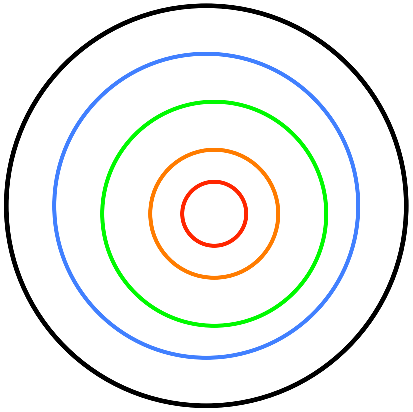
Real Plasma with n, T ~ p

Equilibrium: $$\nabla p = \mathbf{j} \times \mathbf{B}$$

<u>B lines must lie in isobaric surfaces.</u>
Since $\nabla \bullet \mathbf{B} = 0$, only possible if isobaric surfaces are topological tori. Magnetic field lines must form nested tori.

Equilibrium **must** also be stable. **Much** more complex considerations (Shafronov), established that the only **stable, toroidally-symmetric** equilibria must have a toroidal plasma **current** in addition to the **toroidal magnetic field**, a tokamak.



Isobars

Note that the <u>magnetic field lines</u> in the nested surfaces, resulting from the combination of toroidal field and the field from the plasma current, will be helices.

Macroscopic plasma measurements

(for almost any magneticly confined plasma)

I. Magnetic fields, surfaces (Time-dependent)

II. Currents (Time-dependent in **closed** loops)

III. Voltages (Around **closed** loops)

Standard circuit theory approaches useless
No "network" approximation

Magnetic pickup coils

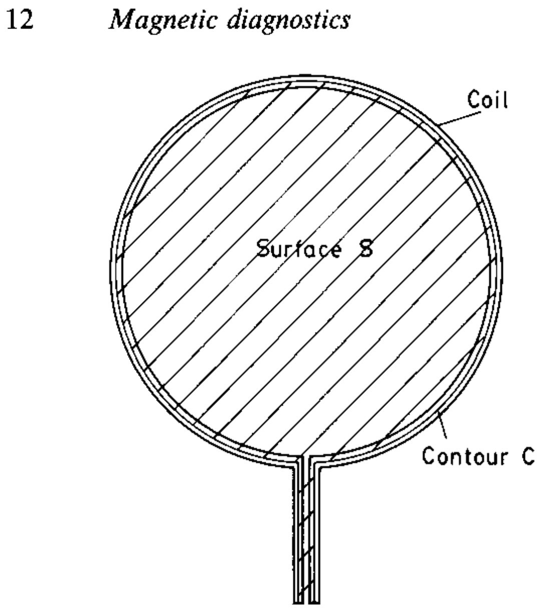
- Applying Faraday's Law

$$\oint \vec{E} \bullet d\vec{l} = -\int_S \dot{\vec{B}}\, d\vec{s}$$

- The contour integral of electric field

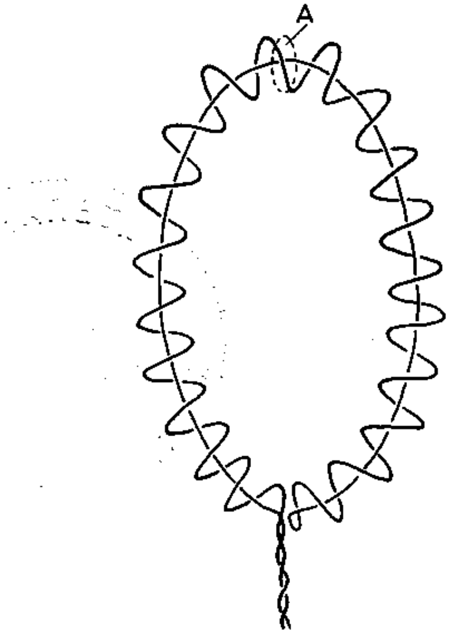
$$\oint_C \vec{E} \bullet d\vec{l} = \oint_{coil} \vec{E} \bullet d\vec{l} + \cancelto{0}{\oint_{ends} \vec{E} \bullet d\vec{l}} = -\int_S \dot{\vec{B}}\, d\vec{s}$$

