Class Meetings:  M, W, F at 1:00-2:00 PM in RLM 7.104
Introduction to High Energy Physics, Perkins, Addison Wesley,  
Prerequisites:  PHY 373
Homework:  A total of ten homework assignments will be given during the term. Typically, homework problems will be given out at the Friday lecture and they will be due the following Friday at the beginning of lecture. Late homework will be accepted for one additional week. Late homework will receive a maximum of one-half credit; homework more than one week overdue will receive no credit.
Exams:  Three quizzes will be given during the term. No makeup tests will be given. There will be a final examination, scheduled by the university. A makeup final examination will be given only in documented cases of illness or emergency. The quizzes and final examination will be closed-book; a single 8 1/2” x 11” page of your notes and calculators may be used.
Grading:  Grades will be determined from points accumulated during the term. Points are given for performance on assigned homework, tests given during the term, participation in class and the final examination. The goal is to accumulate 100 points. Four points (maximum) will be given for each satisfactory homework assignment. Each of the scheduled tests will have a maximum score of 16 points. Up to 12 points will be given for class participation based on attendance, performance in "pop" quizzes, etc. The maximum point value of the final exam will be determined for each student as the difference between 100 and the total points acquired through homework, test scores and participation. Thus, for students who keep up with the homework (40 points possible), have a perfect score on both tests (48 points) and have a good record of class participation (12 points), the final examination will be worth 0 points and they don't have to show up for it. For the student who had to miss one test and lost 24 points on homework and the other things, the final exam will be worth 40 points. The student who blows off the entire semester (not recommended!) can still, in principle, reach 100 points by having a perfect score on the final exam alone.
Early History

• Legacy of “cathode rays”
  – 1895 X-rays Roentgen
  – 1896 Radioactivity Becquerel
  – 1897 Electron J.J. Thompson

• Nuclear atom
  – 1911 Atomic nucleus Rutherford
  – 1913 Semi-classical theory Bohr

• Relativity
  – 1905 Special Relativity Einstein

The “problem” of cathode rays launched modern physics at the end of the 19th century through a series of experimental discoveries that could not be understood in terms of the “classical” physics of Newton, Maxwell, ...
On the *Frontier* of “Modern” Physics

- **Quantum Mechanics**
  - Essential tool for describing atomic/subatomic phenomena
  - Combined with relativity, modern quantum field theory appears capable of describing all known forces except *gravity*
- **Nuclear Physics**
  - Began with Rutherford atom and neutron discovery (1932)
  - Fundamental questions focused on “strong” force
- **Particle Physics**
  - What are the basic building blocks of matter?
  - What are the basic forces in Nature?

Some Early Highlights:

**Quantum Mechanics:**
- 1900: Black-body radiation Planck
- 1905: Photon Einstein
- 1926: Development of QM de Broglie, Schroedinger, de Broglie, Schroedinger, de Broglie, Schroedinger, Heisenberg, Dirac, ...

**Nuclear Physics:**
- 1932: Neutron Chadwick
- Early 1900s: Details of radioactive decays Rutherford, Curie, ...
- 1933: Artificial radioactivity Fermi, ...
- 1939: Nuclear fission Hahn, Strassmann, Meitner, Frisch

**Particle Physics:**
- 1933: Positron Anderson
- 1935: Meson hypothesis Yukawa
- 1947: Pion Occhialini, Powell, ...
- 1947: Strange particles Rochester & Butler
- 1956: Parity violation Lee & Yang
- 1950’s: Dominance of accelerator-based experiments
- 1971-74: Emergence of Standard Model
The Standard Model

- **Elementary Forces:**
  - Strong:
    - Quantum Chromodynamics (QCD)
    - exact gauge symmetry
    - acts on color
    - 8 colored, vector gluons
  - Unified electro-weak
    - hidden gauge symmetry
    - acts on electric charge/weak isospin
    - massive $W^\pm$, $Z^0$
    - photon
    - new Higgs field
  - Gravity
    - “classical” general relativity
    - acts on mass/energy
- **Elementary Particles**
  - Quarks
    \[
    \begin{pmatrix}
    u \\
    d \\
    c \\
    s \\
    t \\
    b
    \end{pmatrix}
    \times 3 \text{ colors}
  - Leptons
    \[
    \begin{pmatrix}
    \nu_e \\
    \nu_\mu \\
    \nu_\tau
    \end{pmatrix}
    \begin{pmatrix}
    e \\
    \mu \\
    \tau
    \end{pmatrix}
    \]
- **Parameters**
  - coupling strengths
    \[
    \alpha_s, \alpha_{em}, \alpha_{weak}
    \]
  - “mixing” angles
    $\theta_W$, CKM mixing of $q=-1/3$ quarks
  - fermion masses

Material on the standard model can be found at:
http://www.cipepweb.org/cipep_sm_large.html
http://www-pdg.lbl.gov/outreach.html
Simple Toolkit

• Units, Magnitudes, Scales
  – use “natural” units e.g. \( h, c = 1 \)
  – scale of atoms--- \( \approx 10^{-8} \text{ cm} \) ---set by interplay of Coulomb attraction and “quantum-pressure”
  – scale of nuclear phenomena--- \( \approx 10^{-13} \text{ cm} = 1 \text{ fm} \)---set by range of strong force

• Special Relativity
  – Observers, clocks, abstracted to “space-time”
  – Moving clocks appear to run slow: \( t = \gamma \tau \)
  – Moving rods appear short: \( l = l_0 / \gamma \)

Useful conversion factor for dealing with “natural” units: \( \hbar c \approx 197 \text{ MeV} \cdot \text{ fm} \)

1 fermi = 1 fm = 10^{-13} cm = 10^{-15} m

Some constants to know:

- fine structure constant \( \alpha = \frac{e^2}{4 \pi \varepsilon_0 \hbar c} \approx \frac{1}{137} \)
- electron mass \( m_e \approx 0.511 \text{ MeV}/c^2 \)
- proton mass \( m_p \approx m_n \approx 940 \text{ MeV}/c^2 \)

Mass, momentum, energy measured in units of MeV/c^2, MeV/c, MeV

Lorentz transformations along the x-axis:

\[
\begin{align*}
x &= \gamma (x' + \beta t') \\
t &= \gamma (t' + \beta x') \\
y &= y' \\
z &= z'
\end{align*}
\]

\[
\begin{align*}
x' &= \gamma (x - \beta t) \\
t' &= \gamma (t - \beta x) \\
y' &= y \\
z' &= z
\end{align*}
\]

Any set of four quantities that transform like \( x,y,z,t \) is called a 4-vector. We shall use the notation, \( a = (a_0, \vec{a}) \), to indicate 4-vectors.

The space-time position 4-vector is: \( x = (t, \vec{x}) \)

The 4-vector “dot” product, \( a \cdot b \equiv a_0 b_0 - \vec{a} \cdot \vec{b} \), is “Lorentz-invariant”, meaning it has the same value in all inertial frames.
Relativistic Kinematics

- Nuclear and particle interactions involve:
  - *Formation*  "beam" particles striking a "target"
  - *Scattering*  elastic or inelastic (different final particles)
  - *Decay*  subject to "conservation laws"

- Relativistic energy-momentum conservation
  always valid and usually provides a helpful guide.
  - Energy-momentum 4-vector:  \( p = (E, \vec{p}) \)  \( \vec{v} = \vec{p} / E \)
  - *Invariant* mass or rest mass:  \( p \cdot p = m^2 = E^2 - \vec{p} \cdot \vec{p} \)
  - Energy-momentum conservation:
    \[ \sum p_i = \sum p'_i = P_{\text{center of mass}} \]

Use natural units where \( c = 1 \). This means that \( m, p, E \) measured in units of MeV/c\(^2\), MeV/c, MeV, (or GeV, or TeV) respectively. \( v \) will be relative to \( c \).

