# Wave Propagation in a Plasma -- Interferometry

References:

Infrared & millimeter waves, Vol. 2 article by D. Veron, edited by K. J. Button (Academic Press) (1979)

Hutchinson, Principles of plasma diagnostics.

# Wave propagation and transmission

For a hot, dense plasma, observational constraints are similar to those in astronomy; one can only observe the phenomena from a distance.

For both astronomy and laboratory plasmas, one can observe radiation from the object.

(In principle, this could include emitted particles as well as waves, but we are treating principally EM waves here.)

For laboratory plasmas, one can also use transmission measurements, generally termed **interferometry** from the analysis technique.

**The next two lectures treat transmission;**

**the final two concern radiation.**

# High-frequency EM waves, B=0

High frequency justifies single-particle treatment; frequency too high to allow time for equilibration to a fluid (a very subtle problem in a system with a very low classical collision rate, not a typical gas.)

In the first lecture, we found for $E(t) = E_o e^{i\omega t}$, $v \propto E$, hence $j \propto E$, and

$$\nabla \times B = \mu_o \left( j + \varepsilon_o \frac{dE}{dt} \right)$$ could be written as $$\nabla \times B = \mu_o \varepsilon \frac{dE}{dt}$$

where $$\varepsilon = \left( \frac{ne^2}{-m\omega^2 \varepsilon_o} + 1 \right) \varepsilon_o = \left( 1 - \frac{\omega_{pe}^2}{\omega^2} \right) \varepsilon_o$$

$$E, B \propto e^{i(k \bullet r - \omega t)} \qquad i\mathbf{k} \times \mathbf{E} = i\omega \mathbf{B} \qquad i\mathbf{k} \times \mathbf{B} = -i\omega \varepsilon \mu_o \mathbf{E}$$

Following the standard calculation for waves: Faraday & Ampere

# High-frequency EM waves, B=0 (II)

$$\mathbf{k}\times\mathbf{B} = \mathbf{k}\times\left(\frac{\mathbf{k}}{\omega}\times\mathbf{E}\right) = -\omega\varepsilon\mu_o\mathbf{E} \qquad (\mathbf{k}\bullet\mathbf{E})\mathbf{k} - (\mathbf{k}\bullet\mathbf{k})\mathbf{E} = -\omega^2\varepsilon\mu_o\mathbf{E}$$

For transverse waves, one has the usual dispersion relation

$$\omega = kc' \qquad k^2c^2 - n^2\omega^2 = 0$$

$$n^2 \equiv \frac{\varepsilon}{\varepsilon_o} \qquad c' = \frac{1}{\sqrt{\varepsilon\mu_o}}$$

$$n^2 = 1 - \frac{\omega_{pe}^2}{\omega^2}$$

For $\omega < \omega_{pe}$, k imaginary -- exponentially decaying fields into the plasma, just like a conductor, which it is [For $\omega = \omega_{pe}$, $\varepsilon=0$ and there is an additional solution, k=0, E≠0, B=0, an electrostatic remnant of a <u>wave for T≠0.]</u>

# Propagation with B = 0 (III)

The dispersion relation can also be written $\omega^2 = \omega_{pe}^2 + c^2 k^2$ making clear that waves ($k^2$>0) exist only for $\omega > \omega_{pe}$.

$$v_{ph} = \frac{\omega}{k} = c\sqrt{1 + \left(\frac{\omega}{kc}\right)^2} > c \qquad v_g = \frac{d\omega}{dk} = \frac{kc^2}{\omega} = \frac{c^2}{v_{ph}} < c$$

The plasma frequency, at which k → 0, the phase velocity → ∞, and the group velocity →0, is called a cutoff because the wave cannot propagate beyond that point, in this case as the density increases. The wavelength → ∞ and the wave reflects.

# Propagation with B = 0 (IV)

The maximum density for propagation can also be expressed in terms of vacuum wavelength rather than ω:

$$n_{crit} = \left(\frac{m\varepsilon_o}{e^2}\right)\omega^2 = \left(\frac{4\pi c^2}{\lambda^2}\right)\frac{\varepsilon_o m_e}{e^2} = \frac{1.11\times 10^{15}}{\lambda^2}[m^{-3}]$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\lambda[10^{-3}\mathrm{m}]$ | 2 | 1 | .337 | .195 | .119 | $10^{-4}$ |
| $n_e[10^{20}\mathrm{m}^{-3}]$ | 2.8 | 11 | 98 | 290 | 780 | $10^{9}$ |

Note: n can be either index of refraction or density

**What wavelength would you need for propagation in a plasma with solid density?**

# Propagation with B = 0 (V)

We have looked at propagation in a $T = 0$ plasma. There are corrections for high temperature plasmas, but they are large only in specific circumstances.

