1. Consider a 2-D electron gas being confined in a quantum well, with potential 
\[ V(x) = 0 \]
\[ V(y) = 0 \]
\[ V(z) = 0 \text{ when } -z_o < z < z_o \]
\[ = \infty \text{ when } z > z_o \text{ or } z < -z_o \text{ where } z_o = 75 \text{ Å}. \]

The well width is therefore 150 Å. If we say that for some reasons these electrons have an effective mass of \( m^* = \frac{m_o}{16} \) where \( m_o \) is the free electron mass.

(a) Calculate the density state versus \( E \) up to \( E = 0.2 \text{ eV} \). (35 pts)
(b) If we say that the density of this 2-D electron gas is \( 3 \times 10^{12}/\text{cm}^2 \), calculate the Fermi energy. (30 pts)
(c) Now let us apply a magnetic field along the z direction. At 1 Tesla, what will be the Fermi energy and the density of states at \( E_F \) at 1 Tesla and 2 Tesla respectively? (40 pts)
(d) With the advancement of materials synthesis, one can engineer an artificial 2D materials with very little impurity scattering. Namely, the relaxation time can be very long. As a consequence, at very low temperature, one can observe de-Hass van Alphen effect even at relatively low magnetic field which means that the number of Landau levels being occupied are very high. Let us assume that we perform the experiment at a field of 2000 Gauss. What will be the periodicity (or periodicities) in magnetic field being observed? (40 pts)
2. The thermoelectric power $Q$, is related to the Onsager coefficients $L^{11}$ and $L^{12}$ as

$$ Q = \frac{L^{12}}{L^{11}}. $$

Your textbook page 254-255 describes in details how to calculate the Onsager coefficients.

As $L^{11}$ and $L^{12}$ are 2\textsuperscript{nd} rank tensors, $Q$ is also a 2\textsuperscript{nd} rank tensor. For a nearly free electron system, $Q$ is isotropic. Namely, it is a scalar times the identity matrix.

(a) Show that when the energy dependence of relaxation time is ignored, the thermoelectric power of a nearly free electron gas can be expressed as

$$ Q = -\left(\frac{\pi^2}{2e}\right) k_B T E_F. $$

(b) In the above derivation, one considers that the effective mass tensor to be isotropic. Do you think this relation is still true if the band structure has an anisotropic effective mass tensor? That is,

$$ E(k_x,k_y,k_z) = \frac{\hbar^2}{2} \left( \frac{k_x^2}{m_x} + \frac{k_y^2}{m_y} + \frac{k_z^2}{m_z} \right). $$

Prove your result. (50 pts)
3. Silicon is an indirect band gap semiconductor with a band gap of 1.1 eV. There are 6 conduction band minima located along <100> directions at 3/4 way to the zone boundary. The longitudinal and transverse effective masses are $m_l=0.98 \, m_o$ and $m_t=0.19 \, m_o$ respectively. The valence band maximum occurs at $\Gamma$ point with the effective masses for heavy and light holes as $m_{th}=0.49 \, m_o$ and $m_{lh}=0.16 \, m_o$.

(a) We are performing a cyclotron resonance experiment to measure the effective mass of conduction electrons. We first apply a magnetic field along the [001] direction then turn to the [101] direction (see figure below). Note the magnetic field always lies on the (110) plane. Calculate and plot the effective masses that you expect to measure as a function of angle off the [001] direction. (40 pts)

![Diagram of crystal axes with magnetic field](image)

Now consider an n-type sample with a doping density of $1 \times 10^{15} \, \text{cm}^{-3}$ and that the donor level is 0.05 eV below the conduction band minimum (CBM).

(b) Calculate the carrier density and the chemical potential at 300K and 20K respectively. (30 pts)

(c) Let us say that the conductivity of this sample can be written as $\sigma = \frac{ne^2\tau}{m^*}$ where $n$ is the carrier density, $\tau$ is the relaxation time, and $m^*$ is the effective mass. What should be the appropriate $m^*$? (15 pts)

(d) If we measure the conductivity of this sample at room temperature to be $0.25 \, \Omega^{-1} \, \text{cm}^{-1}$, what is the relaxation time? (15 pts)

(e) What are the mean velocity and mean free path for the carriers at 300K? (15 pts)

(f) What is the electronic contribution of the specific heat at 300K? (15 pts)
4. Consider a two dimensional crystal structure with atoms (circles) forming a honeycomb network as shown below. Assuming that the interatomic distance is 2 Å (labeled as \(a\)), and that the bond angle is 120 degrees, answer the following questions: (note that this structure is exactly the structure for single sheet graphite except for the lattice constant value)

(a) Draw a primitive unit cell directly on the graph, label the primitive translation vectors \(a_1\) and \(a_2\). Express them as \(a_1 = a (\hat{x} + \hat{y})\). How many atoms are there in a primitive cell? [20pts]

(b) Calculate the corresponding primitive vectors in reciprocal lattice \(b_1\) and \(b_2\). Express them as \(b_1 = \frac{2\pi}{a} (\hat{x} + \hat{y})\). [15 pts] Also, draw the reciprocal lattice. [10 pts]

(c) Are there points on the reciprocal lattice that have vanishing structure factors? If yes, label them. If no, why? [25 pts]

(d) Now consider that you are performing a LEED (Low Energy Electron Diffraction) experiment. Assuming a normal incident geometry and that the energy of the incident electron is 100 eV. What will be the angle for the first diffracted beam (here we labeled it as \(\theta_1\))? How about the second diffraction angle (labeled as \(\theta_2\))? Ignoring the angular dependence of the atomic form factor, what will be the ratio of the diffraction intensity, \(I(\theta_1)/I(\theta_2)\)? [30 pts]

In LEED experiment, if the incident beam wavevector is \(k_i\) and the diffracted beam wavevector is \(k_f\), the diffraction condition is satisfied when \(\Delta k_\parallel = G\) where \(G\) is a reciprocal lattice vector and \(\Delta k = k_f - k_i\).

The corresponding Ewald construction is also attached.
5. Consider a face-center-cubic Bravais lattice. If we distort this lattice along the face diagonal direction (the original [1 \bar{1} 0] direction) what kind of lattice it becomes? (45 pts)