Plasma Radiation

- Free electrons
  - Blackbody emission
  - Bremsstrahlung

- Bound electrons (Z>2)
  - Unresolved, multi-line emission
  - Resolved line emission -- Single Z^{+n}

Objective

- Infer a thermodynamic quantity -- density, temperature, flow velocity -- of each species at each location. [At least f(r), sometimes $f(r,\theta,\phi)$]

- Measured quantities are never directly those desired. Varying degrees of calculation and interpretation required.

For an overview of plasma diagnostics with extensive references for specific techniques, see K.W. Gentle, "Diagnostics for magnetically confined high-temperature plasmas," Rev. Mod. Phys. **67**, 809-836 (1995).

Blackbody Radiation

- Hot plasmas are almost transparent -- optically thin, except for a few special cases and stellar interiors.
- A picture of a hot plasma in the visible is empty except for possible bright spots at the edge where incoming neutrals are ionized.
- The principal, and most important, exception is at the electron cyclotron frequency. (For $n>10^{19}/m^3$ and $T_e>100$ eV, the plasma is a black body radiating at T_e, at least in certain directions and polarizations.

$$\Omega_e = \frac{eB}{m}$$

Electrons are in rapid circular motion at Ω_e and radiate ***well****.*

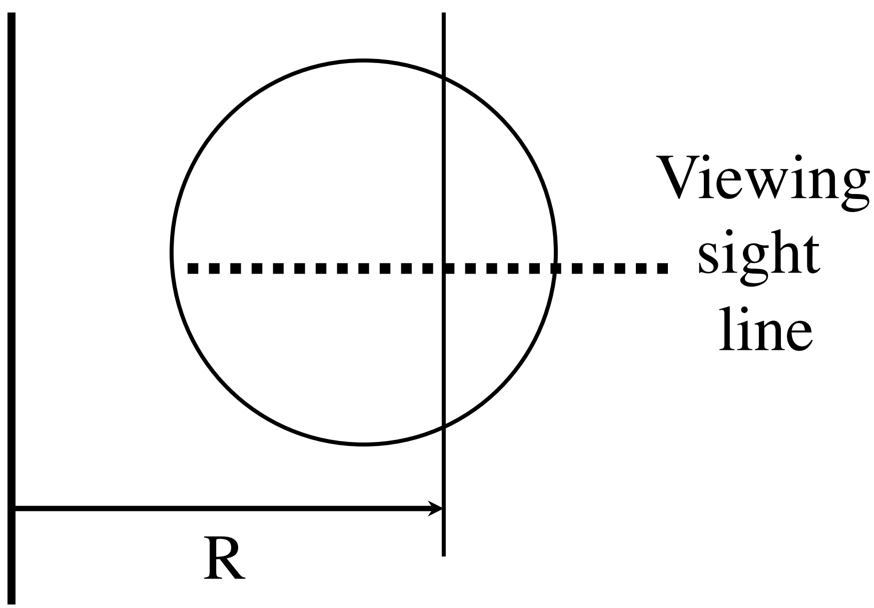
Electron Cyclotron Emission (ECE)

Blackbody radiation at 100 eV (1 million degrees; 200 times the surface of the sun) is quite significant. Were it not confined to a narrow band of frequencies near Ω_e, it would be a very large power.

Blackbody radiation provides the most accurate values of $T_e(r,t)$ now available for a hot, confined plasma.

