroidal Plasmas with Low Collisionality.* †
MICHAEL KOTSCHENREUTHER, *Institute for Fusion Studies, The University of Texas at Austin.*

Neoclassical effects are shown to strongly modify the dynamics of resonant magnetic perturbations, when the mean free path is long. The analysis begins by rigorously deriving reduced nonlinear fluid equations to describe the region near a rational surface using kinetic theory. Novel effects of the neoclassical terms are demonstrated both analytically and by numerical simulation of the fluid equations[1]. These equations are derived using a systematic two scale expansion in the parallel gradients, which is the kinetic analog of previous MHD calculations in toroidal geometry[2]. Strong rotational damping and bootstrap current effects arise, as previously discussed by Callen and Shaing[3]. New effects considered include 1) Large self-consistent plasma currents are shown to arise in magnetic islands and stochasticity when the neoclassical transport is not intrinsically ambipolar. In stellarator fusion reactor regimes, large steady state resonant magnetic perturbations (e.g. from equilibrium Pfirsch-Schluter currents or coil errors) can be strongly reduced or "healed". The linear and nonlinear stability of low and moderate m tearing and interchange modes is also affected. 2) In tokamaks, an analysis of nonlinear Rutherford Island evolution shows that bootstrap current effects contribute a destabilizing term[4]. The destabilizing term dominates for island widths smaller than $q\beta/\Delta'\sqrt{\varepsilon}$. Overlap of the resulting moderate-m islands can seriously degrade confinement for $\beta > 1\%$.

[1]M. Kotschenreuther and A.Y. Aydemir, Institute for Fusion Studies Report.
[2]M. Kotschenreuther, R.D. Hazeltine and P.J. Morrison, Phys. Fluids **28**, 294 (1985).
[3]J.D. Callen and K.C. Shaing, Bull. Am. Phys. Soc. Vol. 30, 1424 (1985).
[4]R. Carrera, R.D. Hazeltine and M. Kotschenreuther, Phys. Fluids **29**, 899 (1986).
*In collaboration with A. Y. Aydemir
†Work supported by the U.S. Department of Energy.

9I 5 Nonlinear Periodic Waves in Plasma Physics.* † E. R. TRACY, *College of William and Mary.*

During the study of the nonlinear aspects of plasma behavior one often turns to simplified models with the hope of gaining deeper insight than can be gotten by a direct approach on the full problem. Such models retain many of the relevant physical properties of the full system, but are more amenable to study. A number of important nonlinear models (the so-called soliton systems) have the added bonus of being exactly solvable so that, in principle, we can answer rigorously whatever physical questions we wish to pose. Many scientists are familiar with soliton systems on the infinite line (i.e. when the wave disturbances are localized in space). However, the recent progress made in the study of these systems with periodic boundary conditions, which are also physically important, is much less familiar. Many important new physical affects appear and new techniques have been developed for their investigation. For example: the non-linear Schrodinger equation arises commonly in the study of modulational problems. On the infinite line this system is stable, but with periodic boundary conditions solutions exhibit instabilities. Recent progress in the study of important nonlinear boundary value problems will also be discussed. Examples to be discussed include the Sinh-Poisson equation[1] - which arises in the study of two dimensional guiding center plasmas, and the Liouville equation - which occurs in the study of ideal two dimensional MHD equilibria. Also to be discussed are nonintegrable models which exhibit chaotic solitons[2].

1) A. C. Ting, PhD. Thesis, University of Maryland (1984).
2) S. N. Qian, PhD. Thesis, University of Maryland (1986).

*This work was carried out in collaboration with H. H. Chen and Y. C. Lee of the University of Maryland.
†This work was supported by NSF, ONR and the College of William and Mary.

$\delta^2 F$: A Generalized Energy Principle

Philip Morrison
IFS & Phys. Dept.
UT Austin

Nov. 7, 1986
APS Baltimore

Collaborators: R. Hazeltine, C. Hsu &
M. Kotschenreuther

Overview

Noncanonical Mechanics

Stability

Negative Energy Modes

Examples

emphasis reduced fluid models w/ flow

Why Noncanonical Mechanics?

Harmonic Oscillator

Energy: $H = \frac{1}{2}(p^2 + q^2)$ quadratic

Dynamics: $\ddot{q} = -kq$ linear

Eulerian Variable Field

Energy: $H = \int \frac{1}{2} \rho v^2 dz$ quadratic

Dynamics: $\vec{v}_t \sim \vec{v} \cdot \nabla \vec{v}$ quadratic

---

General feature of media described by Eulerian variables: Noncanonically Hamiltonian invicid fluids, ideal MHD, ideal 2-fluid Maxwell-Vlasov, Liouville, ...

