Holt SF 17Rev 21
001 (part 1 of 2) 10.0 points

Three positive point charges are arranged in a triangular pattern in a plane, as shown below.

Find the magnitude of the net electric force on the 7 nC charge. The Coulomb constant is $8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

1. $1.33978 \times 10^{-8}$
2. $2.01278 \times 10^{-8}$
3. $5.23443 \times 10^{-9}$
4. $1.74007 \times 10^{-9}$
5. $3.88861 \times 10^{-9}$
6. $2.53508 \times 10^{-9}$
7. $2.13158 \times 10^{-9}$
8. $2.64460 \times 10^{-9}$
9. $2.83544 \times 10^{-9}$
10. $1.48865 \times 10^{-9}$

Correct answer: $2.26446 \times 10^{-9}$ N.

Explanation:

Let: $q_1 = +3 \text{ nC} = +3 \times 10^{-9} \text{ C}$, $q_2 = +7 \text{ nC} = +7 \times 10^{-9} \text{ C}$, $q_3 = +5 \text{ nC} = +5 \times 10^{-9} \text{ C}$,

$(x_1, y_1) = (0 \text{ m}, 9 \text{ m})$,
$(x_2, y_2) = (9 \text{ m}, 0 \text{ m})$,
$(x_3, y_3) = (0 \text{ m}, -9 \text{ m})$, and

$k_C = 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

The distances between the 7 nC charge and $q_1$ and $q_2$ are

$r_1^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 = (x_2 - 0)^2 + (0 - y_1)^2 = x_2^2 + y_1^2 = (9 \text{ m})^2 + (-9 \text{ m})^2 = 162 \text{ m}^2$, and

$r_3^2 = (x_3 - x_2)^2 + (y_3 - y_2)^2 = (0 - x_2)^2 + (y_3 - 0)^2 = x_2^2 + y_3^2 = (-9 \text{ m})^2 + (-9 \text{ m})^2 = 162 \text{ m}^2$.

Consider the magnitudes of the forces. The repulsive force

$F_1 = k_C \frac{q_2 q_1}{r_1^2}$

$= 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times \frac{(7 \times 10^{-9} \text{ C})(3 \times 10^{-9} \text{ C})}{162 \text{ m}^2}$

$= 1.16505 \times 10^{-9} \text{ N}$

acts downward and to the right along the line connecting $q_2$ and $q_1$, and the repulsive force

$F_3 = k_C \frac{q_2 q_3}{r_3^2}$

$= 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times \frac{(7 \times 10^{-9} \text{ C})(5 \times 10^{-9} \text{ C})}{162 \text{ m}^2}$

$= 1.94175 \times 10^{-9} \text{ N}$
acts upward and to the right along the line connecting \( q_2 \) and \( q_3 \).

Now, let’s consider the directions of the forces.

Since \(|x_2| = |y_1| = |y_3|\),

\[
\theta_{21} = -\theta_{23} = 45^\circ ,
\]

\[
F_x = F_1 \cos 45^\circ + F_3 \cos 45^\circ \\
= (1.16505 \times 10^{-9} \text{ N}) \cos 45^\circ \\
+ (1.94175 \times 10^{-9} \text{ N}) \cos 45^\circ \\
= 2.19684 \times 10^{-9} \text{ N} \\
F_y = -F_1 \sin 45^\circ + F_3 \sin 45^\circ \\
= -(1.16505 \times 10^{-9} \text{ N}) \sin 45^\circ \\
+ (1.94175 \times 10^{-9} \text{ N}) \sin 45^\circ \\
= 5.49211 \times 10^{-10} \text{ N} \\
F_{\text{net}} = \sqrt{F_x^2 + F_y^2} \\
= \left[ (2.19684 \times 10^{-9} \text{ N})^2 \\
+ (5.49211 \times 10^{-10} \text{ N})^2 \right]^{\frac{1}{2}} \\
= 2.26446 \times 10^{-9} \text{ N}.
\]

\[
\theta = \arctan \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{5.49211 \times 10^{-10} \text{ N}}{2.19684 \times 10^{-9} \text{ N}} \right) = 14.0362^\circ.
\]

The net force on the 7 nC charge is 2.26446 \times 10^{-9} \text{ N acting } 14.0362^\circ above the positive x-axis.

---

**002 (part 2 of 2) 10.0 points**

What is the direction of this force (measured from the positive x-axis as an angle between 
\(-180^\circ \) and \( 180^\circ \), with counterclockwise positive)?

1. 18.4349
2. 30.9638
3. 26.5651
4. -23.1986
5. 11.3099
6. 14.0362
7. -15.9454
8. 12.9946
9. 21.8014
10. -32.4712

Correct answer: 14.0362°.