**How would propagation change if there are collisions?**

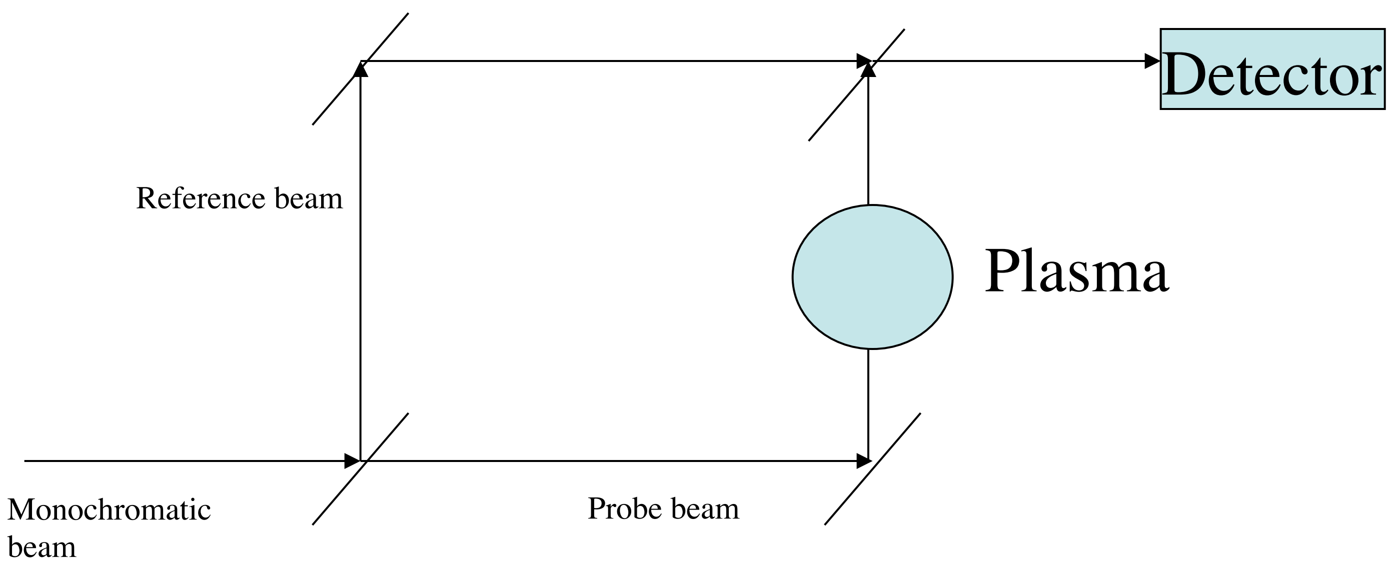
**This analysis is also correct with a $B_o$ when E is polarized $\parallel B_o$, a very important practical case -- the ordinary wave. (Note that a transverse wave must have $k \perp B_o$ for this polarization.)**

# Interferometer

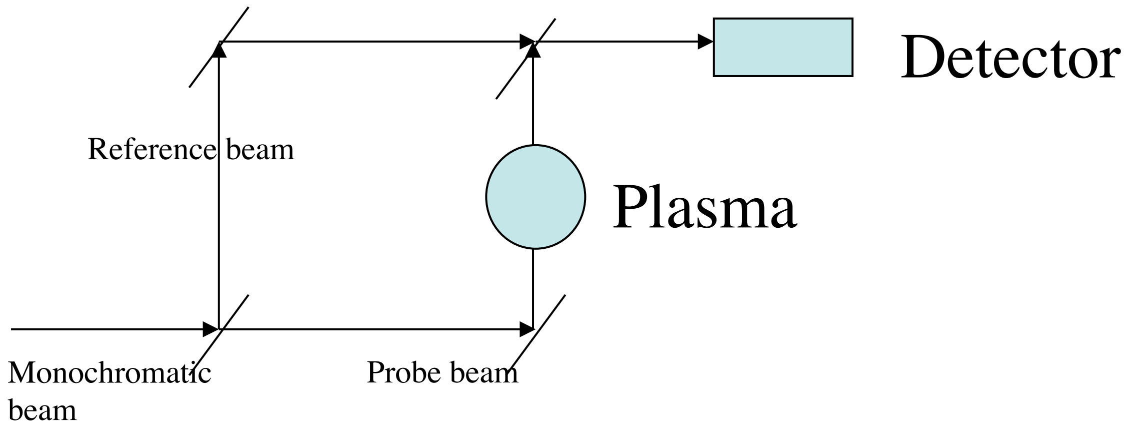
We can also write the index of refraction as