Nonlinearity hides in Poisson bracket.

Noncanonical or S. Lie (1890), Dirac & others

II. Generalized Hamiltonian Mechanics (finite N)

Hamiltons Eqs. :

$$\dot{q}_k = \frac{\partial H}{\partial p_k} = [q_k, H]$$

$$\dot{p}_k = -\frac{\partial H}{\partial q_k} = [p_k, H] \qquad k = 1, 2, \ldots N$$

Poisson Bracket :

$$[f, g] = \sum_{k=1}^{N} \left(\frac{\partial f}{\partial q_k} \frac{\partial g}{\partial p_k} - \frac{\partial g}{\partial q_k} \frac{\partial f}{\partial p_k} \right)$$

Cosymplectic Form :

let

$$z^i = \begin{cases} q_k & i = 1, 2, \ldots \; N = k \\ p_k & i = k + N = N+1, \ldots \; 2N \end{cases}$$

obtain

$$[f, g] = \frac{\partial f}{\partial z^i} J_c^{ij} \frac{\partial g}{\partial z^j}$$

$$(J_c^{ij}) = \begin{bmatrix} 0 & I_N \\ -I_N & 0 \end{bmatrix}$$

$$\dot{z}^i = J_c^{ij} \frac{\partial H}{\partial z^j} = [z^i, H]$$

kinematics or phase space

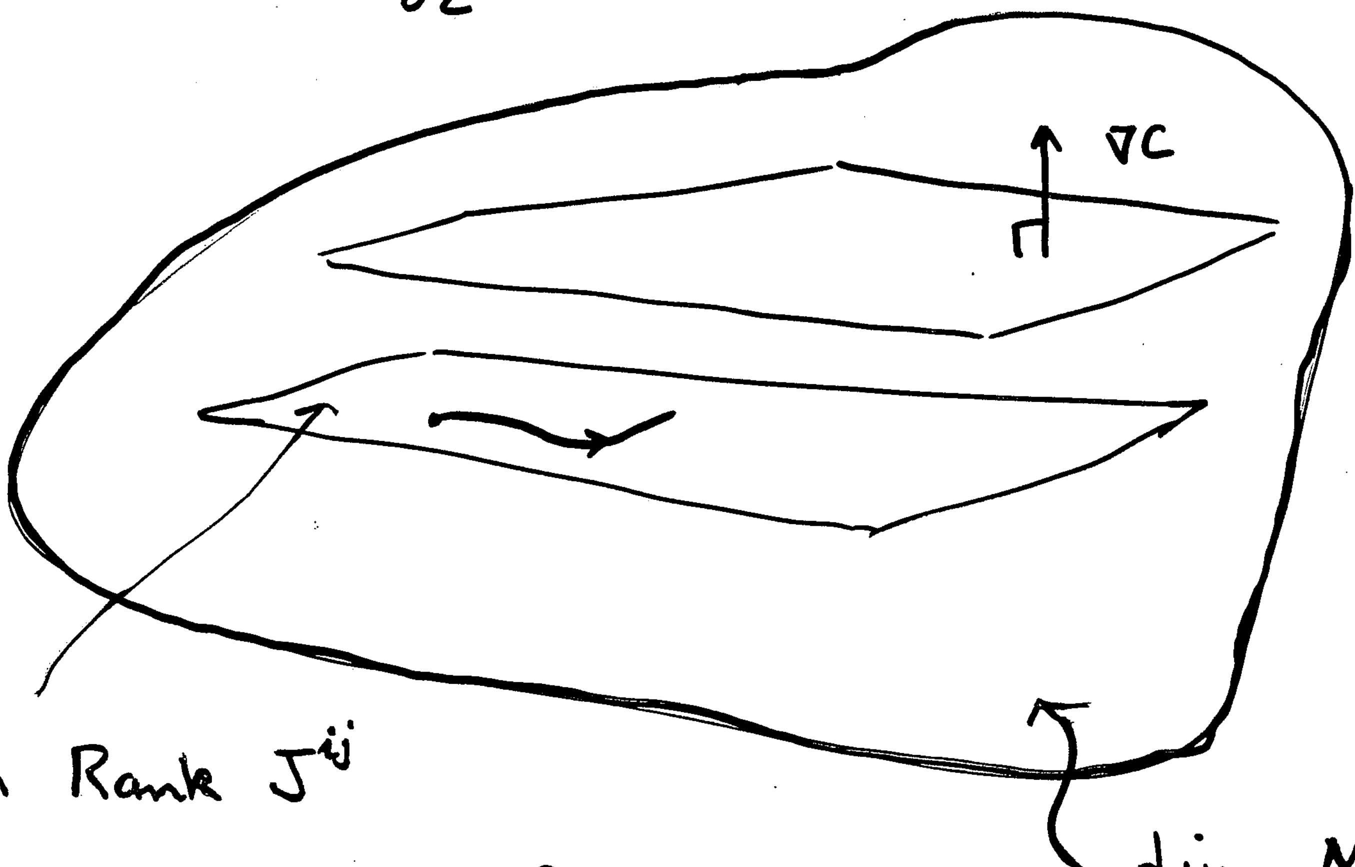
dynamics

Generalized Phase Space

$$J_c^{ij} \longrightarrow J^{ij}(z)$$