- The coil measures the mean value of B normal to coil
- Surface integral spans space between leads. Twist or shield leads to minimize contribution
- Need adequate input impedance for detector (R/L > $\upsilon_{\mathrm{maxresponse}}$)
- Use analog or digital integration for B(t)



Measuring current I

- The plasma current can be measured by a Rogowski coil, which is a linear multiturn solenoidal coil <u>whose ends are brought together to form a loop</u>. Can be looped around any conductor.
- Coils should have uniform cross section A and have n turns/unit length. (Wound on cylinder.)
- Signal does not depend on current distribution, to a good approximation
- Frequency response from coil L and detector R.



Measuring current II

- Total flux linkage can be written as an integral rather than a sum over individual terms (dN=ndl)

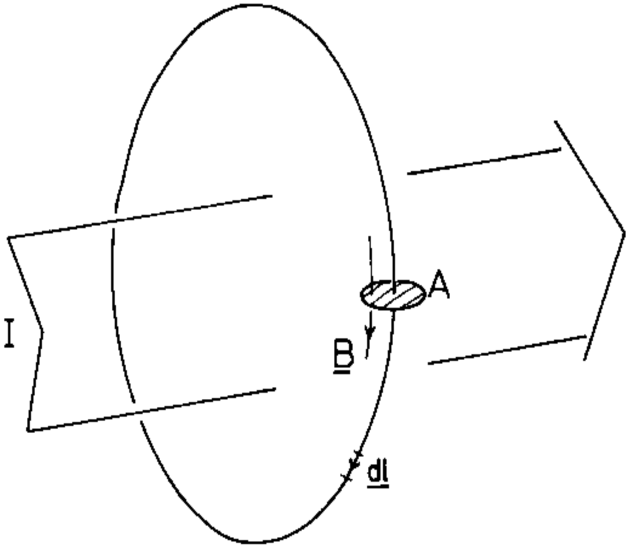
$$\dot{\Phi} = n\oint_l \int_A dA\, \dot{\mathbf{B}} \bullet d\mathbf{l}$$

- Amperes law gives

$$\oint B \bullet d\mathbf{l} = \mu_o I$$

- The voltage out of a Rogowski coil is thus

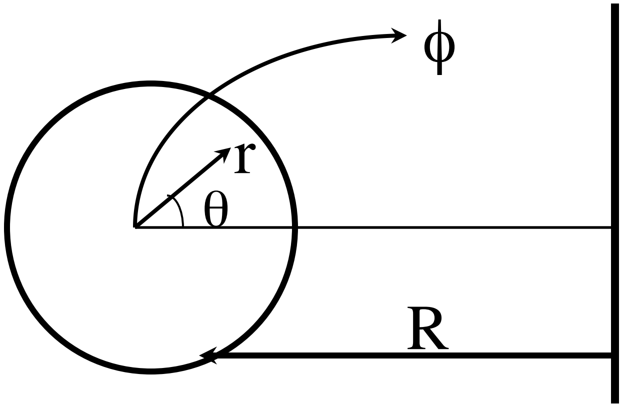
$$V = \dot{\Phi} = nA\mu_o \dot{I}$$



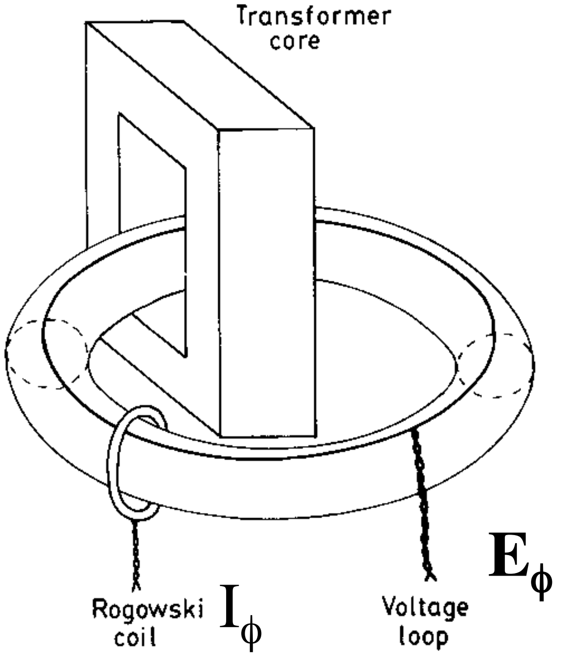
The signal can be integrated passively, actively, or digitally.