$$n_{ref} = \left[1 - \frac{n_e}{n_{crit}}\right]^{1/2} \approx 1 - \frac{n_e}{2 * n_{crit}} \qquad k = \frac{n_{ref}\,\omega}{c}$$

This leads naturally to a technique for inferring plasma density using interferometry to measure the phase difference between paths through the plasma and through "vacuum."



# Interferometer II




The phase difference between the two arms is changed by the presence of the plasma.

$$\varphi = \frac{2\pi}{\lambda}\int_{z_1}^{z_2}\left[n_{ref,plas}\right]dz$$

The phase difference in the probe beam due to the plasma is then

$$\Delta\varphi = \frac{\pi}{\lambda n_{crit}}\int_{z_1}^{z} n(z)dz = \left[\frac{\lambda e^2}{4\pi c^2 \varepsilon_o m_e}\right]\int_{z_1}^{z_2} n(z)dz = 2.8\times 10^{-15}\lambda\int_{z_1}^{z_2} n(z)dz$$

The number of fringes is (n = density)

$$F = \frac{\varphi}{2\pi} = 4.49\times 10^{-16}\lambda\int_{z_1}^{z_2} n(z)dz$$

# Detection: Phase modulation

The electric field amplitudes on the detector in an interferometer are of the form

$$x_{probe} = a\cos(\omega t - \varphi)$$

$$x_{ref} = b\cos(\omega t)$$

Combining these two fields on the detector gives a resulting power proportional to

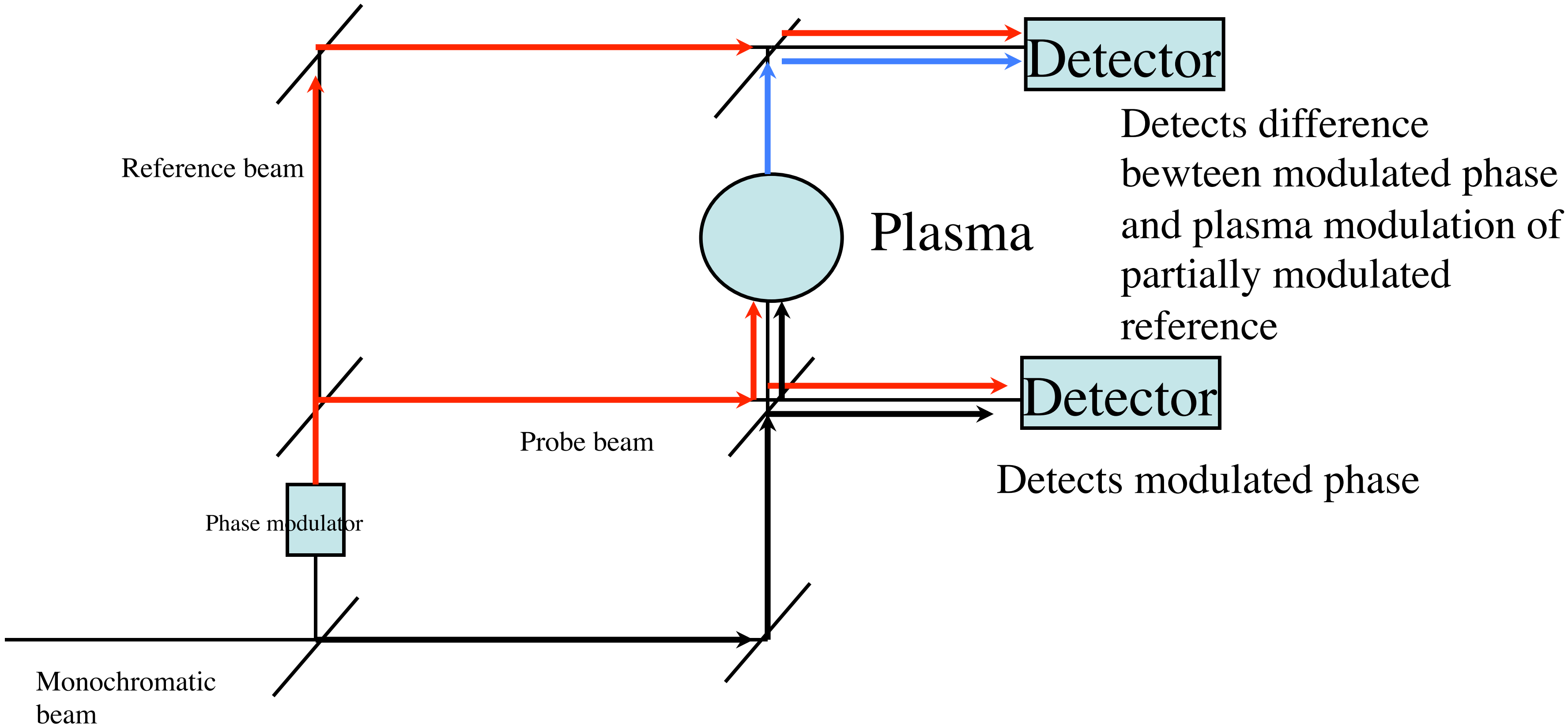
$$\left[x_{probe} + x_{ref}\right]^2 \propto a^2\cos^2(\omega t - \varphi) + b^2\cos^2\omega t + ab\left[\cos(\omega t - \varphi)\right] \bullet \cos(\omega t)$$