$$\Omega_e \approx \frac{eB_T}{m}$$

$$B_T \propto \frac{1}{R}$$

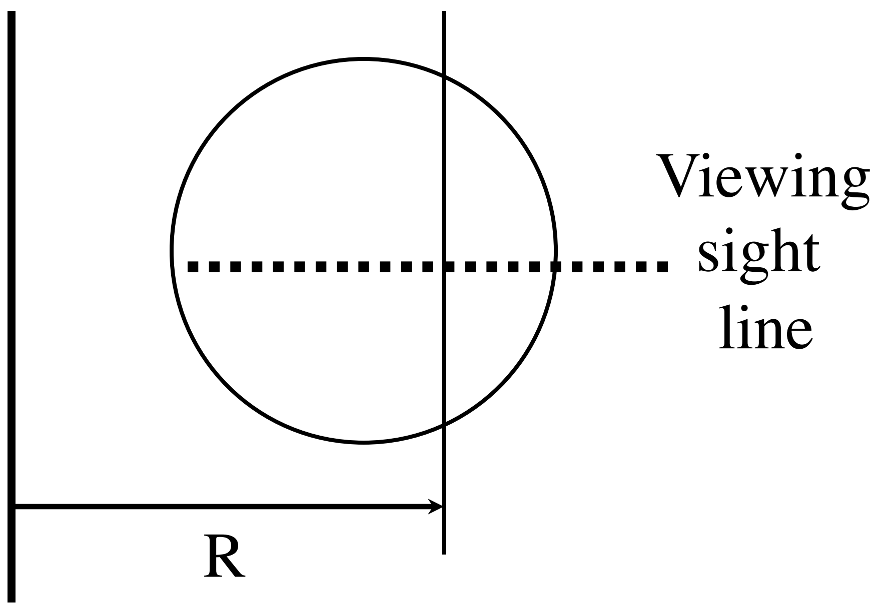


Thus the power at each frequency indicates T_e at the intersection of the sight line with the corresponding R.

Electron Cyclotron Emission (ECE)

- Radiation typically at microwave frequencies
- Sight line determined by microwave horn antenna
- Transmitted to detector ("radiometer") by waveguide
- Detector either heterodyne receiver or grating polychromator

Polychromator is quasi-optical: Grating disperses frequencies with an array of bolometer power detectors. Absolute intensity calibration comparatively easy, but low sensitivity, limited spatial resolution, and poor time response.



Heterodyne is conventional rf technology: Excellent frequency (spatial) resolution, high sensitivity (good S/N), fast (MHz) time response.

Bremsstrahlung

- From e-ion close collisions, large scattering angle
- Since hot plasma "collisionless", intensity far less than blackbody -- detectable only because it extends over a broad frequency range

$$P_{Bremsstrahlung} \approx \left(5.35x10^{-37}\right)Z^2 n_e n_i T_{keV}^{1/2}\left[W/m^3\right]$$

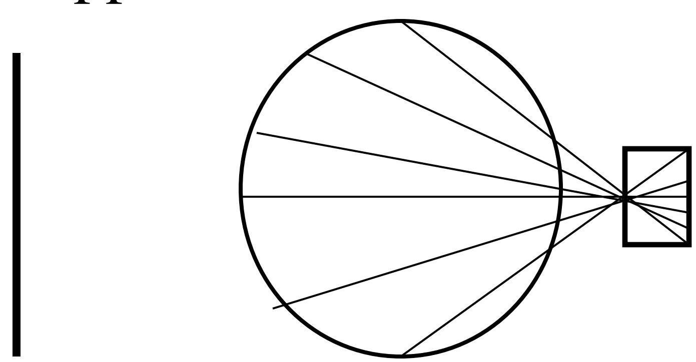
for each ion species with n_i and ionization state Z, where the numerical factor uses T in units of keV.

The radiation is broad-band, but peaks at $h\nu \sim T_e$, like blackbody radiation, because there are fewer electrons at higher energies capable of emitting such photons, and the density of states decreases with decreasing frequency.

Bremsstrahlung Measurements

$$P_{Bremsstrahlung} \approx \left(5.35x10^{-37}\right) Z^2 n_e n_i T_{keV}^{1/2} \left[W/m^3\right]$$

- Intensity has strong density and Z dependence
- Weaker temperature dependence is nevertheless obvious because the temperature profile is strongly peaked on axis
- Chord-integrated intensity measurement (pin-hole camera)
- X-rays detected by broad-band diode arrays ~keV
- Lower bound set by entrance window transmission >100eV
- Upper limit from detector absorbtion <10keV



Compact cameras with >20 channels, good time response, and several views (top, bottom, intermediate) are common.