Application to a tokamak -- Global

- Objective: Calculate Ohmic power input into the plasma.
- Write Poynting's theorem as applied to a volume V bounded by a toroidal surface S, vacuum vessel, where the measuring loops are placed.



$$\int_V \left[\ \mathbf{E}\bullet\mathbf{j}+\frac{1}{2\mu_o}\frac{\partial}{\partial t}\left(B_\theta^2\right)\ \right]d^3x = -\frac{1}{\mu_o}\oiint_S\left(\mathbf{E}\times\mathbf{B}\right)\bullet d\mathbf{S}$$



$B_\theta \propto I_\phi = I_{plasma}$

Time derivative only during current changes.

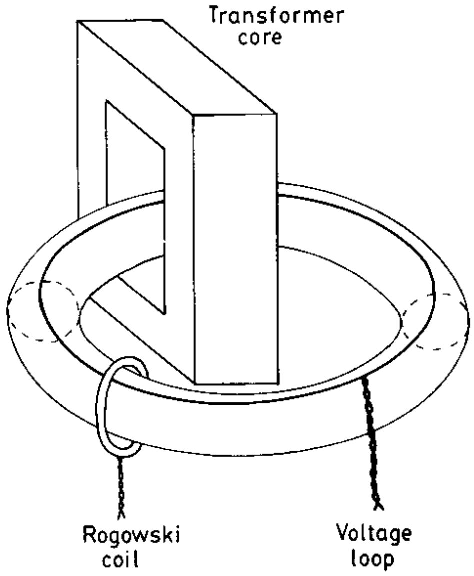
Tokamak II -- Global

We can write the energy equation as

$$P \equiv \int_V \mathbf{E} \bullet \mathbf{j} d^3x \cong \iiint R d\phi E_\phi j_\phi r dr d\theta = V_\phi I_\phi - \frac{\partial}{\partial t}\left(\frac{1}{2} L I_\phi^2\right)$$

where the inductance is $L \equiv \frac{1}{\mu_o I_\phi^2} \int_V B_\theta^2 d^3 x$

and $V_\phi \cong 2\pi R E_\phi$, simple measurements for the power input. Note L depends on the distribution of the toroidal current density.



Using Ohm's law as $j_\phi = \sigma E_\phi$, where both j_ϕ and σ depend strongly on minor radius r, the plasma current can also be written as

$$I_\phi = \int d\theta \int_0^a j_\phi r dr = \int d\theta \int_0^a \sigma r dr E_\phi = \pi a^2 < \sigma > E_\phi$$ and

$$V_\phi = V_{loop} = \frac{1}{< \sigma >} \frac{2R}{a^2} I_\phi \equiv R_{plasma} I_{plasma}$$

Plasma resistance as an average conductivity

Tokamak III -- Internal

Although the basic calculation is usually for σ as $\langle v \rangle \propto E$, it is more often given as resistivity η and credited to Spitzer for the proper combination of the Rutherford cross-section with the screening of the 1/r potential at the Debye length by the plasma:

$$\eta = \frac{m\nu_c}{ne^2} = 5.2x10^{-5}\,\frac{Z_{eff}\ln\Lambda}{T_{eV}^{3/2}}\left[\Omega - m\right]$$

where Z_{eff} is a mean ion charge for complex plasmas. With two circuits to measure current and loop voltage, we have an estimate of (mean electron) temperature. The quantity Λ is approximately the ratio of the Debye length to the impact parameter for 90° scattering. For $T_e > 10$ eV

$$\ln\Lambda \cong 31 - \ln\left(n_e^{1/2}/T_{eV}\right)$$

$\ln\Lambda \sim 15$ for hot plasmas

Tokamak IV -- Internal

Nested flux surfaces

Where is the center (magnetic axis)?
What is shape and position of outermost (not touching vessel) one?