After expanding the trig functions, one can ignore the high frequencies, ω and 2ω, and the detected signal is

$$ab\cos(\varphi) + f(a,b)$$

By following this signal from a time with no plasma, one might obtain the phase, but never the sign. More ingenuity is needed.

# Phase modulation-one version

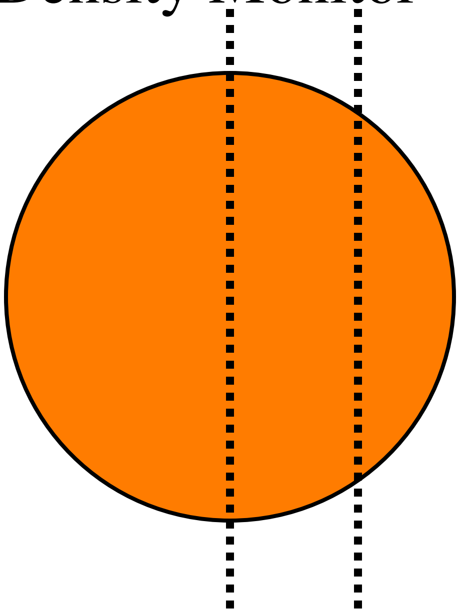



From changing difference between detectors, can infer plasma effect.

One can also use frequency modulation to infer density.

# Typical Interferometer Systems

Density Monitor

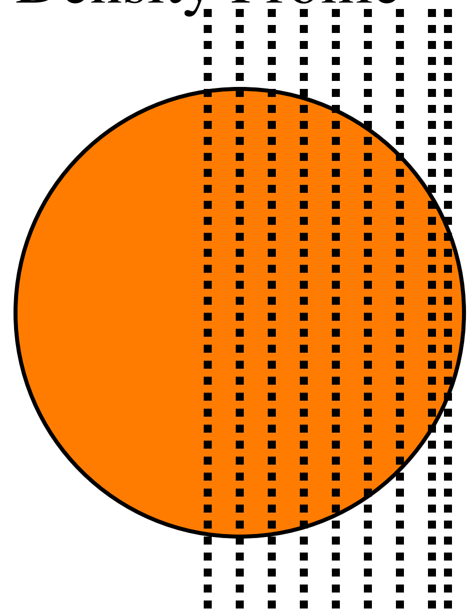


One or two channels

$\int n\,dl$ -- often described as

$\langle n \rangle = \int n\,dl / L_{plasma}$

Density Profile



Many channels (chords)

