Solar Sails - A Discussion

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Abstract

The mechanisms governing solar sails are discussed. Theoretical calculations are shown demonstrating how radiation pressure is used to levitate objects on Earth. This idea is extended to space, where solar sails harness the force due to radiation pressure to accelerate objects to speeds greater than Earth's escape velocity. Resonating cavities as they relate to solar sails are discussed as proposed by Meyer et al., along with a short discussion of IKAROS, the first solar sail propelled spacecraft.

Radiation Pressure

Radiation pressure was first postulated in the early 1600s by Johannes Kepler, who observed that a comet's tail always pointed away from the Sun[1]. Kepler concluded that pressure from sunlight was pushing particles from the comet, which caused their identifying stellar tail. Kepler's interest in the subject waned after his initial theory, and interest in radiation pressure didn't pick up again until Maxwell was working on his theory of electromagnetism in 1873[2]. Maxwell concluded that radiation pressure was a completely real phenomenon, associated with any electromagnetic wave[i]. This pressure, $\mathcal{P}$, can be calculated by the relation[3]

$$\mathcal{P} = \frac{\langle \vec{S} \rangle}{c}, \tag{1}$$

where $\langle \vec{S} \rangle$ is the time average of the electromagnetic wave's Poynting vector[ii] and c is the speed of light. When the time averaging is carried out, this relation is equivalent to

$$\mathcal{P} = \frac{\epsilon_0 {E_0}^2}{2c}, \tag{2}$$

where ϵ_0 is the permittivity of free space, and E_0 is the magnitude of the electric field. The fact that electromagnetic waves and photons exert a pressure is useful for a wide range of reasons. First and foremost, it allows for the levitation of actual particles in a laboratory. For example the glass sphere shown in Figure 1[4] has a mass (m) of 1 gram and a radius (r) of 1 mm. To levitate this sphere with a laser, the force due to

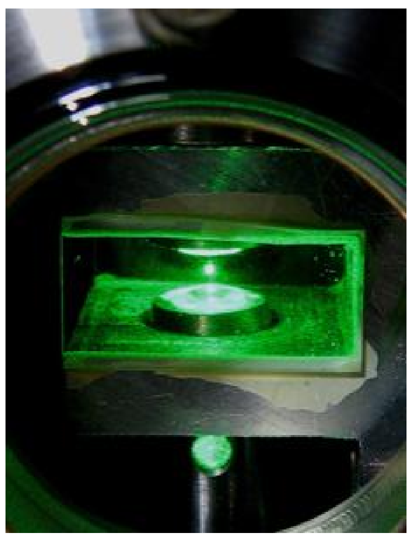

Figure 1: *The optical levitation of a glass bead by a* 532 nm *laser.*

gravity must be balanced with a force from the radiation pressure from the laser:

$$\vec{F}_{net} = \vec{F}_{laser} - \vec{F}_g = 0, \tag{3}$$

with $F_{laser} = \mathcal{P} \cdot \pi r^2$ and $F_g = mg$ where g is the acceleration due to gravity. Substituting in the known values will allow for the calculation of E_0.

$$\frac{\epsilon_0 {E_0}^2}{2} \pi r^2 = mg \rightarrow E_0 = \sqrt{\frac{2mg}{\pi \epsilon_0 r^2}} \tag{4}$$

[i] For this discussion, calculations will be taken with electric field $\vec{E} = E_0 e^{i(kz-\omega t)}\,(\hat{x})$ and magnetic field $\vec{B} = \frac{E_0}{c} e^{i(kz-\omega t)}\,(-\hat{y})$[3]

[ii] $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{\epsilon_0 {E_0}^2}{c} \cos^2(kz - \omega t)\hat{z}$

This shows an electric field strength of 2.655 $\frac{N}{C}$. Incorporating the conservation of energy (and the intensity I of the laser) allows for the calculation of the power of the laser using the equation

$$P = \int I \, \mathrm{d}a = \frac{c\epsilon_0 {E_0}^2}{2} \pi r^2 = c\, m\, g. \tag{5}$$

This gives a laser power of 2.94 MW, which is an extremely large power for a laser to be sustaining for longer than a few seconds. For the sphere to be levitated for one hour, the total energy expended would be 10.58GJ, or, at 13¢ per kilowatthour, a total cost of $382.19. Quite obviously, this is highly impractical, especially since this calculation was for a mass of 1 gram. For a mass of 1 kg, the required power increases to 2.94GW, and the cost for one hour of levitation increases to $382 200. It is important to keep in mind that these calculations were made for the levitation of a particle in Earth's gravitational field. Without the force due to gravity, the particle would accelerate away from the laser.