**Explanation:**

\[
\tan \theta = \frac{F_y}{F_x}
\]

\[
q = \frac{q_A + q_B}{2}
\]

There are two identical small metal spheres with charges 64.3 \( \mu \text{C} \) and \(-41.622 \mu \text{C} \). The distance between them is 6 cm. The spheres are placed in contact then set at their original distance.

Calculate the magnitude of the force between the two spheres at the final position. The Coulomb constant is 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2.

1. 2952.86
2. 67.3856
3. 320.989
4. 9.64534
5. 8.28741
6. 57.464
7. 133.051
8. 45.6884
9. 3.22108
10. 39.5515

Correct answer: 320.989 N.

**Explanation:**

Let: \( q_A = 64.3 \mu \text{C} \)

\[
= 6.43 \times 10^{-5} \text{ C}
\]

\( q_B = -41.622 \mu \text{C} \)

\[
= -4.1622 \times 10^{-5} \text{ C}, \quad \text{and}
\]

\( d = 6 \text{ cm} = 0.06 \text{ m} \).

When the spheres are in contact, the charges will rearrange themselves until equilibrium is reached. Each sphere will then have half of the original total charge:
The force between the two spheres is

\[ F = k \frac{q_1 q_2}{d^2} \]

\[ = \left( 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \times \frac{(1.1339 \times 10^{-5} \text{ C})^2}{(0.06 \text{ m})^2} \]

\[ = 320.989 \text{ N}. \]

The force between the two spheres is

\[ F = k \frac{q_1 q_2}{d^2} \]

\[ = \left( 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \times \frac{(1.1339 \times 10^{-5} \text{ C})^2}{(0.06 \text{ m})^2} \]

\[ = 320.989 \text{ N}. \]

The force between the two spheres is

\[ F = k \frac{q_1 q_2}{d^2} \]

\[ = \left( 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \times \frac{(1.1339 \times 10^{-5} \text{ C})^2}{(0.06 \text{ m})^2} \]

\[ = 320.989 \text{ N}. \]

Let : \( L = 18.8 \text{ cm} = 0.188 \text{ m} \) and \( q = -8.63 \mu \text{C} = -8.63 \times 10^{-6} \text{ C}. \)

Call the length of the rod \( L \) and its charge \( q \). Due to symmetry

\[ E_y = \int dE_y = 0 \]

and

\[ E_x = \int dE \sin \theta = ke \int dq \sin \theta \]

where \( dq = \lambda dx = \lambda r d\theta \), so that

\[ E_x = -k_e \lambda \int_{\pi/2}^{3\pi/2} \cos \theta \, d\theta \]

\[ = -k_e \lambda \frac{\sin \theta}{r} \Bigg|_{\pi/2}^{3\pi/2} \]

\[ = 2 \frac{k_e \lambda}{r}, \]

where \( \lambda = \frac{q}{L} \) and \( r = \frac{L}{\pi} \). Therefore,

\[ E_x = \frac{2 k_e q \pi}{L^2} \]

\[ = \frac{2 \left( 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right)}{(0.188 \text{ m})^2} \times \left( -8.63 \times 10^{-6} \text{ C} \right) \pi \]

\[ = \boxed{-1.37885 \times 10^7 \text{ N/C}}. \]

Since the rod has negative charge, the field is pointing to the left (towards the charge distribution). A positive test charge at \( O \) would feel an attractive force from the semicircle, pointing to the left.
An Oblique Field
005 (part 1 of 2) 10.0 points
A charged cork ball is suspended on a light string in the presence of a uniform electric field as in the figure. The ball is in equilibrium.
\[ \vec{E} = (2.5 \times 10^5 \text{ N/C}) \hat{i} + (3.4 \times 10^5 \text{ N/C}) \hat{j}. \]
The acceleration of gravity is 9.8 m/s².

Find the charge on the ball.
1. 11.1438
2. 6.80226
3. 11.5353
4. 12.0853
5. 15.6364
6. 17.9275
7. 19.3324
8. 9.39524
9. 4.79127
10. 10.6053

Correct answer: 17.9275 nC.

Explanation:
Let:
\[ E_x = 2.5 \times 10^5 \text{ N/C}, \]
\[ E_y = 3.4 \times 10^5 \text{ N/C}, \]
\[ \theta = 34^\circ, \]
and
\[ m_b = 1.3 \text{ g} = 0.0013 \text{ kg}. \]