Infer local $n(\rho)$

Spatial separation and resolution requires good optics: $d\theta \sim \lambda$

"Uncertainty relation" between beam diameter and divergence

*Task of inferring local quantity from chord integrals quite general*

# Hologram

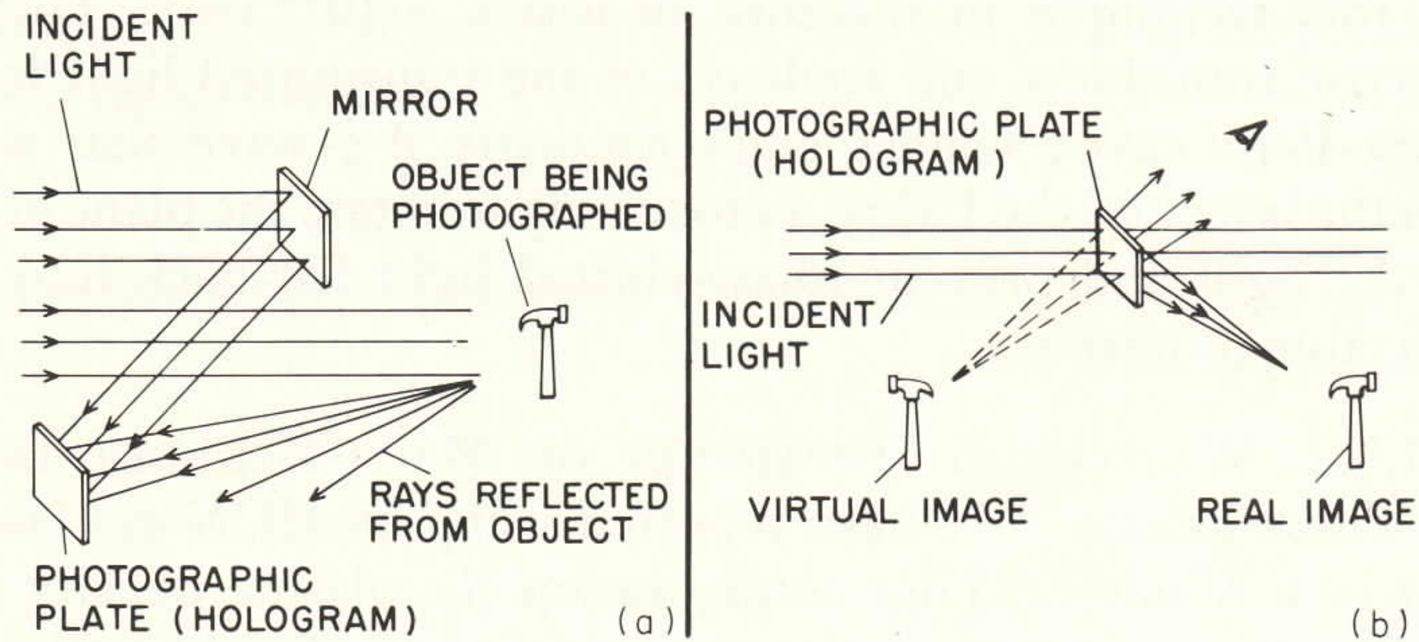



FIG. 11. (a) Diagram of hologram recording and (b) hologram reconstruction.