Just like Newtonian kinematics, it is often convenient to introduce the "center of mass" frame where \( \sum \vec{p} = 0 \). The center-of-mass velocity is easily found in other frames by:

\[
\vec{v}_{\text{cm}} = \frac{\sum \vec{p}}{\sum E}
\]

The *rapidity* variable, \( y \), is the analog of non-relativistic velocity in that it "adds" for successive Lorentz transformations.

\[
y = \frac{1}{2} \ln \left( \frac{1 + \beta}{1 - \beta} \right) \quad \gamma = \cosh y \quad \beta y = \sinh y
\]

The *rapidity* of a particle is defined as the rapidity of the Lorentz frame where the particle is moving purely transversely, perpendicular to the axis of Lorentz "boost". Together with rest mass, \( m \), and "transverse momentum", \( p_t \), \( y \) completely specify the kinematics of particles; the rapidity of relativistic particles is approximately equal to an angle variable, the *pseudo-rapidity*, \( \eta \), given by:

\[
y \approx \eta \equiv \ln(\cot(\theta / 2))
\]

where \( \theta \) is the polar angle of the particle with respect to the axis along which rapidity is defined.
Moving, electrically charged particles lose energy in matter by ionization. Their electric field will act on atomic electrons, causing some to be ejected from atoms under appropriate conditions. Generally, the energy loss per unit length is a function of the charge and velocity of the moving particle. When particles (of charge $|q| = e$) become relativistic ($\beta \gamma \geq 1$), the energy-loss varies slowly with momentum.

The range of a particle is the distance it will travel in some medium before losing all its kinetic energy to ionization. The range can be found from integrating the Bethe-Bloch equation. The figure gives particle ranges (divided by the mass of the particle) in various materials.

Neutral particles, such as photons (gamma rays) and neutrons are “seen” via the ionization of secondary charged particles produced when they interact directly with atoms or nuclei in the material.
Decays

- Very common in nuclear and particle physics—basis of “radioactivity”
- Decay probability/unit time is constant: $\gamma$
  - Exponential decay law: $N(t) = N_0 \exp(-\gamma t)$
  - Mean lifetime, “half-life”: $\tau = \frac{1}{\gamma}$, $\tau_{1/2} = \tau \ln 2 = 0.69 \tau$
  - Uncertainty in energy (or mass) of decaying state related to lifetime via Heisenberg relations: $\Delta E \sim \hbar / \tau$
  - Decays to several final states possible: $\gamma = \sum \gamma_i$, Branching Ratio $= \gamma_i / \gamma$
- Huge range of lifetimes seen in Nature

General rule of nature seems to be that states will decay to states of lower energy as fast as possible, unless there is some principle inhibiting the decay.

Products of decays may also be unstable, giving rise to networks of decays with various components building-up and decaying with different rates.

Comparing objects, such as different isotopes of the same element, with different decay rates permits “dating” of significant biological or nuclear events. A common example is “$^{14}$C dating”, which is useful for time-scales comparable with the half-life of $^{14}$C, 5730 years.

Some common lifetimes:

- “strongly-interacting” particles $\sim 10^{-23}$ s
- Muons 2.2 $\mu$s
- Neutrons 887 s
- $^{238}$U $\sim 6 \times 10^9$ yr
- Protons $> 10^{32}$ yr

Activity of unstable material is defined as the number of decays per unit time; it depends on the amount of material and decay rate. Activity is measured in becquerels (1Bq = 1 disintegrations/s) or curies (1Ci = 3.7$\times$10$^{10}$ disintegrations/s).

Dose is the energy absorbed in material from ionizing radiation. It is measured in units of greys (1Gy = 1 joule/kg) or rads (1 rad = 10$^{-2}$ Gy).

Equivalent Dose is established as the measure of ionizing radiation received by humans. Biological effects depend on the type of particle and dose absorbed. Equivalent dose is measured in units of sieverts (Sv).
Cross Sections

- Useful “meeting-ground” of experiment and theory
  - can measure number of particles lost from a beam when passing through a thin target: \( N_{\text{lost}} = N_0 nt \sigma_{\text{abs}} \)
  - or the number of particles scattered into a given element of solid angle:
    \[
    N_{\text{scat}} = N_0 nt \frac{d\sigma}{d\Omega} \Delta\Omega
    \]
- Usual unit of cross section is the barn, \( 10^{-24}\text{cm}^2 \)
- Observed numbers of events—scattered, transmitted, or from decays follow Poisson statistics

We will encounter a vast range in the magnitude of nuclear and subnuclear cross sections, from barns to femto-barns \((10^{-39} \text{cm}^2)\). The high-energy proton-proton total cross section is in the range 40 - 100 mb, for example.

Poisson statistics tells us that the number of counts observed, \( n \), for a given process, which is necessarily an integer, will generally fluctuate around the mean number expected, \( x \), with a probability distribution given by:

\[
P_n(x) = \frac{x^n}{n!} e^{-x}
\]

Technically, this is the probability that \( n \) events are observed when the number expected is \( x \). If many separate measurements are made, the mean of \( n \) will approach \( x \), with an RMS spread about the mean of \( x^{1/2} \). When one measures \( N \) events in some experiment, \( N \) is taken to be the estimator for \( x \), and the estimated one-standard deviation error is typically taken to be \( N^{1/2} \).
Nuclear Sizes

• “Standard” technique: electron scattering
  – critical kinematic quantity is momentum transfer (to target): \( q = k - k' \) \( k, k' \) incident, scattered electron momenta
  – probes charge density at distance scales \( \sim 1/q \)

• Distribution of charge extracted from measurements of the form-factor, \( F(q) \)
  – “point-like” cross section depends on type of particle used for probe and target
  – \( F(q) \) is the fourier-transform of the charge density, \( \rho(r) \)

In quantum mechanics, the differential scattering cross section is related to the scattering amplitude, \( f(\theta) \), by:

\[
\frac{d\sigma}{d\Omega} = |f(\theta)|^2
\]

In the Born approximation, the scattering amplitude can be computed from the fourier-transform of the scattering potential.

\[
f_{\text{Born}}(\theta) \propto \tilde{V}(q) \equiv \int d^3 r e^{iqr} V(r)
\]

When applied to electron scattering, where the beam particle (probe) is structureless, Coulomb’s law can be used to “factor out” the \( 1/r^2 \) part of the Coulomb force, leaving the scattering amplitude with the form:

\[
f(\theta) \sim \frac{1}{q^2} F(q)
\]

where the form factor, \( F \), has been introduced to describe the (unknown) charge distribution, \( \rho(r) \).

\[
F(q) = \frac{1}{Ze} \int d^3 r e^{iqr} \rho(r) = 1 - \frac{1}{6} q^2 \langle r^2 \rangle
\]

The approximate form of \( F \) is valid when \( q < 1/r_{\text{RMS}} \), the root-mean-square “size” of the target.

The “point-like” cross section appropriate for many electron-nucleus scattering experiments is the Mott cross section for scattering of relativistic electrons (speed \( \beta \)) by a scalar target (in natural units; \( \alpha \) is the fine-structure constant):

\[
\frac{d\sigma}{d\Omega}|_{\text{Mott}} = \frac{4Z^2 \alpha^2 k^2}{q^4} \left[ 1 - \beta^2 \sin^2(\theta/2) \right] \times \left[ 1 - \beta^2 \sin^2(\theta/2) \right] \times \frac{d\sigma}{d\Omega}|_{\text{Rutherford}}
\]
Sizes (contd.)

- $e$-nucleus scattering characterized by *diffraction dips*, analogous to diffraction minima in optics
  - sharp minima in $|F(q)|^2$ when $q \sim n \frac{2\pi}{D}$
  - not seen in $e$-p or $e$-n scattering
- muons, having masses $200 \times$ electrons, form very small Bohr orbits, which also probe nuclear size, through x-rays emitted from *muonic* atoms.
- “Rule-of-thumb” picture:
  - approx. constant density out to a radius, $R \sim 1.2 \text{ fm } A^{1/3}$
  - outer edge “smeared” over $t \sim 0.75 \text{ fm}$

Condition for diffraction minimum by a slit of width $D$, for light of wavelength $\lambda$ is:

$$\frac{D \sin(\theta)}{2} = n \frac{\lambda}{2}$$

Translating to our scattering variables, this becomes:

$$q = 2k \sin(\theta / 2) \approx \frac{2\pi}{\lambda} \sin(\theta) = n \frac{2\pi}{D}$$
Nuclear Masses

• Quantitative description most convenient with introduction of binding energy, \( B(Z,A) \):
  \[ B = Zm_p + Nm_n - M(Z,A) \quad N + Z = A \]
• Strongest binding—largest \( B/A \)—occurs near Fe, where \( B/A \sim 9 \text{ MeV} \)
  – lighter nuclei will tend to fuse
  – heavier nuclei will tend to fission
• The semi-empirical mass formula, derived from the liquid-drop model, provides a useful, but approximate description of nuclear masses.