In the \( \hat{i} \) and \( \hat{j} \) directions, force equilibrium tells us
\[ q \ E_x - T \sin \theta = 0 \quad (1) \]
\[ q \ E_y + T \cos \theta - m \ g = 0, \quad (2) \]
\[ T \sin \theta = q \ E_x \]
\[ T \cos \theta = m \ g - q \ E_y \]
\[ \tan \theta = \frac{T \sin \theta}{T \cos \theta} = \frac{q \ E_x}{m \ g - q \ E_y} \]
\[ (m \ g - q \ E_y) \tan \theta = q \ E_x \]
\[ m \ g - q \ E_y = \frac{q \ E_x}{\tan \theta} \]
\[ q \ E_y = m \ g - \frac{q \ E_x}{\tan \theta} \]
\[ q = \frac{m \ g}{E_y \tan \theta} - \frac{q \ E_x}{E_y \tan \theta + E_x} \]
\[ q = \frac{(0.0013 \text{ kg}) (9.8 \text{ m/s}^2) \tan 34^\circ}{(3.4 \times 10^5 \text{ N/C}) \tan 34^\circ + (2.5 \times 10^5 \text{ N/C})} = 17.9275 \text{ nC}. \]

006 (part 2 of 2) 10.0 points
Find the tension in the string.
1. 0.00976323
2. 0.00685318
3. 0.00384903
4. 0.00303988
5. 0.00432643
6. 0.00962097
7. 0.0080149
8. 0.00511639
9. 0.0086204
10. 0.00491469

Correct answer: 0.0080149 N.

Explanation:
The first equation for force equilibrium gives
\[ T \sin \theta = q \ E_x \]
\[ T \cos \theta = m \ g - q \ E_y \]
\[ \tan \theta = \frac{T \sin \theta}{T \cos \theta} = \frac{q \ E_x}{m \ g - q \ E_y} \]
\[ (m \ g - q \ E_y) \tan \theta = q \ E_x \]
\[ m \ g - q \ E_y = \frac{q \ E_x}{\tan \theta} \]
\[ q \ E_y = m \ g - \frac{q \ E_x}{\tan \theta} \]
\[ q = \frac{m \ g}{E_y \tan \theta} - \frac{q \ E_x}{E_y \tan \theta + E_x} \]
\[ q = \frac{(17.9275 \text{ nC}) (2.5 \times 10^5 \text{ N/C})}{\sin 34^\circ} = 0.0080149 \text{ N}. \]
A cubic box of side \( a \), oriented as shown, contains an unknown charge. The vertically directed electric field has a uniform magnitude \( E \) at the top surface and \( 2E \) at the bottom surface.

How much charge \( Q \) is inside the box?

1. \( Q_{\text{encl}} = 0 \)
2. \( Q_{\text{encl}} = \varepsilon_0 E a^2 \) correct
3. \( Q_{\text{encl}} = 2 \varepsilon_0 E a^2 \)
4. \( Q_{\text{encl}} = 2 \varepsilon_0 E a^2 \)
5. insufficient information
6. \( Q_{\text{encl}} = \frac{E}{\varepsilon_0 a^2} \)
7. \( Q_{\text{encl}} = 3 \frac{E}{\varepsilon_0 a^2} \)
8. \( Q_{\text{encl}} = 3 \varepsilon_0 E a^2 \)
9. \( Q_{\text{encl}} = 6 \varepsilon_0 E a^2 \)
10. \( Q_{\text{encl}} = \frac{1}{2} \varepsilon_0 E a^2 \)

Explanation:
Electric flux through a surface \( S \) is, by convention, positive for electric field lines going \textit{out of} the surface \( S \) and negative for lines going in.

Here the surface is a cube and no flux goes through the vertical sides. The top receives

\[ \Phi_{\text{top}} = -E a^2 \]

(inward is negative) and the bottom

\[ \Phi_{\text{bottom}} = 2E a^2 . \]

The total electric flux is

\[ \Phi_E = -E a^2 + 2E a^2 = E a^2 . \]

Using Gauss’s Law, the charge inside the box is

\[ Q_{\text{encl}} = \varepsilon_0 \Phi_E = \varepsilon_0 E a^2 . \]

Energy gained from A to B

A proton is released from rest in a uniform electric field of magnitude \( 1.9 \times 10^5 \) V/m directed along the positive \( x \) axis. The proton undergoes a displacement of 0.2 m in the direction of the electric field as shown in the figure.

Apply the principle of energy conservation to find the amount of the kinetic energy gained after it has moved 0.2 m.

1. 8.65176e-15
2. 9.13241e-15
3. 9.61306e-15
4. 8.33132e-15
5. 1.92261e-15
6. 6.08827e-15
7. 1.53809e-14
8. 6.72914e-15
9. 4.80653e-15
10. 5.28719e-15

Correct answer: 6.08827 \times 10^{-15} \text{ J.}

Explanation:
The change in the potential energy of the proton is
\[ \Delta U = q_p \Delta V = (1.60218 \times 10^{-19} \text{ C}) (-38000 \text{ V}) = -6.08827 \times 10^{-15} \text{ J}. \]

Conservation of energy in this case is
\[ \Delta K + \Delta U = K_f - K_i + \Delta U = 0. \]

So
\[ K_f - K_i = -\Delta U = 6.08827 \times 10^{-15} \text{ J}. \]

Uniformly Charged Sphere

Find the total flux passing through the Gaussian surface (a spherical shell) with radius \( r \).