# Apply holographic techniques to plasma

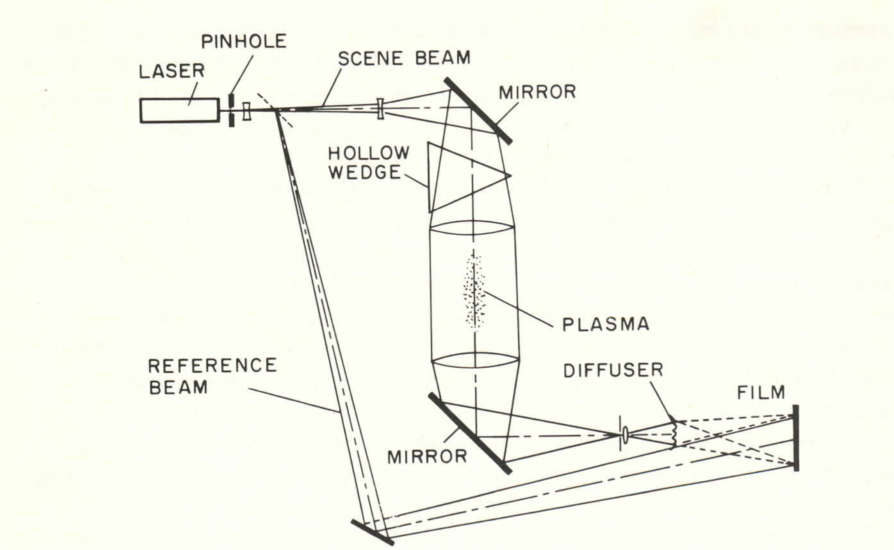
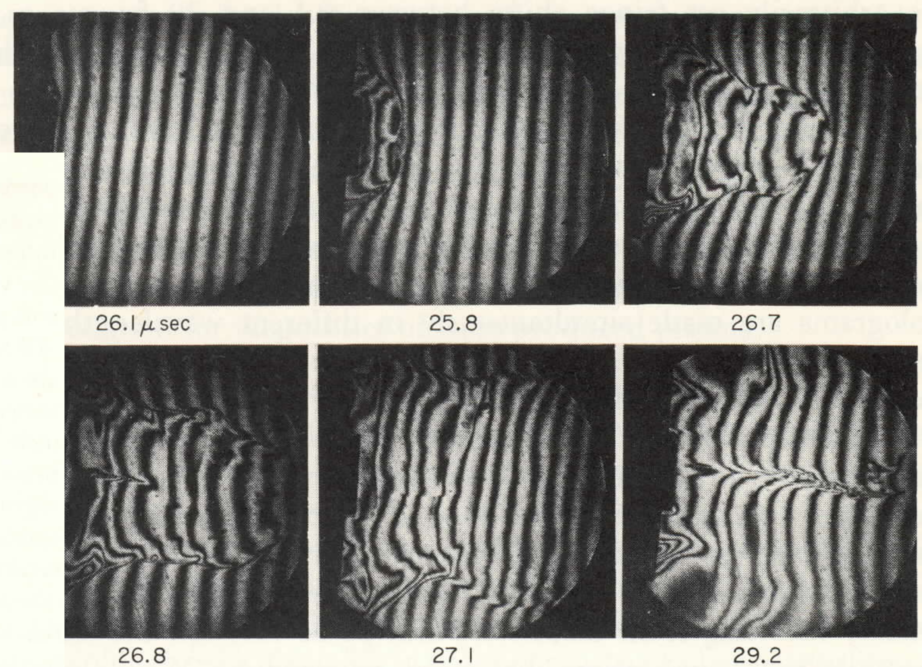



FIG. 12. Diagram of holographic interferometer to measure plasma density.




olographic interferograms of coaxial gun neon plasma at different times after
n.

Excellent images and pictures, but inferences imprecise

# Wave propagation with $\mathbf{B}_o$

Repeat calculation of the dielectric constant: $i\omega m\mathbf{v} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}_o)$

Formally, this is a set of coupled linear equations for the three components of **v** as a function of the components of **E** in tensor (matrix) form. In terms of **j**, this becomes

$$\mathbf{j} = \sigma\mathbf{E}$$

where $\boldsymbol{\sigma}$ is now a tensor (matrix). However, the same procedure applies to construct the dielectric constant, which is also a tensor. The vector manipulations to obtain the wave equation and dispersion relation k(ω) likewise apply, but the dispersion relation and index of refraction now depend also on the direction of **k** and the polarization -- NO degeneracy. Both electrons and ions must also be included.

Because the particle orbits are circles around $\mathbf{B}_o$, it is often necessary to consider modes with circular polarization, not linear.

# Wave propagation with $\mathbf{B}_o$

The subject is extremely complex, with the results depending on the specifics of frequency and direction in a multi-dimensional parameter space. It is the subject of several full textbooks. Moreover, thermal effects cannot be ignored under many circumstances. As a single example, consider a wave propagating perpendicular to $\mathbf{B}_o$ with $\mathbf{E}$ also perpendicular to $\mathbf{B}_o$. For this case, the index of refraction at high frequency is

$$n_{ref} = \left\{ 1 - \left( \frac{\omega_p^2}{\omega^2} \right) \left[ \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_p^2 - \omega_c^2} \right] \right\}^{1/2} \qquad \omega_c = \frac{eB}{m}$$

where the electron cyclotron frequency now enters. This introduces a new effect, a <u>resonance</u>, a frequency at which $n \to \infty$, $\lambda \to 0$, and the derivation fails. Why? Thermal effects (finite gyroradius) must be included and <u>absorbtion</u> occurs.

# Faraday rotation

Consider a linearly polarized beam propagating along the magnetic field direction. We must think of this wave as two circularly polarized waves which in fact propagate at different velocities. One circular wave rotates in same direction as electrons rotate. The other wave is rotating in the opposite direction. The Faraday rotation is approximately the line integral of $n_e B$ along the line of propagation. Usually you would measure $n_e$ by interferometry simultaneously along the same path (phase shift as well as rotation of plane of polarization.