Semi-empirical mass (binding-energy) formula:
\[
B(Z,A) = +a_V A \quad \text{volume binding} \\
- a_S A^{2/3} \quad \text{surface tension} \\
- a_C Z^2/A^{1/3} \quad \text{Coulomb repulsion} \\
- a_A (A-2Z)^2/A \quad \text{“Pauli” asymmetry} \\
\begin{cases} 
-a_p/A^{1/2} & \text{odd-odd nuclei} \\
0 & \text{odd-even nuclei} \\
+ a_p/A^{1/2} & \text{even-even nuclei}
\end{cases}
\]

A useful set of values for the various parameters is:
\[
\begin{align*}
ap & = 15.56 \text{ MeV} \\
as & = 17.23 \text{ MeV} \\
c & = 0.697 \text{ MeV} \\
n & = 23.285 \text{ MeV} \\
p & = 12.0 \text{ MeV}
\end{align*}
\]

To obtain the atomic rest mass, change the proton mass \( m_p \) to the mass of atomic hydrogen \( m_H \)
\[
\begin{align*}
m_p & = 938.280 \text{ MeV}/c^2 \\
n & = 939.573 \text{ MeV}/c^2 \\
H & = 938.791 \text{ MeV}/c^2
\end{align*}
\]
Nuclear Instability

- A principal force that shapes the Periodic Table
  - \((Z, A)\) can change, but the total number of protons and neutrons present does not change.
  - “Cascade” decays occur when daughter nucleus formed in excited state.
  - decays will always take place unless prohibited or inhibited by some conservation principle.

- The main categories of nuclear decays are:
  - \(\alpha\) “alpha” radiation
  - \(\beta\) “beta” radiation
  - \(\gamma\) “gamma” radiation

The “Q-value” of a decay, the total kinetic energy available to the decay products, is a crucial factor in determining the decay rate.

\[ Q = \left( M_{\text{initial}} - \sum_{\text{decay products}} m_i \right) c^2 \]

Characteristics of nuclear decays:

\(\alpha\)-decay: \( (Z, A) \rightarrow (Z-2, A-4) + \alpha \)
  - “line” spectrum
  - rate strongly dependant on \(Q\)-value (fractions of sec. to billions of yrs.)
  - process is quantum-mechanical tunneling through the Coulomb “barrier”

\(\beta\)-decay: \( (Z, A) \rightarrow (Z+1, A) + e^- + \text{antineutrino} \)
  - broad spectrum of electron energies up to the \(Q\)-value. This fact and principles of momentum and angular-momentum conservation led Pauli to introduce the idea of neutrinos (and antineutrinos) participating in \(\beta\)-decays.
    - relatively slow decay rates
    - related processes: \(\beta^+\)-decay \( (Z, A) \rightarrow (Z-1, A) + e^+ + \text{neutrino} \)
      - electron-capture \( (Z, A) + e^- \rightarrow (Z-1, A) + \text{neutrino} \)
      - represents a “new” force—the weak force—now known to be related to the electromagnetic force.

\(\gamma\)-decay: \( (Z, A) \rightarrow (Z, A) + \gamma \)
  - “line” spectrum
  - relatively fast decay rates
  - process is electromagnetic radiation from excited nuclear states, just as in ordinary atomic transitions, but involving higher energies.
Nuclear Stability

- Role of $\alpha$- and $\beta$-decays in achieving stable nuclei
  - Interplay of $A$, $Z$, and binding energy $B(Z, A)$:
    - $\alpha$-decay is energetically allowed for $A > 151$
    - The “Pauli” asymmetry in binding makes it energetically favorable for nuclei away from $Z=N$ to approach symmetry via $\beta$-decay, $\beta^+$-decay, or electron-capture
  - Spontaneous fission occurs in heavy nuclei (e.g. U), but is generally quite rare compared to $\alpha$-decay.

- Some consequences:
  - odd-$A$ nuclei have one stable isotope (against $\beta$-decay)
  - there are almost no stable odd-odd nuclei ($^2$H, $^6$Li, $^{10}$B, $^{14}$N are only exceptions)
  - even-even nuclei can have more than one stable isobar

Examples showing possible $\beta$-decay sequences derived from the semi-empirical mass formula.
# Nuclear Fission

- Discovered/explained in 1939—great technological and historical importance.
- Liquid-drop picture: elongation with energy barrier $E_b$ followed by fission and strong repulsion, yielding energy.

### Key numbers:

<table>
<thead>
<tr>
<th>Process</th>
<th>$E_b$ (MeV)</th>
<th>$B_n$ (MeV)$^*$</th>
<th>$\sigma_f$ (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n+^{235}<em>{92}U$ and $n+^{239}</em>{94}Pu$ <strong>exothermic</strong></td>
<td>5.2</td>
<td>6.5</td>
<td>584</td>
</tr>
<tr>
<td>- $\sim$170 MeV released per atom</td>
<td>5.7</td>
<td>4.8</td>
<td>$2.7 \times 10^{-6}$</td>
</tr>
<tr>
<td>- $\sigma_f$ for thermal neutrons $&gt;&gt; \sigma_{geom}$</td>
<td>4.8</td>
<td>6.4</td>
<td>742</td>
</tr>
<tr>
<td>$n+^{238}_{92}U$</td>
<td>5.2</td>
<td>6.5</td>
<td>584</td>
</tr>
</tbody>
</table>

- Fission products are asymmetric in mass and are **neutron-rich**.

While energetically possible in essentially all nuclei heavier than iron, fission is important in only a few elements because of the interplay of barrier energy, binding energy, and rates for other, competing processes, such as $\alpha$-decay.

The fission cross section in $^{238}_{92}U$ (and Pu) is much larger than the geometric cross section at very low (corresponding to room-temperature) energies because of the large deBroglie wavelength of low-momentum neutrons. At the mean fission energy, fission cross sections of these “fissile” materials for “fast neutrons” is still appreciable $\sim 1.22$ b for $^{235}_{92}U$.

Because heavier nuclei are relatively more neutron-rich than lighter nuclei, when fission occurs, it leads to excess neutrons which may not be bound in the daughter nuclei. Excess neutrons are emitted with a spectrum of energies, characteristic of a hot object with a temperature in the range: $kT \sim $ MeV. The mean neutron energy in U fission is $\sim 2$ MeV.

The neutron excess in fission leads to a **multiplication** by a factor $\nu$ in the number of neutrons present after each generation of fissions: $N_g = \nu N_{g-1}$

The mean number of neutrons emitted in the fission of $^{235}_{92}U$ is $\nu \sim 2.5$

Because each new generation of neutrons may also cause fissions to occur, the possibility exits for a **chain-reaction** of exponential growth in the number of neutrons and energy released. In a chain-reaction, the total number of fissions $N(G)$ after $G$ generations is:

$$N(G) = \frac{\nu^G - 1}{\nu - 1}$$

It takes about 70 generations to fission a mole of $^{235}U$. 

