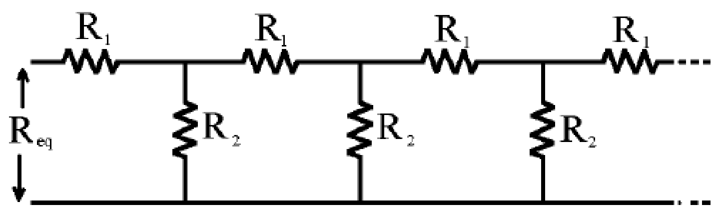
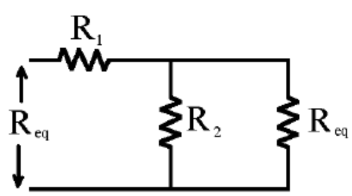
Equivalent for an Infinite Array of Resistors

In practice, of course, one cannot have an infinite number of resistors. However, sometimes an infinite set of resistors can be used as a model for a real system. One example is a model for certain transmission lines (long cables). Consider the following infinite array:



The trick for solving infinite systems is to exploit a symmetry of the system. Here the basic symmetry is translational. If we remove the first pair of resistors (or the first n pairs), the remaining array is still infinite and looks exactly the same as what we started with. Therefore, we can replace it with R_{eq}, which we don't know, but we presume to exist. Then the circuit looks like:



Using circuit reduction, you get

$$R_{eq} = R_1 + R_2 \parallel R_{eq} = R_1 + \frac{R_2 R_{eq}}{R_2 + R_{eq}}$$

$$R_{eq} = \frac{R_1 \left(R_2 + R_{eq} \right)}{R_2 + R_{eq}} + \frac{R_2 R_{eq}}{R_2 + R_{eq}}$$

and then multiplying to get rid of the denominator on the right, and rearranging, one gets the quadratic equation

$${R_{eq}}^2 + R_1 R_{eq} - R_1 R_2 = 0$$

where clearly one must choose the plus sign (or else R_{eq} is negative, which means it supplies energy rather than dissipates energy).
One special case is where $R_1 = R_2$, in which case

$${R_{eq}}^2 + R_1 R_{eq} - {R_1}^2 = 0$$

$$R_{eq} = R_1 \left(\frac{1 + \sqrt{5}}{2} \right)$$

Incidentally, the ratio on the right is known as the Golden Ratio (φ) which is claimed to have some aesthetic appeal in the Arts.