1. \( \Phi = \frac{Q}{\varepsilon_0} \)
2. \( \Phi = \frac{Q}{\varepsilon_0} \left( \frac{r^2}{R^3} \right) \)
3. \( \Phi = \frac{Q}{\varepsilon_0} \left( \frac{R}{r} \right)^3 \)
4. \( \Phi = \frac{Q}{\varepsilon_0} \left( \frac{r}{R} \right) \)
5. \( \Phi = \frac{Q}{\varepsilon_0} \left( \frac{R^2}{r^3} \right) \)
6. \( \Phi = \frac{Q}{\varepsilon_0} \left( \frac{r}{R} \right)^3 \) correct
7. \( \Phi = \frac{Q}{\varepsilon_0} \left( \frac{R}{r} \right)^2 \)
8. \( \Phi = \frac{Q}{\varepsilon_0} \frac{r}{R} \)
9. \( \Phi = \frac{Q}{\varepsilon_0} \left( \frac{r}{R} \right)^2 \)
10. \( \Phi = \frac{Q}{\varepsilon_0} \left( \frac{r}{R} \right) \)

Explanation:
Gauss' law states that
\[ \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0} = \Phi. \]

The net charge inside the sphere of radius \( r \) is
\[ q_{\text{in}} = \rho \frac{4}{3} \pi r^3 = \frac{Q}{4} \frac{4}{3} \pi r^3 = Q \left( \frac{r^3}{R^3} \right), \]
so the total flux through the surface is
\[ \Phi = \frac{Q}{\varepsilon_0} \left( \frac{r^3}{R^3} \right). \]
From Gauss’ law,
\[ E \cdot 4\pi r^2 = \Phi, \]
so the electric field is
\[ E = \frac{\Phi}{4\pi r^2} = \frac{Q r^3}{\epsilon_0 4\pi R^3} \]
\[ = \frac{Q r}{4\pi \epsilon_0 R^3} \]
\[ = \frac{k Q r}{R^3}. \]

Tipler PSE5 23 24
011 10.0 points
The distance between the K\(^+\) and Cl\(^-\) ions in KCl is 2.8 \(\times\) 10\(^{-10}\) m.
Find the energy required to separate the two ions to an infinite distance apart, assuming them to be point charges initially at rest. The elemental charge is 1.6 \(\times\) 10\(^{-19}\) C and the Coulomb constant is 8.99 \(\times\) 10\(^9\) N \(\cdot\) m\(^2\)/C\(^2\).

1. 8.91 eV
2. 6.77 eV
3. 101.3 eV
4. 709.1 eV
5. 4.37 eV
6. 5.14 eV correct
Explanation:
Let : \( q = 1.6 \times 10^{-19} \) C, 
\( k = 8.99 \times 10^9 \) N \(\cdot\) m\(^2\)/C\(^2\), and 
\( d = 2.8 \times 10^{-10} \) m.
The energy is 
\[ W = \Delta K + \Delta U = 0 - U_i \]
\[ = -\frac{k (-e) e}{d} = \frac{k q^2}{d} \]
\[ = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)}{2.8 \times 10^{-10} \text{ m}} \]
\[ \times (1.6 \times 10^{-19} \text{ C})^2 \left( \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) \]
\[ = 5.14 \text{ eV}. \]
Then the total electric potential is

\[
V_{tot} = V_1 + V_2 + V_3
= k_e \frac{q_1}{r_1} + k_e \frac{q_2}{r_2} + k_e \frac{q_3}{r_3}
= 2 k_e \left[ \frac{q_1}{\sqrt{4a^2 - b^2}} + \frac{2q_2}{b} \right]
= 2 \times 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times 1.5 \times 10^{-5} \text{ C} \times \frac{1}{0.0405 \text{ m} + 0.0605 \text{ m}}
= -16657.1 \text{ V}.
\]

**Connected Spheres 02 013 (part 1 of 2) 10.0 points**

Two charged spherical conductors are connected by a long conducting wire, and a charge of 15 µC is placed on the combination. One sphere has a radius of 4.05 cm and the other a radius of 6.05 cm.

What is the electric field near the surface of the sphere with the smaller radius?

1. 31520800.0
2. 40954200.0
3. 85309100.0
4. 36123200.0
5. 44828500.0
6. 35028000.0
7. 29420600.0
8. 32957700.0
9. 19693800.0
10. 44828500.0

Correct answer: 3.29577 \times 10^7 \text{ V/m}.