Only semi-quantitative results

Line Radiation I

The upper states of an atom are generally populated by an inelastic electron collision

$$e + N^z \text{-->} e + N^{z*} \text{-->} e + N^z + \text{photon}$$

The emission coefficient is

$$\varepsilon(\omega) = \frac{\hbar\omega}{4\pi} L(\omega) A_{nm}^{z,a} N_m^{z,a} \sim \frac{\hbar\omega}{4\pi} L(\omega) A_{nm}^{z,a} N^{z} N_e \times \textit{rate}\ \text{coefficient}$$

$$\propto N_e N^z F(T)$$

<ins>Line radiation can be intense</ins>. For fixed N_e, it is density of target atoms times σ, where σ is nearly a step function at the threshold excitation energy to a constant value of the atomic size. **Much** bigger than Bremsstrahlung, a cross-section little bigger than nuclear.

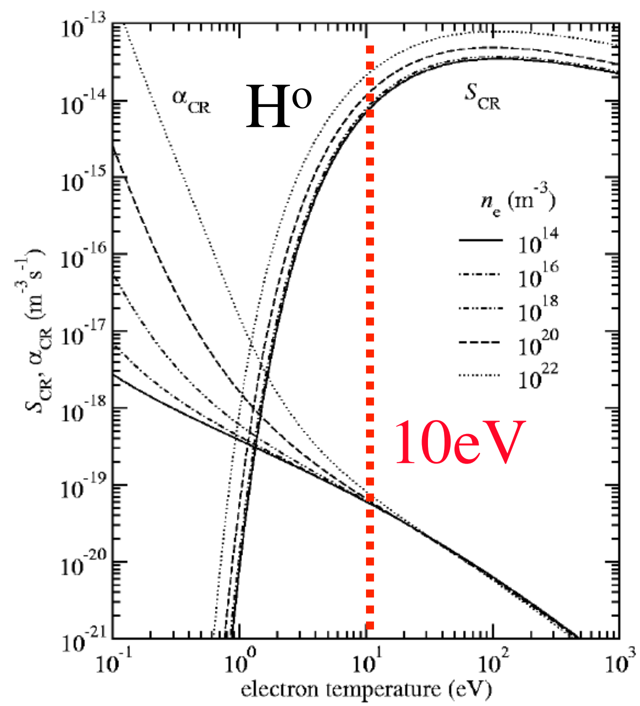


FIG. 2. T_e dependences of S_{CR} and α_{CR} for several n_e values for neutral hydrogen.

Line Radiation II

History: Fusion research made almost no progress 1960-1970 because no experiment could reach T>20eV! Artsimovich solved the problem with a tokamak, although the tokamak configuration proved incidental.

Cause: "Clean" metal surfaces in vacuum bind multiple monolayers of H_2O. Although the experiments used pure H_2, as soon as the plasma formed and began to heat, it desorbed the water. Line radiation of O at 20 eV is intense, sufficient to radiate as much power as anyone had available. We had bright UV lamps.

Solution: Use special (plasma) cleaning processes to remove the layers of H_2O (Artsimovich "kitchen physics"). *The processes are the same as those now universal in the semi-conductor industry for surface cleaning*.