Conditions for a Chain-Reaction in $^{235}$U

- Growth in the density $N$ of neutrons inside a sphere due to fission: $\frac{dN}{dt} = \frac{v-1}{\rho} N(t)$ where $t$ is the mean-time between fissions (~10 ns for “fast” neutrons).
- Diffusion of neutrons—flow determined by “random walk” of collision length $l_c$ and gradient of density. $j = -\frac{1}{3} l_c \langle \psi \rangle \nabla N$ where $\langle \psi \rangle$ is the mean (RMS) speed of neutrons.
- Continuity—conservation of neutrons: $\frac{dN}{dt} = \frac{\partial N}{\partial t} + \nabla \cdot j$

and continuity at the boundary of the sphere: $\frac{1}{2} \langle \psi \rangle N(R) = -\frac{1}{3} l_c \langle \psi \rangle \frac{dN}{dr}|_{r=a}$
- Leads to a critical radius $R_{\text{crit}}$, above which the system will sustain exponential growth. $R_{\text{crit}} \approx 10 \text{ cm in } ^{235}\text{U}$
  - Note: $M_{\text{crit}} \propto \rho R_{\text{crit}} \propto \frac{1}{\rho}$ where $\rho$ is the mass density of $^{235}\text{U}$

There are two general approaches to controlling the fission of materials such as U to achieve a chain reaction: 1) by slowing down the neutrons in suitable low-A materials—called a moderator—to thermal velocities to take advantage of the very large fission cross sections or 2) providing an explosive reaction in nearly pure $^{235}\text{U}$ with fast (~MeV) neutrons. The first approach is the basis for nuclear reactors; the second is the basis for nuclear weapons. Note that natural U is stable against an explosive chain reaction because ~99% is $^{238}\text{U}$, which requires neutrons of energy > 1 MeV to fission—the effective number of neutrons generated in $^{238}\text{U}$ is $\nu \sim 0.4$.

Because of the greater conceptual simplicity of fission in pure $^{235}\text{U}$ and its scientific and historic value, the preliminary calculations that were the basis for the US atomic bomb project (Manhattan Project) are given, patterned after the original introductory lectures by Robert Serber, reproduced and annotated in the book, *The Los Alamos Primer* by Robert Serber, Univ. of California Press, 1992. Modern nuclear weapons use $^{239}\text{Pu}$, in some cases, rather than $^{235}\text{U}$, because the critical mass is smaller and it can be easier to produce the fissile isotope in pure form.

Other key numbers for $^{235}\text{U}$: density $n = 19 \text{ gm/cm}^3$
- collision length $l_c = 1/n \sigma_t = 5 \text{ cm}$ $\sigma_t$ is the “transport” cross section
The shape of the neutron density inside the sphere is given by: $N(r) \propto \frac{\sin kr}{r}$

The condition on $k$ to achieve exponential growth is:

$$kl_c \leq \sqrt{\frac{3(\nu-1)\sigma_t}{\sigma_t}} \Rightarrow a \approx 1.18$$

The numerical solution to the boundary condition gives:

$$M_{\text{crit}} \approx 70 \text{ kg}$$

$$R_{\text{crit}} \approx \left( \frac{3.20}{a} - 0.80 \right) l_c \approx 9.6 \text{ cm}$$
Isospin

- The strong nuclear force is *charge independent*
  - patterns of nuclear energy levels in nuclei of different $Z$, but same $A$ (isobars)
  - $pp, pn, nn$ scattering indicate same force for same spatial state--spin, orbital angular momentum
- Introduce an *internal* (and *approximate*) symmetry, isospin, to describe charge-independence
  - follows same rules—algebra—as angular momentum
  - 2 relevant eigenvalues:
    - $I$ gives the multiplicity $(2I+1)$ of charge states at $\approx$ same energy
    - $I_3$ labels each charge state $-I \leq I_3 \leq +I$

The *algebra* of isospin is described by the commutation relations:

$$[I_1, I_2] = i I_3, \quad [I_2, I_3] = i I_1, \quad [I_3, I_1] = i I_2$$

Charge-independence of the strong force implies that the Hamiltonian describing nuclear forces will commute with the two possible commuting operators of isotopic spin, $I^2$ and $I_3$

$$I^2 |I, m_i\rangle = I(I+1) |I, m_i\rangle, \quad I_3 |I, m_i\rangle = m_i |I, m_i\rangle$$

In this picture, the proton and neutron are viewed as different isospin states of a single entity, the *nucleon*;

$$I_3 |p\rangle = + \frac{1}{2} |p\rangle, \quad I_3 |n\rangle = - \frac{1}{2} |n\rangle$$

Allowed nucleon states must obey the *generalized Pauli Principle*:

$$\Psi_{\text{total}} (1,2) = \Psi_{\text{space}} \Psi_{\text{spin}} \Psi_{\text{isospin}} = - \Psi(2,1)$$

This restricts the number of possible states that can be formed from protons and neutrons. As a consequence, “triplet” charge states ($I=1$, which is symmetric) must be associated with anti-symmetric space-spin states, while “singlet” ($I=0$) charge states will be associated with symmetric space-spin states. The deuteron and alpha particle are examples of the second half of this rule.

For a nucleus comprising $A$ nucleons, $I_3 = (Z-N)/2$ and $Q/e = Z = I_3 + A/2$

Empirical rule: all nuclear ground states take on the *lowest* value of $I$ consistent with $I_3$; i.e. $I = |Z-N|/2$
The Pion

- Discovered in cosmic rays in 1947, based in part on 1935 suggestion by H. Yukawa to explain the range of the strong force.
- Ubiquitous in strong interactions with protons and neutrons; $\pi^\pm$ decay through weak force, $\pi^0$ through electromagnetic force.
- Properties measured in low-energy reactions:
  - Spin = 0 comparing $\sigma(\pi^+d \rightarrow pp)$ with $\sigma(pp \rightarrow \pi^-d)$
  - Parity = - from observed reaction: $\pi^-d \rightarrow nn$
  - Isospin = 1 three charge states: $\pi^+$, $\pi^0$, $\pi^-$

Pions are the lowest-mass members of the class of strongly interacting particles (collectively called hadrons) with integral values of spin (bosons) called mesons. Hadrons with half-integral spin (fermions), such as nucleons, are called baryons.

Some properties of pions are listed below:

<table>
<thead>
<tr>
<th>Property</th>
<th>$\pi^\pm$</th>
<th>$\pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rest Mass</td>
<td>139.6 (MeV/c^2)</td>
<td>135.0</td>
</tr>
<tr>
<td>Lifetime (s)</td>
<td>$2.60 \times 10^{-8}$</td>
<td>$8.4 \times 10^{-17}$</td>
</tr>
<tr>
<td>Principal Decay modes</td>
<td>99.99% $\mu^+\nu$</td>
<td>98.8 % $\gamma\gamma$</td>
</tr>
<tr>
<td></td>
<td>1.2 % $e^+e^-\gamma$</td>
<td></td>
</tr>
</tbody>
</table>
Pion-Nucleon Resonances

- Broad “bumps” found in $\sigma(\pi p)$ or $\sigma(\pi n)$ at various collision energies
  - interpreted as “particle” at the $\pi$-nucleon invariant mass
  - width of mass peak related to lifetime $\tau \Delta m \sim 1$
    - 200 MeV width indicates a typical “strong” decay, $\tau \sim 10^{-23}$ s
- First system studied: the “3-3” resonance at 1232 MeV
  - $J^P$ determined from angular distribution of scattered particles: $1+3\cos^2(\theta) \Rightarrow J^P = 3/2^+$
  - Isospin assignment ($I=3/2$) proved by cross section ratios
- Many other families of resonances also exist

Reactions studied at pion kinetic energies near 190 MeV:

a) $\pi^+ p \rightarrow \pi^+ p$ elastic scattering
b) $\pi^- p \rightarrow \pi^- p$ elastic scattering
c) $\pi^- p \rightarrow \pi^0 n$ charge-exchange

plus others involving neutrons (bound in nuclear targets such as deuterium).

Angular distribution: determined by the orbital angular momentum $L$ of the pion-nucleon and spin ($S=1/2$) of the proton (pion spin=0). If the resonance has $J^P=1/2^+$, then $L=0$ (“$S$-wave”) and the angular distribution would be uniform. If $J^P=1/2^+$ or $J^P=3/2^+$, then $L=1$ and the angular distribution is given by appropriate sums of squares of the $Y_{1m}$ functions. The $J^P=3/2^+$ case gives the observed angular distribution.