Even for the (symmetric) tokamak, solving $\nabla p = \mathbf{j} \times \mathbf{B}$ is a <u>very</u> difficult problem theoretically and computationally, and inferring the solution from experiment is equally demanding.

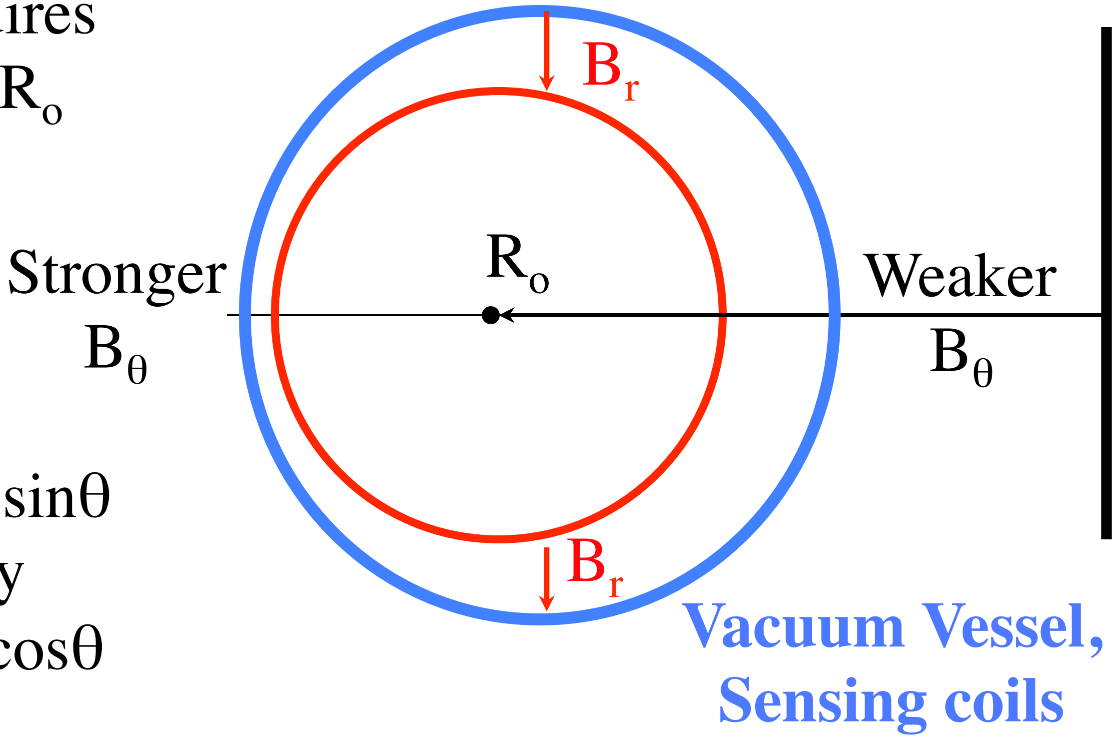
The scope of the problem can be appreciated by considering the simplest case -- a tokamak with a toroidal field, a plasma current, and <u>no</u> other significant contributors to **B** (unlike any modern tokamak). As might be expected, the nested flux surfaces have circular cross-sections. The primary quantities of importance are only the positions of the axis and outermost flux surface:

$$\mathbf{R_A, z_A, a, R_o, z_o}$$

Center of Outermost (last) surface

MHD equilibrium requires
z_o=z_A and value of R_A-R_o

Magnetic flux surface,
Lines of "B_θ"



ΔR: $\Delta B_\theta \cos\theta$; $\Delta B_r \sin\theta$
and by symmetry
Δz: $\Delta B_\theta \sin\theta$; $\Delta B_r \cos\theta$

From a set of coils specificly constructed to measure I_p and the sine and cosine components of B_r and B_θ, one can determine the position of the center of the outermost flux surface and, with less accuracy, R_A-R_o.