**Explanation:**

Let:

\[
\begin{align*}
q_1 &= \frac{r_1 q_2}{r_2} \quad \text{(1)} \\
Q &= q_1 + q_2 \quad \text{(2)}
\end{align*}
\]

We are also given the total charge of the combination

\[
Q = q_1 + q_2
\]

With two equations and two unknowns, we can solve for the charge \( q_1 \) on the smaller sphere.

From (1),

\[
q_1 = \frac{r_1 q_2}{r_2} = \frac{r_1 (Q - q_1)}{r_2}
\]

\[
q_1 r_2 = r_1 Q - r_1 q_1
\]

\[
q_1 (r_1 + r_2) = r_1 Q
\]

\[
q_1 = \frac{r_1 Q}{r_1 + r_2}
\]

Hence the electric field near the surface of the sphere with radius \( r_1 \) is

\[
E = k_e \frac{q_1}{r_1^2}
= \frac{Q}{(r_1 + r_2) r_1}
= \frac{(8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times 1.5 \times 10^{-5} \text{ C}}{(0.0405 \text{ m} + 0.0605 \text{ m}) (0.0405 \text{ m})}
= 3.29577 \times 10^7 \text{ V/m}.
\]
014 (part 2 of 2) 10.0 points
What is the potential of the larger sphere?
1. 1051180.0
2. 1334780.0
3. 1158920.0
4. 2662000.0
5. 1517900.0
6. 3300280.0
7. 2015180.0
8. 1948630.0
9. 1781320.0
10. 2105990.0

Correct answer: 1.33478 $\times 10^6$ V.

Explanation:
Since the two spheres are at the same potential, it suffices to consider the potential of either sphere. Since $q_1$ was obtained in the first part of the question, we may simply determine the potential of the smaller sphere.

$$V = k_\epsilon \frac{q_1}{r_1} = k_\epsilon \frac{Q}{r_1 + r_2} = \left(8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \times \frac{1.5 \times 10^{-5} \text{ C}}{0.0405 \text{ m} + 0.0605 \text{ m}} = 1.33478 \times 10^6 \text{ V}.$$
Find the potential at \( O \), where \( R_O < R_1 \).

1. \( V_O = 0 \)

2. \( V_O = \frac{k Q}{R_3} - \frac{k Q}{R_2} + \frac{k Q}{R_1} \) correct

3. \( V_O = \infty \)

4. \( V_O = \frac{k Q}{R_3} + \frac{k Q}{R_1} \)

5. \( V_O = \frac{2k Q}{R_1} \)

6. \( V_O = \frac{k Q}{R_2} + \frac{k Q}{R_1} \)

7. \( V_O = \frac{\sqrt{2} k Q}{R_1} \)

8. \( V_O = \frac{k Q}{R_1} \)

9. \( V_O = \frac{2k Q}{R_1 + R_2} \)

10. \( V_O = \frac{2 \sqrt{2} k Q}{R_1} \)

**Explanation:**

We are still inside the spherical shell. The potential due to the shell requires knowing something about where charge exists on the shell. To this end, consider a Gaussian spherical surface inside the shell. This surface can have no flux through it since no electric field can be upheld inside a conductor. By Gauss’s Law, there can therefore be no charge enclosed. But we are already enclosing \( Q \) on the sphere, so a charge \(-Q\) must reside on the inner surface of the shell:

\[ q_{inner} = -Q. \]

Since the net charge on the shell is zero, the charge on the outer surface must be \(+Q\) for the inner and outer charges to add up to zero:

\[ q_{outer} = +Q. \]

Now that we know the exact distribution of charge, we can utilize the expression for potential inside of a spherical charge distribution:

\[ V = k \frac{Q}{a}. \]

for a thin shell of radius \( a \).

Thus the potential due to the inner shell (radius \( R_2 \)) is

\[ V_2 = k \frac{q_{inner}}{R_2} = -k \frac{Q}{R_2} \]

and that due to the outer shell (radius \( R_3 \)) is

\[ V_3 = k \frac{q_{outer}}{R_3} = k \frac{Q}{R_3}, \]

so the contribution from the two surfaces of the shell is

\[ V_1 = k \frac{Q}{R_3} - k \frac{Q}{R_2}. \]

However, we are now looking at a point inside the sphere. The charge is all on the surface of the sphere, so similarly to the situation for the shell we have

\[ V = k \frac{Q}{R_1} \]

everywhere inside the sphere.

Thus the total potential at \( O \) is

\[ V_O = k \frac{Q}{R_3} - k \frac{Q}{R_2} + k \frac{Q}{R_1}. \]

This potential is the same at \( O \) as it is everywhere inside the sphere. A conductor is an equipotential object.