Using similar techniques, isospin can be used to determine ratios of the cross sections for the processes listed above, assuming the resonance has a definite value of isospin. Because $I_\pi=1$ and $I_p=1/2$, the possible values are 1/2 and 3/2. The former would give $\sigma_a : \sigma_b : \sigma_c = 0 : 4 : 2$, while the $I=3/2$ hypothesis predicts $\sigma_a : \sigma_b : \sigma_c = 9 : 1 : 2$, as observed. Thus, these resonances are interpreted as an isospin quadruplet, $\Delta^+ \Delta^+ \Delta^+ \Delta^+$, of baryon number $B=1$ states with $J^P=3/2^+$ at a mass of 1232 MeV.

Many other multiplets of resonances were subsequently found at higher masses with various sets of quantum numbers.
Strange Particles

- **Discoveries of “V” particles** (Rochester & Butler, others):
  - Strong production cross sections, but relatively long-lived
  - Seen as mesons ($K^\pm, K^0$) and baryons ($\Lambda^0, \Sigma^-, \ldots$) “hyperons”
  - Gell-Mann, Pais proposal of “associated production”
    - New additive quantum number, strangeness, assigned; ordinary particles ($p, n, \pi, \ldots$) have $S=0$, new particles have $S=\pm1, \ldots$
    - Strangeness conserved by strong, EM forces; not conserved in particular ways by weak force
- Many “strange” relatives of “ordinary” hadrons are now known
  - form multiplets when plotted on $I_3-Y(=B+S)$ plane

Initial assignments of strangeness were purely arbitrary; afterwards, assignments made assuming strangeness conservation in the (strong) interactions where new particles are produced.

Distinct patterns of observed families of particles are found. For a given spin-parity assignment, particles of nearly equal mass form “multiplets”. A convenient and illuminating way to display these patterns is by plotting coordinates of the particle’s values of $I_3$ and $Y$ (called “hypercharge”), where $Y=B+S$. In terms of the hypercharge, the relationship between electric charge (in units of the elementary charge $e$), $Q$, and isotopic spin for all hadrons, mesons and baryons, is: $Q = I_3 + Y/2$
Some Hadron Multiplets

\[ ^{\text{JP}=0^-} \text{Mesons} \]
\[
\begin{array}{c}
(495 \text{ MeV}) K^0 \\
(140 \text{ MeV}) \pi^- \\
(495 \text{ MeV}) K^- \\
(892 \text{ MeV}) K^{0*} \\
(770 \text{ MeV}) \rho^- \\
(892 \text{ MeV}) K^{*-} \\
\end{array}
\]
\[ K^+ \quad \pi^+ \quad \eta \quad K^0 \quad \Sigma^- \quad \sum^+ \quad \Xi^- \quad \Xi^0 \]

\[ ^{\text{JP}=1/2^+} \text{Baryons} \]
\[
\begin{array}{c}
\pi^- \\
\eta \\
K^- \\
K^{0*} \\
\rho^- \\
K^{*-} \\
\end{array}
\]
\[ \eta^+ \quad \pi^0 \quad \pi^+ \quad \eta \quad \phi \quad \rho^+ \quad \Omega^- \]

\[ ^{\text{JP}=1^-} \text{Mesons} \]
\[
\begin{array}{c}
(495 \text{ MeV}) K^0 \\
(140 \text{ MeV}) \pi^- \\
(495 \text{ MeV}) K^- \\
(892 \text{ MeV}) K^{0*} \\
(770 \text{ MeV}) \rho^- \\
(892 \text{ MeV}) K^{*-} \\
\end{array}
\]
\[ K^+ \quad \pi^+ \quad \eta \quad K^0 \quad \Sigma^- \quad \sum^+ \quad \Xi^- \quad \Xi^0 \]

\[ ^{\text{JP}=3/2^+} \text{Baryons} \]
\[
\begin{array}{c}
\pi^- \\
\eta \\
K^- \\
K^{0*} \\
\rho^- \\
K^{*-} \\
\end{array}
\]
\[ \Delta^- \quad \Delta^0 \quad \Delta^+ \quad \Delta^{++} \]

\( (940 \text{ MeV}) \)
\( (1189 \text{ MeV}) \)
\( (1116 \text{ MeV}) \)
\( (1315 \text{ MeV}) \)
\( (1232 \text{ MeV}) \)
\( (1385 \text{ MeV}) \)
\( (1530 \text{ MeV}) \)
\( (1672 \text{ MeV}) \)

\( (495 \text{ MeV}) \)
\( (892 \text{ MeV}) \)
\( (770 \text{ MeV}) \)
\( (892 \text{ MeV}) \)
\( (140 \text{ MeV}) \)
\( (1530 \text{ MeV}) \)
Discovery of the $\Omega^-$

- First found at Brookhaven National Lab, 1963
- Based on one bubble chamber event!
- Another nice example:

$$K^- p \rightarrow \Omega^- K^+ K^0$$

$$\Omega^- \rightarrow \Lambda^0 K^-$$

$$\Lambda^0 \rightarrow p \pi^-$$

Bubble Chambers

Invented by Don Glaser in 1952 – 1960 Nobel Prize; extensively developed by L.W. Alvarez (also Nobel Prize winner based on work using bubble chambers).

Widely used until fairly recently when largely superceded by new technology and different research directions in high energy physics.

The working liquid of the chamber is also the beam target for accelerated particles. The subsequent interactions of beam particles and collision products formed in the chamber can be “tracked” or viewed in photographs.

Common working fluids: LH$_2$, LD$_2$, freons

Principle of operation: Ionization by charged particles passing through the liquid of the chamber can nucleate bubble growth. After some sensitive time during which the chamber is exposed to high energy charged particles, a piston-like device on the chamber effects a rapid expansion of the working liquid in the chamber and small bubbles start to form where nucleated by ionization left behind by charged tracks. At an appropriate time after the expansion, the chamber is photographed (usually with multiple stereo views). After the photograph is taken, the nominal working pressure is reestablished in the liquid and the chamber is again sensitive to another batch of particles.

Advantages: fine-grained images of moderately complex events

Disadvantages: poor “duty-cycle” because of need to expand chamber; limited to relatively common events; scanning of photographs tedious/expensive
Early Quark Model

- Gell-Mann and, independently, Zweig (1964) realized that all known (at the time) hadrons could be “built” out of 3 spin-1/2 objects, called quarks by Gell-Mann
  - Followed earlier observation of “the 8-fold way” by Gell-Mann and Ne’eman that the Y-J patterns of hadrons are representations of the group SU(3)
  - At first, quarks were thought to be mathematical objects, not “real”
- Properties of the 3 quarks: symbol name charge $I$ $I_3$ $S$
  
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>up</td>
<td>+2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>$d$</td>
<td>down</td>
<td>-1/3</td>
<td>1/2</td>
</tr>
<tr>
<td>$s$</td>
<td>strange</td>
<td>-1/3</td>
<td>0</td>
</tr>
</tbody>
</table>

- Mesons are formed from quark-antiquark pairs; baryons are formed from three quarks

Some other successes of the early “static” quark model:

Pion-nucleon cross sections follow from “quark counting”: \[ \frac{\sigma(\pi N)}{\sigma(NN)} = \frac{2}{3} \]

Magnetic moments of the “stable” baryons:

<table>
<thead>
<tr>
<th>Baryon</th>
<th>Quark Moments</th>
<th>Predicted (n.m)</th>
<th>Observed (n.m.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$\frac{4}{3} \mu_u - \frac{1}{3} \mu_d$</td>
<td>2.79</td>
<td>2.793</td>
</tr>
<tr>
<td>$n$</td>
<td>$\frac{4}{3} \mu_d - \frac{1}{3} \mu_u$</td>
<td>-1.86</td>
<td>-1.913</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$\mu_s$</td>
<td>-0.58</td>
<td>-0.614 +/- 0.005</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>$\frac{4}{3} \mu_u - \frac{1}{3} \mu_s$</td>
<td>2.68</td>
<td>2.33 +/- 0.13</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>$\frac{2}{3} (\mu_u + \mu_d) - \frac{1}{3} \mu_s$</td>
<td>0.82</td>
<td>—</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>$\frac{4}{3} \mu_d - \frac{1}{3} \mu_s$</td>
<td>-1.05</td>
<td>-1.00 +/- 0.12</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>$\frac{4}{3} \mu_s - \frac{1}{3} \mu_u$</td>
<td>-1.40</td>
<td>-1.25 +/- 0.014</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>$\frac{4}{3} \mu_u - \frac{1}{3} \mu_d$</td>
<td>-0.47</td>
<td>-1.85 +/- 0.75</td>
</tr>
</tbody>
</table>
Quarks, contd.