The radiation pressure of light from the sun at a distance of 1AU is $4.7 \cdot 10^{-6}$ Pa[5], which is a very small pressure, although it is crucially important to consider in interplanetary travel. Had NASA scientists ignored the solar radiation pressure when planning the Viking's travel to Mars, the spacecraft would have missed the planet by approximately 15 000 km[6].

Solar Sails

The idea of a spacecraft propeled by radiation pressure was first proposed by Friedrich Zander in 1924[7]. The momentum p of a photon can be related to its total energy $\mathcal{E}$ by

$$p = \frac{\mathcal{E}}{c}. \tag{6}$$

Because a photon has an associated momentum, it can exert a force on an object. This is useful in the application of space propulsion as an alternative to rocketry. Solar sails, which are essentially giant mirrors in space, use the ideas of the conservation of momentum and radiation pressure to propel objects using light, illustrated by Figure 2[8]. As shown in the previous section, solar sails would be completely useless on Earth because photon pressure is such a weak effect other forces easily dominate motion. However in the vacuum of space photon pressure can dominate the motion with an energy source that is essentially free[iii]. At first

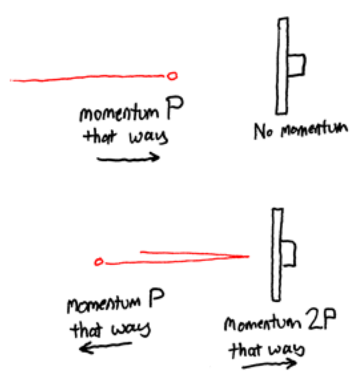

Figure 2: *An simple illustration of the mechanics behind solar sails.*

[iii]This energy is, of course, supplied by some source of electromagnetic wave. The name *solar sail* implies that the source is the sun, although as Figure 2 implies, they work with any source of electromagnetic wave.

glance, Figure 2 may seem to violate the conservation of energy, and at a *glance* it does. Momentum is conserved because the total momentum is the same before the collision as it is after the collision. Before the collision the photon has energy $\mathcal{E}$ and therefore momentum $p = \frac{\mathcal{E}}{c}$. During the collision, the photon transfers a tiny amount of energy to the solar sail, which the photon loses due to anti-Stokes scattering. The photon's energy loss (in the form of a decrease in its frequency) is a bit lower than its initial energy, although the photon has a new momentum $-p$. The attractive quality this effect has on the solar sail is that its new momentum is $2p$, because it is built from reflective material. Given that there can be a huge amount of photons striking the solar sail, it is easy to see that the sail's momentum will increase very quickly. The use of more than one collision per photon was proposed by Meyer et al.[9] in their article *Laser Elevator: Momentum Transfer Using an Optical Resonator*. The general idea of their proposal is illustrated in Figure 3[8]. By using a mirror mounted to the source of the photon, a resonating cavity can be created, and the solar sail can be made many times more effective than it was originally. These hypothetical situations all assert that the

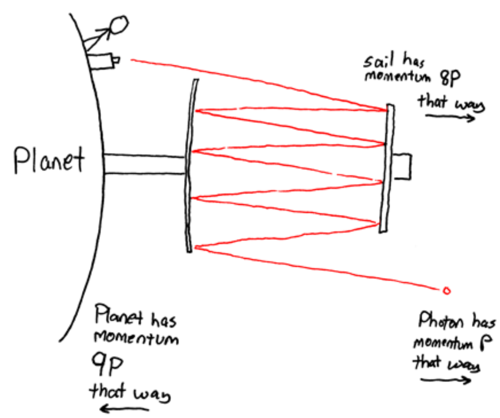

Figure 3: *A hypothetical solar sail, four times more effective than the one in Figure 2 with four times as many photon passes.*

reflectivity of the sail is 1. Meyer et al. calculated that it is possible to improve current theoretical solar sails to be over 1000 times as efficient as before using these mirrors (even with a more realistic reflectivity of 0.9995). They also state, quite impressively, that a system such as they describe would be able to reach Pluto in a time of approximately 6.5 years. This gives an average velocity of $21.5\frac{\text{km}}{\text{s}}$.[iv]

Conclusion

On May 21, 2010, the first experimental spacecraft propelled using solar sails was (IKAROS) launched and reached Venus on December 8, 2010[10][v]. The mission demonstrated that solar sails can be an effective and practical means of interplanetary space propulsion. IKAROS showed the effectiveness of solar sails, and measured its own acceleration due to radiation pressure[11]. This astounding feat serves as a testament to the incredible acheivements of the electromagnetic theory involved and clever engineering involved in the project.

[iv]This is a little less than twice the escape velocity of Earth, or around 0.007% of the speed of light.

[v]unfortunately, it is still impractical to outfit the ship with a resonating cavity like Meyer describes

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