---

**Add a Charge to Four 3017 (part 1 of 2) 10.0 points**

Four charges are placed at the corners of a square of side \( a \), with \( q_1 = q_2 = -q \), \( q_3 = q_4 = +q \), where \( q \) is positive. Initially there is no charge at the center of the square.

\[ q_2 = -q \quad q_3 = +q \]

\[ q_1 = -q \quad q_4 = +q \]
Find the work required to bring the charge \( q \) from infinity and place it at the center of the square.

1. \( W = -\frac{2kq^2}{a^2} \)
2. \( W = -\frac{2kq^2}{a} \)
3. \( W = \frac{4kq^2}{a} \)
4. \( W = \frac{2kq^2}{a^2} \)
5. \( W = \frac{8kq^2}{a^2} \)
6. \( W = -\frac{4kq^2}{a^2} \)
7. \( W = 0 \) correct
8. \( W = -\frac{4kq^2}{a} \)
9. \( W = \frac{4kq^2}{a^2} \)
10. \( W = \frac{2kq^2}{a} \)

Explanation:

Based on the superposition principle, the potential at the center due to the charges at the corners is

\[
V = V_1 + V_2 + V_3 + V_4
= \frac{kq}{r} (-1 - 1 + 1 + 1) = 0,
\]

where \( r \) is the common distance from the center to the corners. The work required to bring the charge \( q \) from infinity to the center is then \( W = qV = 0 \).

---

**018 (part 2 of 2) 10.0 points**

What is the magnitude of the total electrostatic energy of the final 5 charge system? It may be useful to consider the symmetry property of the charge distribution which leads to cancellations among several terms.

1. \( U = 4 \frac{kq^2}{a} \)
2. \( U = \sqrt{2} \frac{kq^2}{a} \)
3. \( U = 4 \sqrt{2} \frac{kq^2}{a} \)
4. \( U = 2 \frac{kq^2}{a} \)
5. \( U = \sqrt{2} \frac{kq^2}{a^2} \)
6. \( U = 4 \sqrt{2} \frac{kq^2}{a^2} \)
7. \( U = 2 \sqrt{2} \frac{kq^2}{a^2} \)
8. \( U = 4 \frac{kq^2}{a^2} \)
9. \( U = 2 \frac{kq^2}{a^2} \)
10. \( U = 8 \frac{kq^2}{a^2} \)

Explanation:

No work is done in bringing in the fifth charge and placing it at the center, so the total electrostatic energy is contributed only by 4 corner charges:

\[
U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}
= \frac{kq^2}{a} \left( +1 - \frac{1}{\sqrt{2}} - 1 - 1 - \frac{1}{\sqrt{2}} + 1 \right)
= -\sqrt{2} \frac{kq^2}{a}.
\]

---

**Cylindrical Capacitor 03 v2**

**019 10.0 points**

A 75 m length of coaxial cable has a solid cylindrical wire inner conductor with a diameter of 1.168 mm and carries a charge of 5.21 \( \mu \text{C} \). The surrounding conductor is a cylindrical shell and has an inner diameter of 6.552 mm and a charge of \(-5.21 \mu \text{C}\).

Assume the region between the conductors is air. The Coulomb constant is \( 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \).

What is the capacitance of this cable?

1. 3.5117
2. 2.62668
3. 1.73136
Correct answer: 2.41954 nF.

**Explanation:**

Let : 

\[ k_e = 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2, \]

\[ Q = 5.21 \mu \text{C}, \]

\[ \ell = 75 \text{ m}, \]

\[ a = 1.168 \text{ mm}, \text{ and} \]

\[ b = 6.552 \text{ mm}. \]

The charge per unit length is \( \lambda \equiv \frac{Q}{\ell} \).

\[ V = -\int_a^b \vec{E} \cdot d\vec{s} \]

\[ = -2 k_e \lambda \int_a^b \frac{dr}{r} \]

\[ = -2 k_e \frac{Q}{\ell} \ln \left( \frac{b}{a} \right). \]

The capacitance of a cylindrical capacitor is given by

\[ C \equiv \frac{Q}{V} \]

\[ = \frac{\ell}{2 k_e} \frac{1}{\ln \left( \frac{b}{a} \right)} \]

\[ = \frac{75 \text{ m}}{2 \times 2 \times 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} \]

\[ \times \frac{1}{\ln \left( \frac{6.552 \text{ mm}}{1.168 \text{ mm}} \right)} \cdot \left( 1 \times 10^9 \text{ nF} \right) \]

\[ = 2.41954 \text{ nF} \]

---

Capacitor Circuit 04 shortest 020 (part 1 of 2) 10.0 points

Consider the group of capacitors shown in the figure.

Find the equivalent capacitance between points \( a \) and \( d \).

Correct answer: 1.35359 \( \mu \text{F} \).