• Isolated or “bare” quarks have never been observed
  – extensive searches for fractionally-charged objects
  – a consequence of the strong force, QCD, called “confinement”
• Fermi statistics “problem”
  – The \( J^P=3/2^+ \) “decuplet” (\( \Delta^+, \Omega^- \), etc.) requires purely symmetric space & spin wave-functions, which should not be allowed for spin-1/2 objects as quarks are supposed to be.
  – Finally resolved with the “color” quantum number, the “charge” of the theory of the strong force, QCD. Real particles must be “color-singlets”, meaning the color part of the wave-function of baryons will be antisymmetric.
  – When applied to \( J^P=1/2^+ \), neutron, proton, etc., the \( uuu, ddd, sss \) states not present, which gives 8 nucleon-like states, an “octet”, as observed.
• Nevertheless, the patterns of hadron multiplets and many of their properties were remarkably well described in the simple quark model

Successes of the static quark model (contd.):

Mass splittings between baryons

Leptonic decays of \( J^P=1^- \) “vector” mesons:

\[
\Gamma(V \to l^+l^-) = \frac{16\pi\alpha^2 Q^2}{M_V^2} \psi(0)^2
\]

where \( Q \) is the charge of quark \( q \). In physical states comprising more than one quark flavor, \( Q^2 = \left| \sum a_i Q_i \right|^2 \)

Vector meson states: \( \rho^0 = \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d}) \), \( \sigma^0 = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \), \( \phi^0 = s\bar{s} \)

Prediction: \( \Gamma(\rho^0) : \Gamma(\sigma^0) : \Gamma(\phi^0) = 9 : 1 : 2 \)

Data: \( 8.8 +/- 2.6 : 1 : 1.70 +/- 0.41 \)
Weak Interactions: Preliminaries

• Originally invoked to describe nuclear β-decay
  – neutrino proposed by Pauli to preserve momentum and angular momentum conservation—subsequently observed directly
  – Fermi showed universality of strength of weak interactions—described by the “Fermi constant” $G \sim 10^{-5}/m_p^2$
  – decay rates scale as $E^5$

• Neutrino properties:
  – spin-1/2, electrically neutral
  – nearly massless (a topic of current research interest)
  – distinct neutrinos/antineutrinos associated with each lepton family, $e$, $\mu$, $\tau$
  – cross sections with ordinary matter grow with lab energy
Weak Interactions II: Downfall of Parity

• The $\theta$–$\tau$ paradox: early days of strange particle decays indicated the same object ($K$ meson) decays to two different parity states $\pi\pi$ and $\pi\pi\pi$, which is not allowed if Parity is conserved.

• Lee and Yang (1957) reviewed basis for parity conservation and found none for weak interactions—OK for strong and EM interactions

• Lee and Yang work soon followed by experimental verifications of parity violation—measurements of expectation values that change sign under Parity (e.g. $J\cdot p$)

• In fact, Parity is violated by the maximum extent possible!
Case Study: Role of Helicity in Pion Decay

- If WI is universal, why does $\frac{\pi \rightarrow e^+\bar{\nu}}{\pi \rightarrow \mu^+\bar{\nu}} \approx 1.3 \times 10^{-4}$?
- Note alignment of spins ($\pi^r$)
  - Pion spin = 0 $\Rightarrow$ $l^+$ spin = $-\nu$ spin $l^+ = e^+, \mu^+$
  - but... WI selects “right-handed” $l^+$ and “left-handed” $\nu$, which prefer spins to be parallel: “wrong helicity”
- Decay rate given by Fermi’s Golden Rule:
  $$\frac{1}{\pi} \left(1 - \frac{v}{c}\right) \sqrt{dp} \, dE_{\nu} = \frac{2m_0^2}{(m_0^2 + m_i)} \left[\frac{m_0^2 + m_i}{8m_0^2} (m_e^2 - m_i^2)^2\right]$$
  - helicity phase-space
- Suppression of wrong helicity electrons (1-$v/c$) dominates larger available phase-space (in agreement with exp.)
Deep-inelastic Electron Scattering (DIES)

- Quark substructure of nucleon was revealed by high energy inelastic electron scattering in late 1960’s
  - only recoiling electron is measured—final hadrons included single protons (elastic scattering) plus all other allowed states
  - Feynman, Bjorken, and others interpreted this as scattering from pieces of the nucleon, which Feynman called “partons”
- The cross section for DIES is analogous to Rutherford scattering with form factors to describe nuclear shapes, but:
  - spin-1/2 character of electron and nucleon accounted for
  - “structure-functions” of two kinematic variables replace form-factors when describing inelastic scattering.
- The data exhibit scaling behavior, implying point-like substructure

Inelastic electron-proton scattering can be described by two “structure functions”, $W_1$ & $W_2$ which correspond to form factors in elastic scattering.

\[ \frac{d^2 \sigma}{dq^2 dv} = \frac{4\pi\alpha^2}{q^4 EM} \left[ W_2(q^2,\nu) \cos^2 \frac{\theta}{2} + 2W_1(q^2,\nu)\sin^2 \frac{\theta}{2} \right], \]

\[ v = E - E' \]

Scaling is indicated by the observation that the related structure functions $F_1, F_2$ depend only on the dimensionless scaling variable $x$:

\[ F_2 \equiv \frac{\nu W_2(q^2,\nu)}{M} \approx F_2(x) \]
\[ F_1 \equiv W_1(q^2,\nu) \approx F_1(x) \]

\[ x \equiv \frac{-q^2}{2M\nu}, \quad 0 < x < 1 \]
Electron-Positron Colliding Beams

- Accelerator and detector technology to explore high energy $e^+e^-$ collisions came of age in the 1970’s and proved to be a powerful tool for studying hadron and lepton physics.
- Properties of the initial state formed by $e^+e^-$ annihilation:
  - $P = 0$, $E = 2E_{\text{beam}}$, $Q = B = S = \ldots = 0$
  - $J^P = 1^-$ “one-photon exchange” $\Rightarrow \frac{d\sigma}{d\Omega} \propto e^+_a e^- b \cos^2 \theta$
  - spin-1/2 objects produced with “known” rate: $Q_x$ is the electric charge of $x$ in units of $e$
    $\sigma_{\alpha,\nu} = \frac{4}{3}\pi \frac{\alpha^2}{E^2} Q_x^2$
- Total cross section for producing hadrons related to quark charges:
  $R = \frac{\sigma_{\text{hadrons}}}{\sigma_{\mu\mu}} = \sum_{\text{quarks}} Q_x^2$

The two critical performance characteristics of colliding beams accelerators—storage rings—are the beam energy $E_{\text{beam}}$ and luminosity $L$. Beam energy determines what states can be produced; luminosity determines the rate at which processes will occur.

For a given process with cross section $\sigma$, the counting rate $R$ is given by:

$$R = L \sigma$$

The luminosity is a property of the storage ring system. Typically, beam particles are “bunched” by a radio-frequency acceleration system into groups—called bunches—of approximately equal population that occupy a fraction of the circumference of the storage ring. In particle-antiparticle colliders, such as $e^+e^-$ and proton-antiproton colliders, the colliding beams follow essentially the same path through a common set of magnetic elements that make up the single storage ring. For two oppositely directed beams of relativistic particles the formula for $L$ is:

$$L = fn\frac{N_1 N_2}{A}$$

Where $N_1$ and $N_2$ are the numbers of particles in each bunch of the two colliding beams, $n$ is the number of bunches in each beam, $f$ is the revolution frequency and $A$ is the cross-sectional area of each beam (assumed to overlap completely).