Magnetic Axis R_A, z_A

I. From the (four) magnetic measurements used to find R_o, z_o, one can also infer Δ_S -- Shafronov shift -- the difference between R_o and $R_A(z_o = z_A)$.

II. From radiation measurements, either transmitted or emitted, one can infer the center (magnetic axis). These are all chord-integrated, $\int dl$, and the values are generally maximum for the chord through the axis because the integrand is maximal on axis. Emission of energetic radiation, typically x-rays ~1keV, occurs only near the axis because only there is the temperature high enough and the result is strongly peaked for the chord through the axis.

Gross Plasma Containment

Beyond MHD -- "Transport"

- Energy

$$W_{Loss} = \frac{E}{\tau_E} = W_{IN} - \frac{dE}{dt}$$

W_{IN} is (measured) power input

E is (measured) energy content

$\tau_E \equiv$ energy confinement time (generally well-known)

- Particles

$$F_{Loss} = \frac{N}{\tau_P} = F_{IN} - \frac{dN}{dt}$$

F_{IN} is (unknown) particle source

$\tau_P \equiv$ particle containment time
(poorly determined)

Quantitative Plasma Confinement

- Conservation Laws $\frac{\partial X}{\partial t} = -\nabla \bullet Flux_X$
- Plasma approximately uniform (n, T, etc.) on a magnetic surface
- Parameters vary in perpendicular "radial" direction

- Particles: n_s, $\Gamma_{s,r} = -D_s \frac{\partial n_s}{\partial r} + V_s n_s$
- Energy: $(3/2) n_s T_s \Rightarrow$ $\frac{3}{2}\frac{\partial T_s}{\partial t} = \nabla_r \bullet \left(\chi_s \frac{\partial T_s}{\partial r} \right)$

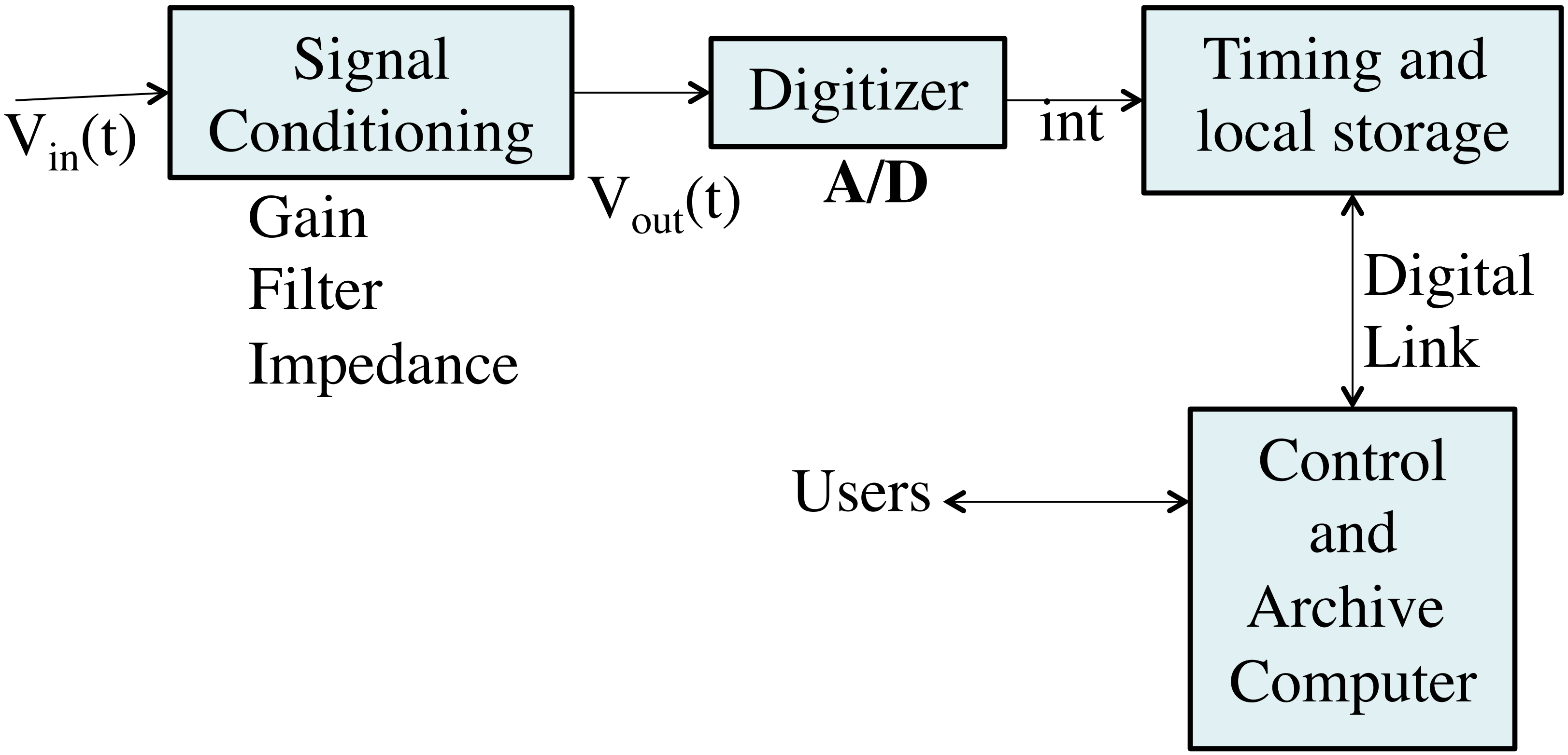
Objective: Measure and understand the transport coefficients, which are different for electrons and each ion species and also include (vector) momentum.