**Explanation:**

Let : 

\( C_1 = 8.41 \mu \text{F}, \)

\( C_2 = 4.43 \mu \text{F}, \)

\( C_3 = 8.66 \mu \text{F}, \)

\( C_4 = 1.84 \mu \text{F}, \)

and \( \mathcal{E}_B = 11.8 \text{ V} \).

For capacitors in series,

\[ \frac{1}{C_{\text{series}}} = \sum \frac{1}{C_i} \]

\[ V_{\text{series}} = \sum V_i, \]

and the individual charges are the same.
For parallel capacitors,
\[ C_{\text{parallel}} = \sum C_i \]
\[ Q_{\text{parallel}} = \sum Q_i , \]
and the individual voltages are the same.

In the given circuit \( C_2 \) and \( C_3 \) are connected parallel with equivalent capacitance
\[ C_{23} = C_2 + C_3 \]
\[ = 4.43 \, \mu F + 8.66 \, \mu F \]
\[ = 13.09 \, \mu F . \]

\( C_1, \ C_{23}, \) and \( C_4 \) are connected in series with equivalent capacitance
\[ C_{ba} = \left( \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4} \right)^{-1} \]
\[ = \left( \frac{1}{8.41 \, \mu F} + \frac{1}{13.09 \, \mu F} + \frac{1}{1.84 \, \mu F} \right)^{-1} \]
\[ = 1.35359 \, \mu F . \]

The remaining voltage is
\[ V_{\text{remain}} = V_{\text{total}} - V_1 - V_4 \]
\[ = 11.8 \, V - 1.8992 \, V - 8.6806 \, V \]
\[ = 1.22019 \, V . \]

This remaining voltage is the same across the parallel capacitors, so
\[ Q_2 = C_2 V_{\text{remain}} \]
\[ = (4.43 \, \mu F) (1.22019 \, V) \]
\[ = 5.40545 \, \mu C . \]

---

**021 (part 2 of 2) 10.0 points**

Determine the charge on the 4.43 \( \mu F \) capacitor at the top center part of the circuit.

1. 34.3037
2. 3.95722
3. 16.2366
4. 16.4891
5. 10.9564
6. 11.6865
7. 5.40545
8. 14.9974
9. 5.05784
10. 10.1086

Correct answer: 5.40545 \( \mu C \).

**Explanation:**

The voltage drops across \( C_1 \) and \( C_4 \) are then
\[ V_1 = \frac{Q_1}{C_1} = \frac{15.9723 \, \mu C}{8.41 \, \mu F} = 1.8992 \, V \]
and
\[ V_4 = \frac{Q_4}{C_4} = \frac{15.9723 \, \mu C}{1.84 \, \mu F} = 8.6806 \, V . \]
7. \( E_0 = \frac{2Q}{\epsilon_0 A} \)

**Explanation:**

**Solution:**

From Gauss’ Law, we know that \( \Phi = \frac{Q_{\text{encl}}}{\epsilon_0} \).

For a flat surface, like a plate, the flux is the normal component of the electric field times the area. Also, for a parallel plate capacitor, the field is only in the gap (neglecting edge effects), so

\[
\Phi = E_0 A
\]

\[
E_0 = \frac{\Phi}{A} = \frac{Q}{\epsilon_0 A}.
\]

---

**023 (part 2 of 3) 10.0 points**

With the dielectrics inserted, the fields in \( \kappa_1 \) and \( \kappa_2 \) are respectively

1. \( E_1 = E_0 \kappa_2 \) and \( E_2 = E_0 \kappa_1 \)
2. \( E_1 = \frac{E_0}{\kappa_2} \) and \( E_2 = \frac{E_0}{\kappa_1} \)
3. \( E_1 = \frac{E_0}{\kappa_1} \) and \( E_2 = \frac{E_0}{\kappa_2} \) correct
4. \( E_1 = \frac{E_0 \kappa_2}{\kappa_1 + \kappa_2} \) and \( E_2 = \frac{E_0 \kappa_1}{\kappa_1 + \kappa_2} \)
5. \( E_1 = E_0 \kappa_1 \) and \( E_2 = E_0 \kappa_2 \)
6. \( E_1 = \frac{E_0 \kappa_1 \kappa_2}{\kappa_1 + \kappa_2} \) and \( E_2 = \frac{E_0 \kappa_1 \kappa_2}{\kappa_1 + \kappa_2} \)
7. \( E_1 = \frac{E_0 \kappa_1}{\kappa_1 + \kappa_2} \) and \( E_2 = \frac{E_0 \kappa_2}{\kappa_1 + \kappa_2} \)

**Explanation:**

In any region with dielectric constant \( \kappa \), the permittivity \( \epsilon_0 \) becomes \( \kappa \epsilon_0 \), so the fields in \( \kappa_1 \) and \( \kappa_2 \) are

\[
E_1 = \frac{Q}{\kappa_1 \epsilon_0 A} = \frac{E_0}{\kappa_1}
\]

\[
E_2 = \frac{Q}{\kappa_2 \epsilon_0 A} = \frac{E_0}{\kappa_2}.
\]

---

**024 (part 3 of 3) 10.0 points**

Let \( \kappa_1 = 3 \), \( \kappa_2 = 4.41 \), \( A = 2.74 \text{ m}^2 \), and \( d = 4.27 \text{ mm} \).