Typical units of $L$ are $\text{cm}^2 \text{s}^{-1}$.
Charmonium

- A new “vector-meson” discovered unexpectedly in 1974: the $J/\Psi$ particle—$m = 3.1 \text{ GeV}, J^P = 1^-$
  - $e^+e^-$ production particularly dramatic—very narrow, large “resonance”
  - also seen in $p + \text{nucleus} \rightarrow e^+e^- + X$
  - related states form narrow (long-lived) atomic-like levels with level-spacings similar to positronium and photon transitions
- Crucial discovery—the “November Revolution”
  - solidified quark picture
  - revealed underlying simplicity of strong force: QCD
  - revealed new, heavier charge $+2/3$ quark: “Charm”
- “Naked” charm—particles formed from the new, $c$-quark, and the original ($u, d, s$) quarks—produced at higher energies

History repeated itself in the late 1970’s at higher energies with the discovery of the Y (Upsilon) family of particles near 10 GeV in mass. The Y is a bound state of a “bottom”-quark and antiquark. The level structure of the Upsilon family is very similar to charmonium.

The heaviest quark known, the top quark, is considerably more massive and decays so rapidly that bound states analogous to the $J/\Psi$ or Y do not form.

The level-structure of charmonium and “bottom-onium” can be well described by the following empirical form of the strong interaction potential:

$$V(r) \approx -\frac{4}{3} \frac{\alpha_s}{r} + cr$$

$\alpha_s$ describes the strong interaction coupling strength and the coefficient of the term linear in $r$ ($c \sim 0.2 \text{ GeV}^2$) results in a constant force at large distances with strength of about 16 tons! This term is called the “confinement” potential—work done against this force will create new particles when the quarks are separated by the order of 1 fermi, thus permanently confining quarks inside hadrons.
Other Significant $e^+e^-$ Discoveries
(at “intermediate” energies)

- Jets: the physical manifestation of quarks and gluons
  - clustering of hadrons around a common “event-axis” that reveals the underlying quark-antiquark production direction.
  - at higher energies a third “jet-axis” forms, in direct analogy to the emission of photons by charged particles. These are heavy, strongly interacting versions of the photons called gluons
- A new “heavy-lepton”, the $\tau$ (tau)
  - discovered amongst the new charmonium and charm events through the distinctive final state: $e^+e^- \rightarrow e\,\mu + \text{missing energy}$
  - mass $\sim 1.77$ GeV
  - cross section and other properties mimic muons

Since the 1970’s, $e^+e^-$ colliders have become one of the most productive and sought-after tools in the study of particle physics. The largest circular collider is called LEP (Large Electron Positron); it was operated over the past decade at CERN near Geneva, Switzerland until being decommissioned in 2000 for a new proton-proton collider (called the LHC for Large Hadron Collider) which will use the 27 km circumference LEP tunnel. The maximum energy of LEP was about 200 GeV in the center-of-mass.

The $b$-quark and quark and gluon jets have been studied extensively using $e^+e^-$ (and other) colliders and LEP has performed extensive studies that verify the standard model. By LEP energies (around 100 GeV), the $Z^0$ boson—the carrier of the neutral weak force—dominates single-photon-exchange as the mechanism for producing quarks and leptons in $e^+e^-$ collisions. Special “$b$-factories” are operating today at SLAC and in Japan to study detailed properties of particles containing $b$ quarks. These facilities comprise two storage rings of different energy for the electron and positron beams. The asymmetric beam energies permit observation of CP symmetry-violating effects.

Circular $e^+e^-$ colliders are severely limited in beam energy by synchrotron radiation. Therefore, the world physics community is studying the possibility of building a linear $e^+e^-$ collider which may permit studies to energies as high as 1000 GeV. Because of the 2000 times greater mass of protons compared with electrons, proton-proton colliders are not yet limited in their peak energy by synchrotron radiation. The LHC center-of-mass energy will be 14 TeV, 14,000 GeV.
Quantum Chromodynamics (QCD)

- The accepted candidate theory of strong interactions
  - proposed on theoretical grounds just before the J/Ψ discovery
  - its descriptions of charmonium, jets, and other phenomena and its internal theoretical consistency led to early, broad acceptance

- Based on SU(3) color gauge symmetry
  - “color” carried by quarks (necessary to preserve the Pauli Principle) is the strong analog of electric charge
  - 8 massless, colored gluons mediate the strong force between quarks
  - because gluons have color, they also interact with each other
  - physical particles—mesons and baryons—must be “colorless”

- Strong coupling constant $\alpha_s$ varies with energy scale

A comprehensive review of QCD can be found at the Particle Data Group web site: http://www-pdg.lbl.gov/2000/qcdrpp.pdf

A theoretical program has been underway for several years to compute detailed properties of hadrons from the QCD equations. This is called “lattice-QCD”; “lattice” refers to the discrete nature of the computational methods which must be employed when attempting to calculate quantum-fields with digital computers. The computational demands presented by QCD are among the most challenging in high-performance computing. Due to the particular nature of lattice-QCD computations, special processors have been developed to optimize calculations. Recently, considerable progress has been made in relating lattice-QCD calculations to approximate “potential” models that are used to describe the spectra of charmonium and bottomonium, for example. An example of a recent calculation of the effective QCD potential between quarks as determined by lattice calculations is shown below:

It is seen to exhibit the $1/r +$ linear form expected from the analysis of experimental data.
Neutral Kaons

- Neutral $K$ mesons are produced in two distinct states of definite strangeness, $S=\pm 1$, but decay via the weak force to states of zero strangeness.
- Gell-Mann and Pais predicted weak decays would "mix" the strangeness states so physical neutral kaon particles are coherent superpositions of $S=+1$ and $S=-1$ amplitudes. Experiments show:
  - $K_S : \tau = 0.89 \times 10^{-10} \text{ s} ; K_L : \tau = 5.2 \times 10^{-8} \text{ s} ; \Delta m/m = 0.7 \times 10^{-14}$
  - $S=-1$ component can be regenerated from an initial pure $S=+1$ state as the $K_S$ particles decay away!

This is a classic example of a quantum-mechanical "2-level" system. The physical particles that one detects propagate through space with definite masses and (significantly different) lifetimes. These are called $K_S$ and $K_L$ (for short- and long-lived). Their masses are 497.7 MeV/c$^2$. The primary decay modes are: $K_S \rightarrow \pi^+\pi^- , \pi^0\pi^0$ and $K_L \rightarrow \pi^+\pi^-\pi^0$.

On the other hand, the strong force produces neutral Kaons in states of definite strangeness and the hamiltonian describing the evolution of these strangeness eigenstates must have a certain form according to basic principles of quantum mechanics. The existence of weak strangeness-changing decays give rise to (small) off-diagonal elements in the hamiltonian that cause "mixing" of the strangeness eigenstates to the physical states. To a very good approximation, the physical states consist of equal contributions (but orthogonal phase admixtures) of $S=+1$ and $S=-1$ states. When a particular strong process (such as $\pi^-p \rightarrow K^0\Lambda^0$) produces kaons in a given strangeness state, they will subsequently evolve to states of mixed strangeness because of the different decay rates and phase advances of the physical $K_{S,L}$ particles. Long after the strong production collision, the propagating kaon (most likely a $K_L$) will contain equal admixtures of $S=+1$ and $S=-1$. Should this beam encounter matter, the different absorption probabilities of the two strangeness states will give rise to regeneration of $K_S$ decays.