The J^{ij} is not arbitrary; it must maintain bracket properties. The generalization does allow $\det J = 0$

$$\Rightarrow \quad J^{ij} \frac{\partial C}{\partial z^j} = 0 \qquad \text{Null eigenvectors}$$



dim Rank J^{ij}

trajectories are confined to leaves.

dim M
($J^{ij} = M \times M$)

IV STABILITY

Spectral $\quad \psi = \psi_e + \delta\psi\, e^{i\omega t}$

linearize - $\mathrm{Im}\,\omega < 0$?

$\mathrm{Im}\,\omega = 0$ stable

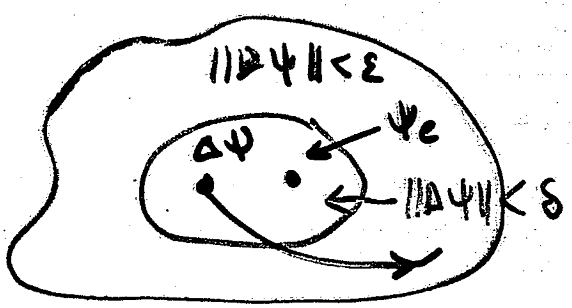
Linear Stability secular growth - Linear eqs. stable

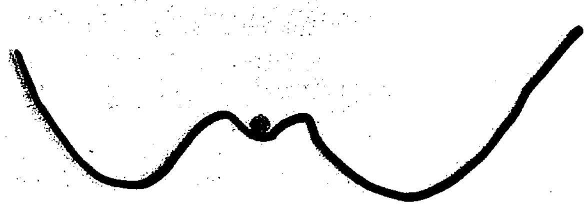
Formal Stability Liapunov Function - $\delta^2 F$ definite

Nonlinear Stability

Definition. An equilibrium ψ_e is nonlinearly stable if for all $\varepsilon > 0$ there is a $\delta > 0$ such that for $\psi(t=0) = \psi_e + \Delta\psi(t=0)$ with $\|\Delta\psi\| < \delta$ (at $t=0$), then $\|\Delta\psi\| < \varepsilon$ for all time.

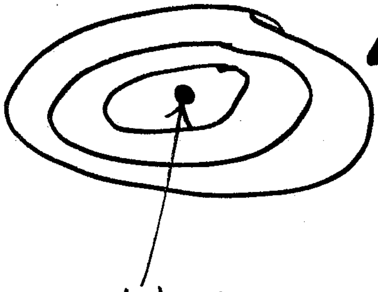
Dynamics determined by nonlinear equations ($\Delta\psi$ finite) requires norm.