N.B. Line radiation from an ion intense for $T_e > E_{ion}/2$ for ion

Line Radiation III

Unresolved line emission

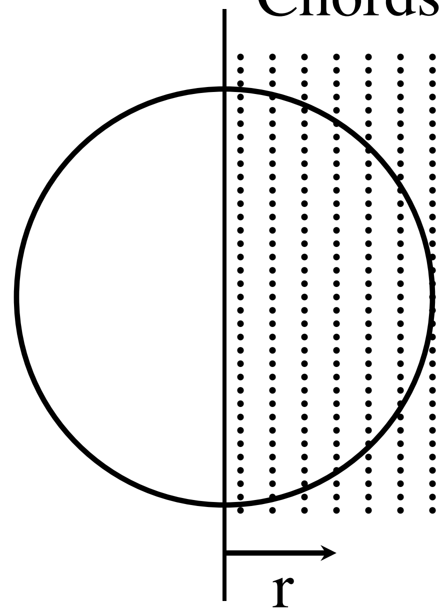
- Molecular emission is the textbook example.
- In a hot plasma, incoming H_2 and He will be ionized immediately; the characteristic lines are observable.
- Higher-Z elements -- C is ubiquitous, B, O, N, Ne are possible -- will be fully stripped far short of the core.
- In this process, they will proceed through a succession of ionization states, each emitting many lines.
- At most, a few strong characteristic lines may be identified, generally from the He- or H-like states.
- The total radiated power may be significant, but broad-band detectors are needed to measure the total power.
- Bolometers for $h\nu<200$eV ideal, but bolometers insensitive.
- AXUV diodes ideal: Block $h\nu<10$eV, but fast and sensitive over range where almost all power radiated -- configured like x-ray diode camera arrays.

Chord-integrated Measurements

- All external measurements are intrinsically chord-integrated.
- A few are naturally localized by nature of measurement, e.g. ECE, x-ray bremsstrahlung, and lines of localized ions.
- Most are simple chord integrals of quantity of interest, e.g. density interferometry and total radiated power.
- Chord-integrated measurements require special analysis.

Formally, the chord integrals are integral transforms of the local quantity. The transform must be inverted to obtain the quantity desired. There is an extensive mathematical literature on integral transforms and inversions, e.g. tomography. In practice, computational algorithms of many sorts are used. Here, we will consider only one example, but one of great historical and experimental importance.

The Abel Inversion



Chord integrals I(r) of a quantity n(r,θ) that depends only on r, where n(r)=0 for r>R:

$$I(r) = 2\int_r^R n(\rho)\frac{\rho d\rho}{\sqrt{\rho^2 - r^2}}$$

which can be put into a standard form as:

$$t = \left(\frac{r}{R}\right)^2 \qquad F(x) = \int_x^1 \frac{f(t)dt}{\sqrt{t - x}}$$

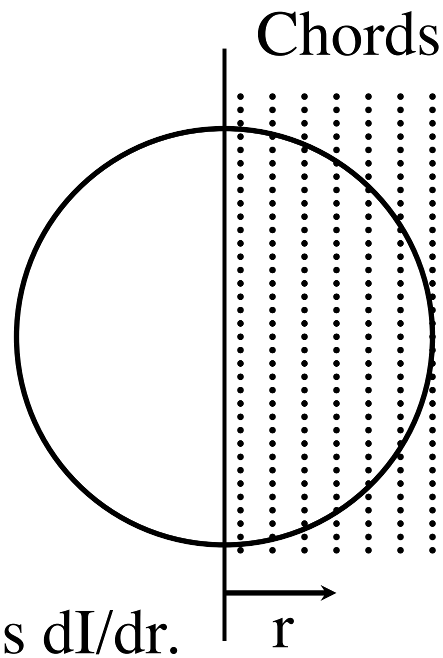
Abel proved that this transform could be inverted as:

$$f(z) = \frac{-1}{\pi}\int_z^1 \frac{dF/dx}{\sqrt{x - z}}dx \qquad n(r) = \frac{-1}{\pi}\int_r^R \frac{dI/d\rho}{\sqrt{\rho^2 - r^2}}d\rho$$

which is straightforward, but not at all simple, to prove.

The Abel Inversion

$$n(r) = \frac{-1}{\pi}\int_r^R \frac{dI/d\rho}{\sqrt{\rho^2 - r^2}}\,d\rho$$



This is an unusual case in which an exact mathematical solution is of little direct, practical use. The reasons are twofold. First, the integrand is singular at $\rho=r$, making actual integration delicate. Second, the integrand requires dI/dr. Although mathematical functions are differentiable, discrete experimental <u>data is not</u>. Numerical differentiation is "noisy" with large error bars. The consequence is that n(r) is unduly sensitive to dI(r)/dr. Several methods are available to mitigate the problem, but care is always required.