To the extent that $K_{S,L}$ contain exactly equal admixtures of $S$ states, they will decay to different eigenstates of the combined operators charge conjugation and parity ($CP$: $CP(\pi^+\pi^- , \pi^0\pi^0) = +1$, $CP(\pi^+\pi^-\pi^0) = -1$). It turns out that $K_L$ has a small ($\sim 0.2\%$) probability to decay to the "wrong" $CP$ state, an effect known as $CP$-violation, which is an active area of current research.
Weak Interactions of Hadrons

- Classes of weak decays involving hadrons:
  - purely leptonic — $\pi \rightarrow \mu \nu$
  - semi-leptonic — ordinary $\beta$-decay
  - non-leptonic — $\Lambda^0 \rightarrow p \pi^-$

- The myriad cases all understood by simple rules at the quark-level:
  - charged-current WI take place between “upper” ($q=+2/3$) quarks and “downer” ($q=-1/3$) quarks
  - “flavor-changing” neutral WI’s do not happen

- Systematics of non- and semi-leptonic decays of strange particles explained:
  - “$\Delta I = 1/2$” rule: follows from $s \rightarrow u$ WI
  - “$\Delta S = \Delta Q$” rule: e.g. $\beta$-decays of strange baryons
Strength of Strange WI: Cabibbo Picture

- Observation: weak decays of strange particles appear about 20× slower than corresponding non-strange particle decays.
- Cabibbo’s ansatz: weak interactions in hadrons take place through the quark interchange $u \leftrightarrow d'$
  
  where $d' = d \cos \theta_c + s \sin \theta_c$

  $d, s$ are the respective strong interaction quark states.
- $\theta_c$ is called the “Cabibbo angle”; it is a parameter that describes the relative strength of WI among strange and non-strange particles.
- With three sets of pairs of quarks, Cabibbo’s ansatz is generalized to a $3 \times 3$ unitary transformation among the $q=-1/3$ quarks which involves three angles and a phase.

Under Cabibbo’s hypothesis, small differences in the apparent strength of weak interactions in nuclear beta decay and muon decay, for example, were resolved because the nuclear reactions are proportional to $\cos^2 \theta_c$.

Reactions that involve the same $J^P$ states, but differ in strangeness can be compared to demonstrate the effect of Cabibbo “mixing” and/or to measure the Cabibbo angle. Examples include:

- Nuclear $\beta$-decay
  
  $\mu \rightarrow e + \nu \bar{\nu}$
  
  $\pi^- \rightarrow \pi^0 e^- \bar{\nu}$
  
  $K^- \rightarrow \pi^0 e^- \bar{\nu}$

  $\propto G^2 \cos^2 \theta_c$
  
  $\propto G^2$
  
  $\propto G^2 \cos^2 \theta_c$
  
  $\propto G^2 \sin^2 \theta_c$

Numerically, $\theta_c \approx 0.25$
Weak Neutral Currents

- Weak interactions known before ~1970 involved “charged currents”: changes in the electric charge of the hadronic components by one unit, for example.
- Subsequent experiments found “neutral current” processes such as: $\nu + N \rightarrow \nu + N'$, $\bar{\nu} + N \rightarrow \bar{\nu} + N'$, where $N, N'$ refer to different nuclear or hadronic states.
  - The process $\bar{\nu}_\mu + e^- \rightarrow e^- + \nu_\mu$, discovered in 1973, is a classic example of a weak neutral current.
  - The above n-nucleon processes are comparable in rate (1/4 - 1/2) to the corresponding charged current processes, $\nu + N \rightarrow \mu + N'$.
- “Flavor-changing” neutral currents, such as the strangeness changing process, $\kappa^- \rightarrow \pi^0 \nu \bar{\nu}$, are highly suppressed.
The Special Role of Charm

- The existence of the charmed quark was predicted before its experimental observation to restore symmetry or balance between the quarks in analogy to the patterns of leptons.
- $s'$ is the “orthogonal” Cabibbo combination, $s' = s \cos \theta_c - d \sin \theta_c$, introduced by Glashow, et al., to prevent strangeness-changing neutral currents through interference: “GIM mechanism”.
- Charmed particles (particles containing a charm quark) were predicted and, later, measured to decay preferentially to strange particles:
  \[
  c \rightarrow s + X, \quad c \rightarrow d + X = \cot^2 \theta_c \approx 20
  \]

Charm was the best initial explanation for the narrow resonance, $J/\psi$, discovered in 1974 which ushered in the “November Revolution” of particle physics. But this is a charm-anticharm system which does not display weak decays or the special connection to strange quarks.

At higher energies, pairs of hadrons containing a single charm quark or antiquark along with the original quarks, $u,d,s$, are formed. The decays of these particles, which start in mass near 1.9 GeV, confirmed the prediction of Glashow, Illiopoulos, and Maiani (GIM) that strange particles would be favored in decays of charmed particles. Furthermore, the correlation between electric charge and strangeness is exactly as predicted by the quark picture.

This is the first example of “quark chemistry”, where a new quark was studied in combinations with the original three quarks. Mesons and new forms of baryons have been studied and all agree with the quark model.
Gauge Invariance

- Powerful symmetry that appears to underlie all known forces.
  - obeyed in electrodynamics—provides useful guide
  - wave-functions that differ from ones describing physically meaningful states by only a phase factor that can vary at any point in space-time must, according to this principle, also describe allowed states
  - forces explicit form describing particle interactions and corresponding fields which mediate forces
  - generally requires massless gauge fields to describe forces
- Yang and Mills extended gauge symmetry ideas to isospin
- QCD extends gauge symmetry to the SU(3) algebra describing color.
Electro-weak Unification

• Weinberg (1967) (and Glashow and Salam, independently) showed that the Electromagnetic and Weak forces could be unified in a common picture based on gauge symmetry.
  – The symmetry exploits “weak isospin”, the complementarity of up/down components of quark types or lepton/neutrinos (SU(2)) and “weak-hypercharge”, $Y/2 = Q - I_3$ (U(1))
  – To maintain the gauge symmetry, three weak fields, $W^+$, $W^-$, $Z^0$, and the photon are present.
  – To describe mass, a new field, the “Higgs” field is introduced, which allows the $W, Z$ to have mass, while retaining exact gauge symmetry. This process is called “spontaneous symmetry breaking”

• Weinberg predicted the $W$ and $Z$ masses, later confirmed

The $W^+$, $W^-$ fields or particles can be viewed as the “carrier” of the “normal” weak force, the weak charged-current involving left-handed particles (fermions). This means, for example, that $W$ particles decay to pairs (particle/antiparticle) of leptons or quarks that are of the same family (SU(2) multiplet), that differ by one unit of electric charge, such as $\nu_e \nu_e$, $\mu \mu$, $ud$, $cs$, etc., and where the particles are created with left-handed (negative) helicity.

The $Z^0$ field is the carrier of the weak neutral current and couples to all particle/antiparticle pairs of quarks and leptons.

The photon is massless and couples to electric charge.

The predictions for the $W, Z$ masses are:

$$M_{W^\pm} = \left( \frac{e^2 \sqrt{2}}{8G_F \sin^2 \theta_W} \right)^{1/2} \approx \frac{37.4 \text{ GeV}}{\sin \theta_W}$$

$$M_{Z^0} = \frac{M_{W^\pm}}{\cos \theta_W} \approx 75 \text{ GeV} \sin 2\theta_W$$

The $W$ and $Z$ were subsequently discovered at the CERN proton-antiproton collider. The $Z$ is produced directly by electron-positron annihilation at the LEP collider at CERN and the SLC collider at SLAC. The measured masses are about 80 GeV, 91 GeV for the $W$, $Z$, respectively.

The electro-weak theory does not predict the mass of any particles that may be associated with the Higgs field.
Tests of Standard Model

Bottom line: no Substantiated Disagreement, but …

- High-energy accelerator experiments
  - LEP—$e^+e^-$ collisions at $Z^0$ resonance peak
  - myriad other tests
- Precision atomic physics—interference of weak neutral currents with EM processes
- Other
  - element formation in early universe
  - all knowledge in physics that has come before
- However, non-zero neutrino masses are outside the scope of the standard model
Beyond the Standard Model?

• Why?
  – complexity of SM
  – open questions
    • Higgs?
    • neutrino masses?
  – dark matter
    • Is it new stuff, not made out of quarks and/or leptons?
  – role of gravity/dark energy?
  – apparent convergence of $\alpha_s, \alpha_{\text{em}}, \alpha_{\text{weak}}$ at very high energy—unification scale?

• How?
  – GUTs — “grand unified theories” that join quarks and leptons $\Rightarrow p$ decay
  – Super Symmetry (SUSY)
    • fermion-boson mirror particles $\Rightarrow$ hosts of new “fundamental” particles to be discovered
    • naturally describes large differences in strengths of strong and electro-weak force
    • can accommodate gravity
  – String Theory ...