Nonlinearly Stable

Liapunov Stability



Surfaces defined by constant of motion

equilibrium

$$F(z) \approx F(z_e) + \cancelto{0}{\frac{\partial F(z_e)}{\partial z}(z - z_e)}$$

$$+ \frac{\partial^2 F}{\partial z^2} \frac{(z - z_e)}{2} + \cdots$$

Hamiltonian Systems Have "Built In" Candidates for Liapunov Functions

Standard Hamiltonian:

$$H = \sum \frac{p_i^2}{2} + V(q) \qquad \frac{\partial^2 V}{\partial q_i \partial q_j} > 0$$

Lagrange Condition

Arbitrary Hamiltonian:

$$\frac{\partial^2 H}{\partial q_i \partial q_j} \text{ definite}$$

Dirchelet Condition

Noncanonical Hamiltonian:

$$F = H + C$$

$$\frac{\partial^2 F}{\partial z_i \partial z_j} \text{ definite}$$

$\delta^2 F$ Stability

$$\frac{dF}{dt} = 0 \quad , \quad \frac{\delta F}{\delta \psi^i}[\psi_e] = 0$$

$$\delta^2 F = \int \delta\psi^i \frac{\delta^2 F}{\delta\psi^i \delta\psi^j} \delta\psi^j$$ definite ?

$$\frac{\partial \psi^i}{\partial t} = \{\psi^i, H\} = \{\psi^i, F\} = O^{ij} \frac{\delta(H+C)}{\delta\psi^j}$$

$$\frac{\delta(H+C)}{\delta\psi^i} = 0 \quad \Rightarrow \quad \frac{\partial\psi^i}{\partial t} = 0$$

More General Equil.

Oberman & Kruskal, Rosenbluth, Gardner

Arnold

Recently

Marsden & Weinstein et al.

Hazeltine & PJM

Kotschenreuther

Two Things of Interest

* Disagreement between
  $\delta^2 F$ and Nonlinear stability
  (only for ∞ dim systems)

* Disagreement between
  $\delta^2 F$ and Spectral stability
  (Negative energy modes)

indefiniteness of $\delta^2 F$ can mean two things:
* linear instability
* negative energy modes

Both can be disastarous

PERTURBED ENERGY

$$\dot{z}^i = J^{ij}(z)\frac{\partial F}{\partial z^j}$$

Linearize : $z = z_e + \delta z$

Equilibrium: $\dfrac{\partial F(z_e)}{\partial z^j} = 0$

Generally $\dfrac{\partial H(z_e)}{\partial z^j} \neq 0$ or

yields trivial equilibria.

$$\delta \dot{z}^i = J^{ij}(z_e)\,\frac{\partial^2 F(z_e)}{\partial z^j \partial z^k}\,\delta z^k$$

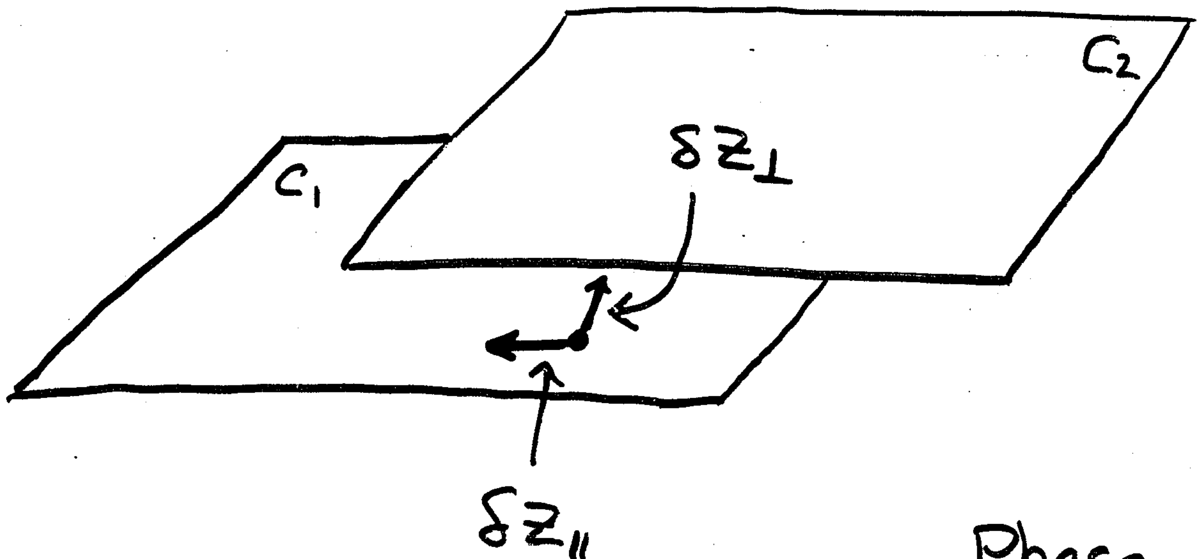
$$= \left\{ \delta z^i , \frac{\delta^2 F}{2} \right\}_L$$

$$\uparrow$$

Perturbed Hamiltonian $\dfrac{\delta^2 F}{2}$

Not $\dfrac{\delta^2 H}{2}$! What is the energy.