# Considerations in design of an interferometer

What wavelength do you choose?

- $n_{crit}$
- Refraction of interferometer beam by non uniform plasma
  - plasma acts like a lens and refracts beam from path
- Compromise -- as $\omega \rightarrow \omega_{pe}$, phase shift increases, but so do
  - refraction errors

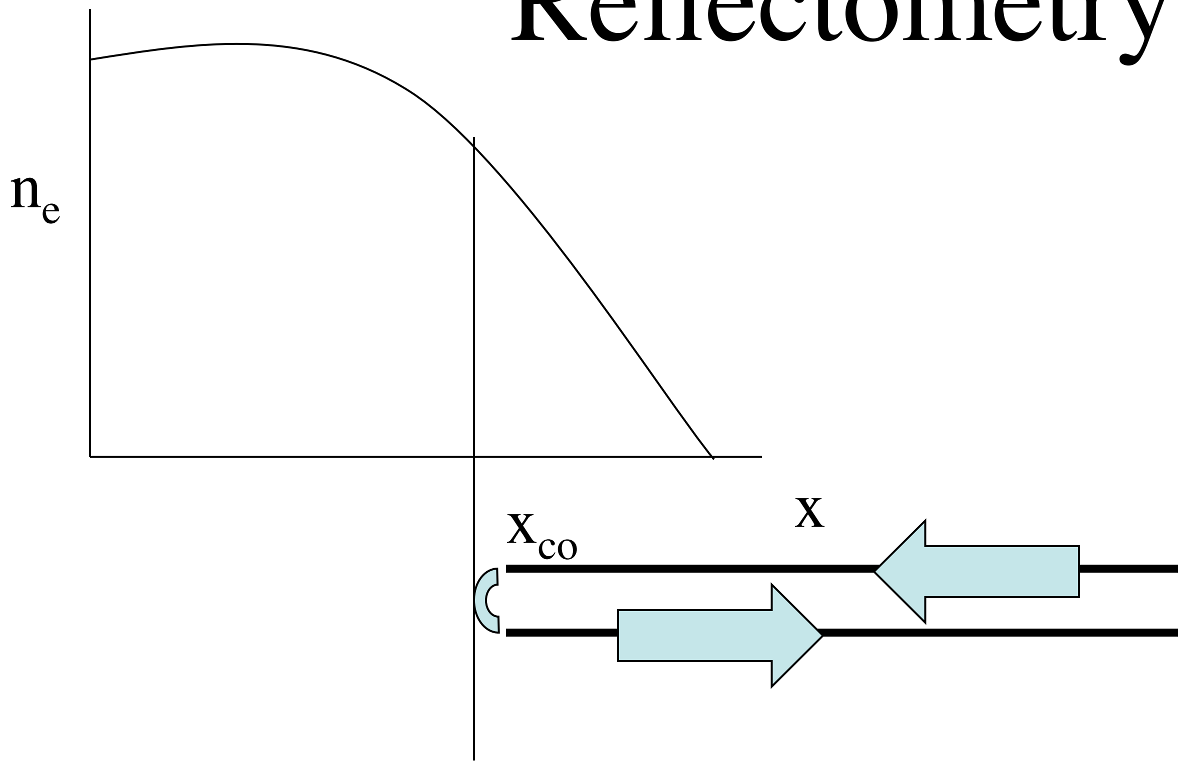
What phase shift can you measure?

What other frequencies are important in plasma?

Vibration

Spatial resolution required

# Reflectometry




Probe beam

*How does reflectometry differ from radar?*
*How do you measure electron density in ionosphere?*

Two forms: One in time domain -- pulse time of flight;
Other in frequency domain -- phase shift interferometry.

*Most useful if gradient significant; fails for flat or hollow profiles.*

# Principles of Scientific Communication

I. The purpose of publication is <u>not</u> to advance science, but to advance the authors.

II. Publication is more a social than a scientific activity.

III. Almost all publications are insignificant -- irrelevant, trivial, repetitious, inconsequential, or insufficient to support conclusions.

IV. Refereeing serves only to prevent publication of absurdly wrong or broadly unpopular results.

V. A significant fraction of published conclusions are quite wrong -- often inconsistent with the data shown.

*These observations are universal, applying to all journals.*