An important practical consequence is that hollow n(r) profiles can never be well resolved.

Line Radiation IV
Resolved Lines

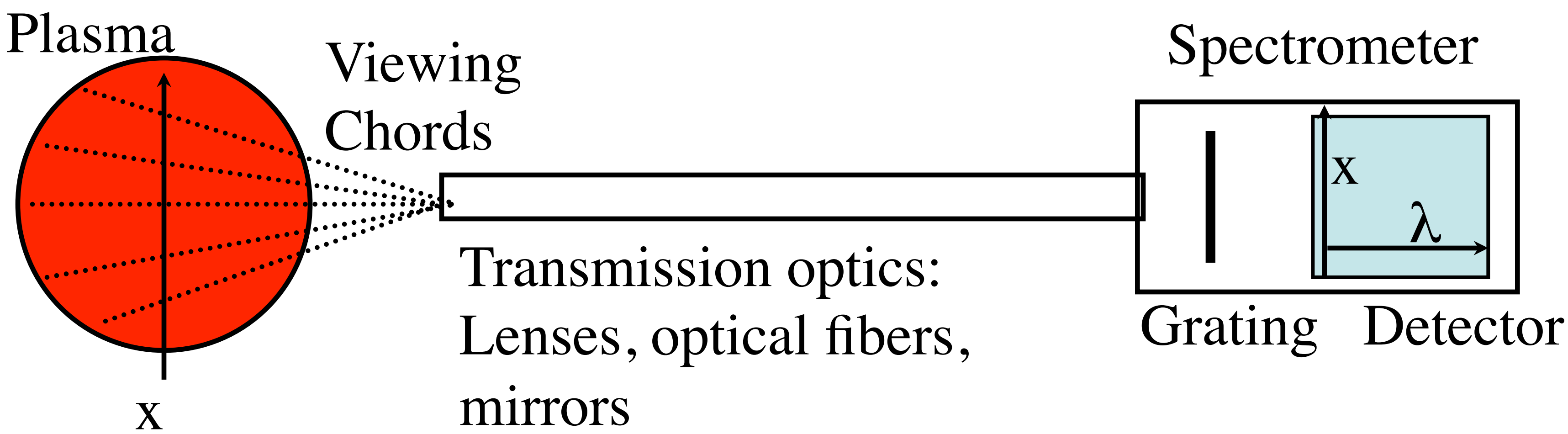
Typical Measurements:

- Line shape -- Position (Doppler shift) and Width (Thermal broadening)
- Line ratios -- Relative intensity of lines from closely-related states. For complex spectra, may depend on local density, temperature, **B**, **E**, etc.
- Absolute line intensity -- Depends on local n_e, T_e, and density of the specific ion, even specific ion energy state

Listed in order of increasing difficulty and uncertainty in result

Line Radiation V
Resolved Lines

Typical Modern Instruments:



Detector is 2-D CCD array optimized for instrument wavelength; Grating blazed to match, often in higher order for resolution. Optics generally low f/ for high throughput/sensitivity. Great variety of instruments available from visible to x-ray.

Line Radiation VI
Typical Instruments

- Broadband VUV to identify strong (low ionization state) lines from major impurities: Qualitative -- what's there
- Common instruments, often visible, to measure (Doppler) line width for ion temperature measurements -- easily done, but depends on localization of observed atomic state for location
- High-resolution instruments, often visible, to measure line shift for ion velocity measurements -- difficult because $V \ll v_{th}$, and depends on localization of observed atomic state for location
- A wide variety of instruments specialized by wavelength and purpose

Where is Z_o^{+n}? Spatial Resolution? Types of equilibria

- Complete thermodynamic equilibrium
  - All processes are balanced by their inverse
  - Does not exist in laboratory plasmas
- Local thermal equilibrium (LTE)
  - Distribution of states is nearly the same
  - Collisional rates >> radiative rates
- Coronal equilibrium (as in Sun's corona)
  - Radiative rates >> collisional rates