$\delta^2 F/2$ = Free Energy



Phase Space

We add a source term and pull the system away from equilibrium.

$\delta z_{\parallel}$ is the only relevant part

Since $\delta z_{\perp}$ changes the equil.

Thermodynamic analogy: $dW = dU + TdS$

Let $H \to H + H_{ext}(t)$ input $\quad \delta z_{\parallel} \quad \delta z$

$H_{ext} = z^j S_j(t) \quad (= q\, F_{ext}(t))$

$$\boxed{\Delta H_c = -\int_0^t \dot{z}^j S_j(t)\, dt = \frac{\delta^2 F}{2}}$$

Negative Energy Modes

For systems with finite # of degrees of freedom or complete discrete spectra $\delta^2 F/2$ can be written in action-angle form

$$\frac{\delta^2 F}{2} = \sum_i \omega_i \; J_i$$

freq

action

The signature of ω_i cannot be changed. A mode has Neg. energy when $\omega_i < 0$. Agrees with $\omega \frac{\partial \varepsilon}{\partial \omega}$ condition.

General Definition of Negative Energy

* Real Spectrum
* Indefinite $\delta^2 F$

THE DANGER OF SPECTRAL THEORY
(Cherry's Example)

o.d.e.'s

$$\dot{z}_1 = z_2 - \alpha(z_2 z_3 + z_1 z_4)$$

$$\dot{z}_2 = -z_1 + \alpha(z_2 z_4 - z_1 z_3)$$

$$\dot{z}_3 = -2 z_4 - \alpha z_1 z_2$$

$$\dot{z}_4 = 2 z_2 + \frac{\alpha}{2}(z_2^2 - z_1^2)$$

Linearly (spectrally) stable yet unstable

---

linear analysis $\Rightarrow$ real frequencies, yet unstable. Solution diverges in finite time. Explosie instab.

$$z_1 = \frac{\sqrt{2}}{\alpha(t-\varepsilon)} \sin(t+\gamma) \qquad z_2 = \frac{\sqrt{2}}{\alpha(t-\varepsilon)} \cos(t+\gamma)$$

$$z_3 = \frac{1}{\alpha(t-\varepsilon)} \sin(2t+\gamma) \qquad z_4 = \frac{-1}{\alpha(t-\varepsilon)} \cos(2t+\gamma)$$

Cherry's Hamiltonian:

$$\boxed{H = \underbrace{\tfrac{1}{2}(p_1^2 + q_1^2)}_{\omega_1 = 1} - \underbrace{(q_2^2 + p_2^2)}_{\omega_2 = -2} + \frac{\alpha}{2}\left\{q_2(q_1^2 - p_1^2) - 2 q_1 p_1 p_2\right\}}$$

two features:

(i) $\mathcal{O}(3)$ resonance: $2\omega_1 + \omega_2 = 0$

(ii) Negative energy mode

$\Downarrow$

generic behavior

$\delta^2 F$ indefinite

VI. Two-Stream Instability (warm ions & electrons)

$$\frac{\partial v_\alpha}{\partial t} + v_\alpha \frac{\partial v_\alpha}{\partial x} = \frac{e_\alpha}{m_\alpha} E - \frac{1}{\rho_\alpha} \frac{\partial p_\alpha}{\partial x}$$

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha v_\alpha) = 0 \qquad \frac{\partial E}{\partial x} = 4\pi e (n_i - n_e)$$