What is the resultant capacitance?

1. \( 2.06894 \times 10^{-7} \)
2. \( 1.44507 \times 10^{-8} \)
3. \( 8.05613 \times 10^{-9} \)
4. \( 1.06711 \times 10^{-8} \)
5. \( 3.21764 \times 10^{-8} \)
6. \( 4.75003 \times 10^{-8} \)
7. \( 9.74561 \times 10^{-9} \)
8. \( 1.68571 \times 10^{-8} \)
9. \( 2.02877 \times 10^{-8} \)
10. \( 2.86385 \times 10^{-8} \)

Correct answer: \( 2.02877 \times 10^{-8} \text{ F} \).

**Explanation:**

The potential difference between the two plates is the sum of the potential differences across the two dielectrics. The potential difference across a region with a constant electric field is the product of the field and the distance, so

\[
V = V_1 + V_2
\]

\[
= \frac{E_0 d}{\kappa_1} + \frac{E_0 d}{\kappa_2}
\]

\[
= \frac{Q d}{\epsilon_0 A} \frac{1}{2} \left( \frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right).
\]

The capacitance is given by

\[
C = \frac{Q}{V}
\]

\[
= \frac{2 \epsilon_0 A}{d} \left( \frac{1}{\frac{1}{\kappa_1} + \frac{1}{\kappa_2}} \right)
\]

\[
= 2 \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} \frac{\epsilon_0 A}{d}
\]

\[
= 2 \left[ \frac{(3)(4.41)}{3 + 4.41} \right] \times \left( 8.854 \times 10^{-12} \text{ F/m} \right) (2.74 \text{ m}^2)
\]

\[
= \frac{0.00427 \text{ m}}{2.02877 \times 10^{-8} \text{ F}}.
\]

---

**Charge in a Wire 025 10.0 points**

The current in a wire decreases with time according to the relationship

\[
I = (2.32 \text{ mA}) \times e^{-at}
\]
where \( a = 0.13328 \text{ s}^{-1} \).

Determine the total charge that passes through the wire from \( t = 0 \) to the time the current has diminished to zero.

1. 0.0114046
2. 0.017407
3. 0.0105792
4. 0.0216086
5. 0.0281363
6. 0.0112545
7. 0.0197329
8. 0.0277611
9. 0.0129802
10. 0.0296369

Correct answer: 0.017407 C.

**Explanation:**

\[
I = \frac{dq}{dt}
\]

\[q = \int_{t=0}^{t} I \, dt = \int_{t=0}^{\infty} (0.00232 \text{ A})e^{-0.13328 \text{ s}^{-1} t} \, dt = (0.00232 \text{ A}) \left( \frac{e^{-0.13328 \text{ s}^{-1} \infty} - e^{-0.13328 \text{ s}^{-1} 0}}{0.13328 \text{ s}^{-1}} \right) = 0.017407 \text{ C}.
\]

Current in Tungsten Wire

A 0.59 V potential difference is maintained across a 1.3 m length of tungsten wire that has a cross-sectional area of 0.72 mm\(^2\) and the resistivity of the tungsten is 5.6 \times 10^{-8} \Omega \cdot \text{m}.

What is the current in the wire?

1. 5.83517
2. 1.72556
3. 11.873
4. 8.71763
5. 13.7143
6. 16.7545
7. 8.0
8. 1.62175
9. 3.69643
10. 14.5357

Correct answer: 5.83517 A.

**Explanation:**

Let:

\[V = 0.59 \text{ V},\]
\[\ell = 1.3 \text{ m},\]
\[A = 0.72 \text{ mm}^2 = 7.2 \times 10^{-7} \text{ m}^2,\]
and
\[\rho = 5.6 \times 10^{-8} \Omega \cdot \text{m}.
\]

The resistance is

\[R = \frac{V}{I} = \frac{\rho \ell}{A},\]

so the current is

\[
I = \frac{V A}{\rho \ell} = \frac{(0.59 \text{ V}) (7.2 \times 10^{-7} \text{ m}^2)}{(5.6 \times 10^{-8} \Omega \cdot \text{m})(1.3 \text{ m})} = 5.83517 \text{ A}.
\]