Digital Signal Acquisition and Processing

- An essential tool for experimental physics
- Used for one-dimensional or multi-dimensional (image) data
- Consider the basic general principles for one-dimensional data -- S(t)
- Use for higher dimensions also quite common but specific to application

Digital Signal Acquisition and Processing

Components of a System



$V_{in}(t)$

Signal Conditioning

Gain

Filter

Impedance

$V_{out}(t)$

Digitizer

A/D

int

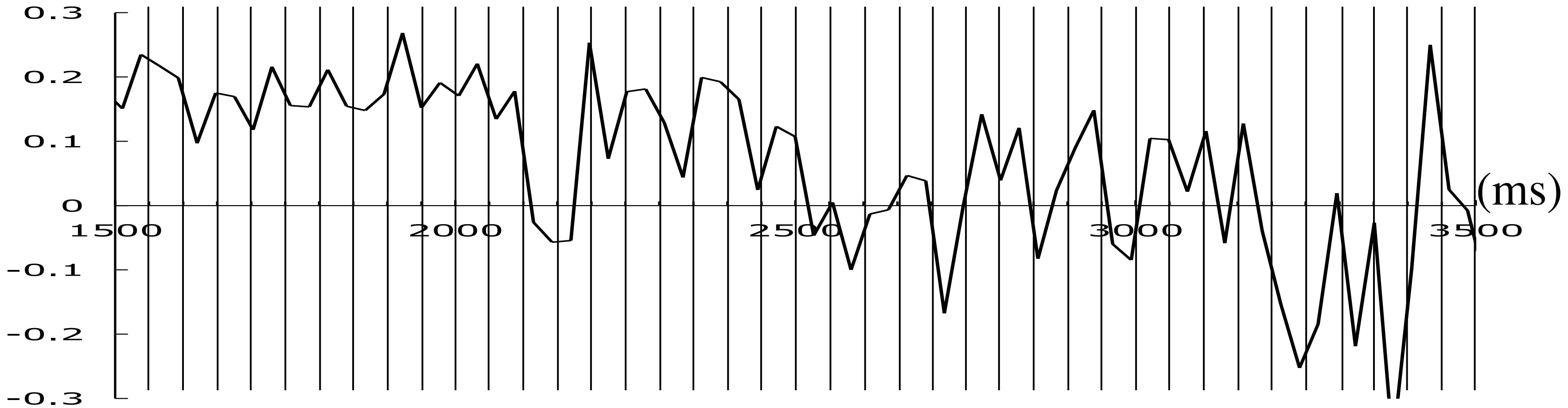
Timing and local storage

Digital Link

Users

Control and Archive Computer

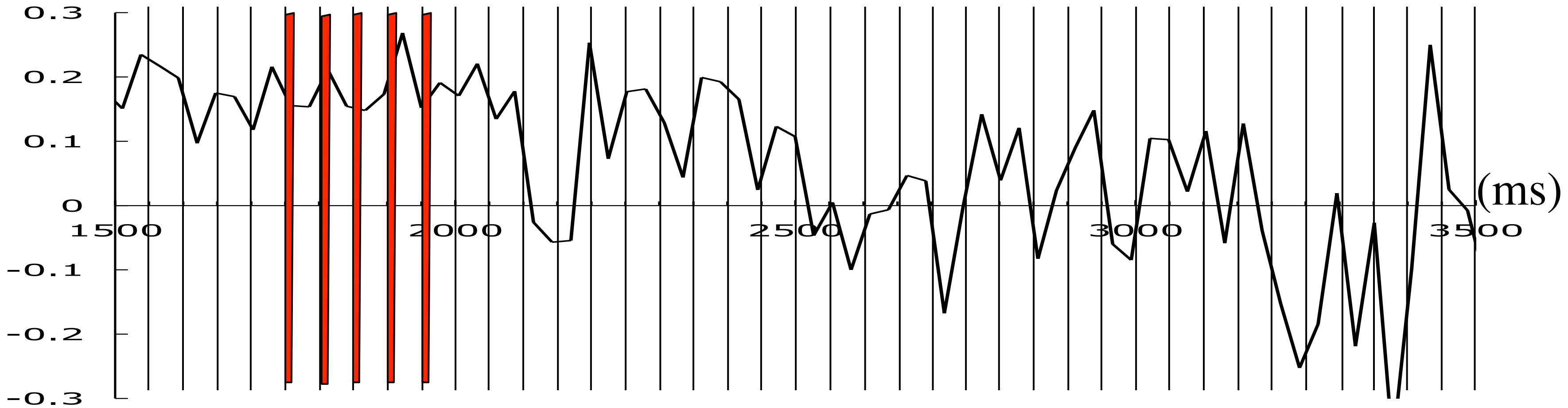
Digital Signal Acquisition and Processing



Sample data periodically, here every 5 ms,
a rate of 200 samples/sec

- What happens next?
- NOT what you might expect!

Digital Signal Acquisition and Processing



Interval between samples 5 ms / Sampling frequency 200/sec

Sampling Time 0.2 ms ("Sample and hold")

Consequences:

- ALIASING – e.g. a signal at 200 Hz appears as a dc signal; 240 Hz appears at 40Hz!
- Poor phase, amplitude data for ν near sampling ν

A/D System Design Requirements

- $\nu_{\mathrm{Samp}} \geq 5\nu_{\mathrm{Data}}$ and $\nu_{\mathrm{Samp}} >> \nu_{\mathrm{Phase}}$
- Signal conditioning -6 db at ν_{Samp} and adequate to charge C_{in} of sample and hold in brief sample τ*
- Keep $|V_{\mathrm{max}}|$ below A/D rating – A/Ds NOT tolerant
- Check digitizer bits (14 bit, 16 bit, etc.); $|V_{\mathrm{min}}|$ should correspond to 4-6 bits to keep resolution
- Total Memory capacity adequate to application

These conflicting demands generally require specific A/D systems for each application.

* Depends somewhat on whether one A/D per channel or a single A/D switched between channels

Advantages of A/D & Computer Systems

- Ease of acquisition, annotation, storage, display, retrieval, and backup
- Highly accurate values and timing
- Complete flexibility in analysis and reanalysis
- Natural data sharing and collaboration

These account for its near-universal use

Disadvantages of A/D & Computer Systems

- High “overhead” in creating and maintaining hardware and software
- Output sometimes misleading: GIGO, misrepresentation of input signal & aliasing, “manufactured” precision
- Lack of robust, versatile electronics
- Limited open-sourced resources
- **Overuse of proprietary software**; “rented” results from temporary licenses may vanish