equil. n_{0i}, n_{0e}, v_D ← drifting electrons

spectral stability condition given via

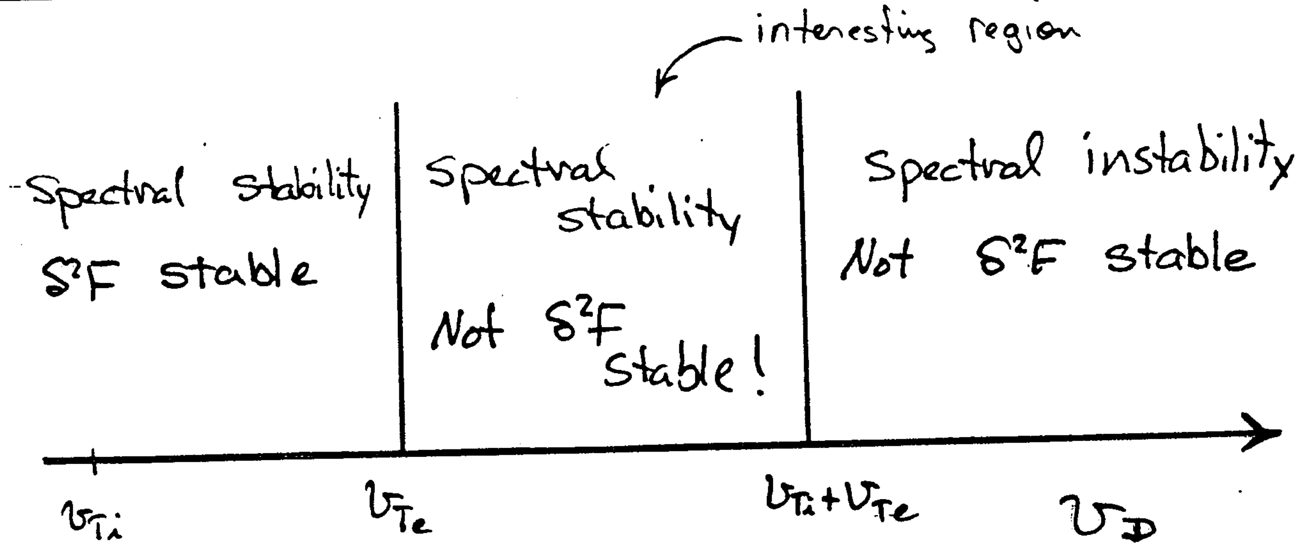
$$\boxed{0 = 1 - \frac{\omega_{pi}^2}{\omega^2 - k^2 v_{Ti}^2} - \frac{\omega_{pe}^2}{(\omega - k v_0)^2 - k^2 v_{Te}^2} = \varepsilon(k, \omega)}$$

Threshold: $v_D > v_{Ti} + v_{Te}$ $\Rightarrow$ instab.

$\delta^2 F$:

Threshold: $v_D < v_{Te}$ $\Rightarrow$ $\delta^2 F$ positive definite

---



2-Dim Euler Equation

$\omega(x,y,t) \equiv$ Vorticity

$\phi(x,y,t) \equiv$ Stream Function

$\nabla^2\phi = \omega \qquad\qquad \vec{v} = \hat{z} \times \nabla\phi$

$\omega_t = -\vec{v}\cdot\nabla\omega = -[\phi,\omega] = \hat{z}\cdot\nabla\phi\times\nabla\omega$

$H = \int \frac{1}{2}|\nabla\phi|^2 \qquad\qquad C = \int \mathcal{F}(\omega)$

$F = H + C \qquad\qquad \delta F = 0 \implies$

$\phi = \mathcal{F}'(\omega) \qquad\qquad \nabla^2\phi = \mathcal{F}'^{-1}(\phi)$

$$\boxed{\delta^2 F = \int |\nabla\delta\phi|^2 + \mathcal{F}''(\omega)(\delta\omega)^2}$$

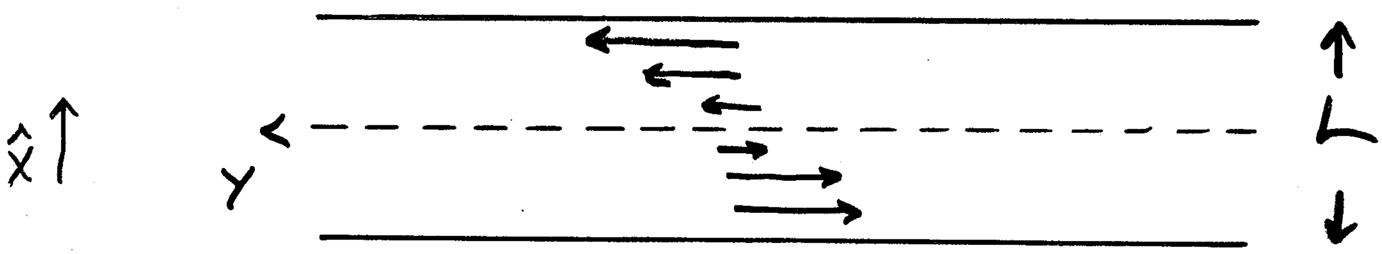
Stability if $\mathcal{F}'' > 0 \iff$

$$\boxed{\frac{\nabla\phi\cdot\nabla\omega}{|\nabla\phi|^2} > 0}$$

Arnold's Theorem

Plane Parallel Flow (K-H instab.)

$$\vec{v} = v(x)\,\hat{y} = v_0 \tanh(\beta x)\,\hat{y}$$



Rayleigh - Arnold Condition

$$\boxed{\frac{v_{xx}}{v} > 0 \quad \text{for stability}}$$

For tanh Always negative
No threshold

Rosenbluth - Simon Condition

$$\boxed{\begin{array}{l} L\beta_c = 2.39 \\ \qquad \text{Unstable iff } \beta > \beta_c \end{array}}$$

For linear / spectral stability

Low-β RMHD (axi on helical sym.)

$\vec{B}_{\parallel} = B_0 \hat{z} = \text{const.}$ $\qquad$ $\vec{B}_p = -\hat{z} \times \nabla\psi$

$$\nabla^2 \psi = J$$

$U_t = [U, \phi] - [J, \psi]$ $\qquad$ vorticity

$\psi_t = [\psi, \phi]$ $\qquad$ $\parallel$-Ohms Law

$H = \int \frac{|\nabla\phi|^2}{2} + \frac{|\nabla\psi|^2}{2}$ $\qquad$ $C_1 = \int \mathcal{F}(\psi)$

$$C_2 = \int U\, G(\psi)$$

Plane Parallel Flow (w/ B)

Modified R-A Condition ($B = -\psi_x$)

$$\boxed{\left(1 - \frac{v^2}{B^2}\right)\frac{B_{xx}}{B} - 2\left(\frac{v}{B}\right)^2\left(\frac{v_x}{v} - \frac{B_x}{B}\right)\frac{B_x}{B} > 0}$$

$v = v_0 \tanh \beta x$ $\qquad$ $\psi = \frac{b_0}{\beta} \cosh^m \beta x$

Subalfvenic: $\quad m^2 > v_0^2 / b_0^2$

$$\boxed{m^2\left(\frac{2 - 3m}{2 - m}\right) < \frac{v_0^2}{b_0^2} < m^2}$$

RMHD (cont.) - General Equil.

$$F = \int \frac{1}{2}|\nabla\phi|^2 + \frac{1}{2}|\nabla\psi|^2 + \mathcal{F}(\psi) + U\,G(\psi)$$

$\delta F = 0 \quad \Rightarrow \quad$ Flow Modified Reduced Grad-Shafranov Eq. ↓

$$\boxed{\begin{gathered}[1 - G'^2(\psi)]\nabla^2\psi - G'(\psi)G''(\psi)|\nabla\psi|^2 = \mathcal{F}'(\psi) \\ \phi = G(\psi)\end{gathered}}$$

Note: (i) $G' = -v/B$ — poloidal Alfven Sing. $\pm G' = 1$

(ii) $\phi = +\psi$ — Nonlinear Alfven wave, $\hat{z} \times \nabla$ wave frame

(iii) $G = 0$

$$\nabla^2\psi = J(\psi) = \mathcal{F}'(\psi)$$

Interesting Transformation:

$$\chi = \int^{\psi} \sqrt{1 - G'^2(\psi')}\,d\psi'$$

$$\nabla^2\chi = \partial\mathcal{F}/\partial\chi = \mathcal{G}(\chi)$$

RMHD (cont.) - $\delta^2 F$

No Flow

$$\delta^2 F = \int |\nabla\delta\phi|^2 + |\nabla\delta\psi|^2 + (\delta\psi)^2 \frac{\vec{B}_p \times \hat{z} \cdot \nabla J_{\parallel}}{B_p^2}$$

Kinetic Energy

Line Bending

Kink

Poloidal Flow

$$\delta^2 F = \int |\nabla\delta\phi - \nabla G'\delta\psi|^2 + |\nabla\delta\psi|^2\left(1 - \frac{v^2}{B_p^2}\right)$$

N.L. Alfven Wave

Line Bending w/ Flow

$$+ \frac{(\delta\psi)^2}{B_p^2}\left\{ \vec{B}_p \times \hat{z} \cdot \nabla J_{\parallel}\left(1 - \frac{v^2}{B_p^2}\right) - \frac{1}{2}\left[\vec{B}_p \times \hat{z} \cdot \nabla \frac{v^2}{B_p^2}\right]\left[\vec{B}_p \times \hat{z} \cdot \nabla B_p^2\right]\right\}$$

Flow Modified Kink

Ram pressure modification

CRMHD

$v \equiv$ parallel velocity

$p \equiv$ pressure

$h \equiv r\cos\theta \qquad \nabla h = \vec{\kappa} \equiv$ curvature

$$U_t = [U, \phi] + [\psi, J] + 2[p, h]$$

$$\psi_t = [\psi, \phi]$$

$$v_t = [v, \phi] + [\psi, p]$$

$$p_t = [p, \phi] + \beta[\psi, v] + 2\beta[h, \phi]$$

$\parallel$ compressibility $\qquad \perp$ compressibility

$$H = \int \frac{1}{2}|\nabla\phi|^2 + \frac{v^2}{2} + \frac{|\nabla\psi|^2}{2} + \frac{p^2}{2\beta}$$

$$C_1 = \int \mathcal{F}(\psi) \qquad C_2 = \int v\, N(\psi) \qquad C_3 = \int L(\psi)\left(\frac{p}{\beta} + 2h\right)$$

$$C_4 = \int G(\psi)\, U - v\, G'(\psi)\,(p/\beta + 2h)$$

$\delta^2 F$ w/o Flow for CRMHD

$$F = H + C_1 + C_3$$

$$F = \int \frac{|\nabla\phi|^2}{2} + \frac{|\nabla\psi|^2}{2} + \frac{v^2}{2} + \frac{p^2}{2\beta} + \mathcal{J}(\psi) + L(\psi)\left(\frac{p}{\beta} + 2h\right.$$

$\delta F = 0 \Rightarrow$ No flow

$$p = -L(\psi)$$

$$\nabla^2 \psi = \mathcal{J}'(\psi) - p'(\psi)\left[\frac{p(\psi)}{\beta} + 2h\right]$$

$$\delta^2 F = \int \underbrace{(\delta v)^2 + |\nabla\delta\phi|^2}_{\text{K. E.}} + \underbrace{|\nabla\delta\psi|^2}_{\text{Line Bending}}$$

$$\underbrace{\frac{1}{\beta}\left[\delta p - \frac{\hat{z}\times\vec{B}\cdot\nabla p}{B^2}\delta\psi\right]^2}_{\text{Sound} \sim \nabla\cdot\xi} + \underbrace{(\delta\psi)^2\left[\frac{\hat{z}\times\vec{B}}{B^2}\cdot\nabla J_{\parallel}\right]}_{\text{Kink}}$$

$$+ \underbrace{(\delta\psi)^2\left[\frac{2(\hat{z}\times\vec{B}\cdot\nabla p)(\hat{z}\times\vec{B}\cdot\nabla h)}{B^2}\right]}_{\text{interchange}}$$

CRMHD w/ Toroidal Flow

$$F = H + C_1 + C_2 + C_3$$

$$F = \int \frac{v^2}{2} + \frac{|\nabla\phi|^2}{2} + \frac{|\nabla\psi|^2}{2} + \frac{p^2}{2\beta} + \mathcal{J}(\psi) + v\,N(\psi)$$
$$+ L(\psi)\left(p/\beta + 2h\right)$$

$$\delta F = 0 \Rightarrow \qquad v = -N(\psi)$$
$$p = -L(\psi)$$

$$\nabla^2\psi = \mathcal{J}'(\psi) - p'(\psi)\left(\frac{p(\psi)}{\beta} + 2h\right)$$
$$- v'(\psi)\,v(\psi)$$

$$\delta^2 F = \int |\nabla\delta\phi|^2 + \left(\delta v - \frac{\hat{z}\times\vec{B}}{B^2}\cdot\nabla v\,\delta\psi\right)^2$$
$$+ \frac{1}{\beta}\left(\delta p - \frac{\hat{z}\times\vec{B}}{B^2}\cdot\nabla p\,\delta\psi\right)^2 + |\nabla\delta\psi|^2$$

$$+(\delta\psi)^2\Big(\underbrace{\frac{\hat{z}\times\vec{B}}{B^2}\cdot\nabla J_{\parallel}}_{\text{kink}} + \underbrace{2\left(\frac{\hat{z}\times\vec{B}}{B^2}\cdot\nabla p\right)\left(\hat{z}\times\vec{B}\cdot\nabla h\right)}_{\text{interchange}}\Big)$$

Flow <u>not</u> destabilizing! But $v \sim v_{Ap}$.