Sample exam.
For your reference, I give you this sample exam which was the final exam for my 1998 Modern Physics Course. The format of our final exam will be very similar to this sample exam. However, you should realize that some problems given in this sample exam are not within the scope of what we have covered.

Phys-355 unique # 56085            Name _______________________

Final Exam            SSN _______________________

I. Close book section (This section has 200 points)
Single-choice only. In this section, you will not get partial credit.

You have 90 minutes to complete this exam. You are allowed to use a self-prepared summary sheet on a 8” x 11” paper (both sides). You can use calculator. If you finish this section earlier, you can turn it in and get the openbook section.

Useful Quantities

c = 3.0 x 10^8 m/s
hc = 1240 eV-nm.
electron mass 9.1094 x 10^{-31} kg or 0.511 MeV/c^2
proton mass 1.6726 x 10^{-27} kg or 938.27 MeV/c^2
neutron mass 1.6747 x 10^{-27} kg or 939.57 MeV/c^2
Planck constant \( h = 6.626 \times 10^{-34} \) J s, \( \hbar = 1.055 \times 10^{-34} \) J s,
Hydrogen ground states \( E_0 = -13.6 \) eV
Rydberg constant \( R_\infty = 1.09737 \times 10^7 \) m^{-1}
Hydrogen Rydberg \( R_H = 1.09678 \times 10^7 \) m^{-1}

1. We wish to measure simultaneously the wavelength and position of a photon. Assume that the wavelength measurement gives \( \lambda = 6000 \) Å with an accuracy of \( \lambda/\lambda = 1 \times 10^{-6} \). What is the minimum uncertainty in the position of the photon? You will have to use propagation of uncertainty in \( \lambda \) to calculate the uncertainty in the momentum. (15 points)

(a) 0.6 m, (b) 1.67 m, (c) 6 m,  (d) 1.67 \times 10^{-1} m, (e) 6 \times 10^{-2} m, (f) 1.67 \times 10^{-3} m, (g) 6 \times 10^{-3} m, (h) 1.67 \times 10^{-5} m, (i) 6 \times 10^{-5} m, (j) 16.7 m.
2. If you know that the de Broglie wavelength of a 100 eV electron is $\lambda_1$, what should the kinetic energy of a proton to have the same de Broglie wavelength? (10 points)

(a) 183.8 KeV, (b) 4.29 KeV, (c) 2.33 eV, (d) 1.838 KeV, (e) 429 eV, (g) 100 KeV, (h) 2.33 KeV

3. Cu is a noble metal with an electron density of $8.47 \times 10^{22}/\text{cm}^3$, and Ag is another noble metal with an electron density of $5.86 \times 10^{22}/\text{cm}^3$. If we know that the Fermi energy of Cu is 7.00 eV, what do you expect the Fermi energy of Ag to be? (12 points)

(a) 4.84 eV (b) 10.1 eV, (c) 4.03 eV, (d) 5.49 eV, (e) 3.35 eV, (f) 11.4 eV, (g) 14 eV, (h) 5 eV.

4. Two space ships A and B are moving toward each other. Their velocities as measured from the earth frame are 0.4 c toward +x direction for A and 0.5 c toward -x direction for B (see figure below). What is the velocity of A as measured by B. (10 points)

(a) 0.9 c (b) 0.64 c, (c) 0.71 c, (d) 0.75 c, (e) 0.31 c, (f) 0.55 c, (g) 0.47 c, (h) 0.97 c

5. If the astronaut in space ship B send a radio signal with frequency $\nu_0$ to A, what will be the frequency measured by A? (10 points)

(a) $1.20 \nu_0$, (b) $0.82 \nu_0$, (c) $1.25 \nu_0$, (d) $2.65 \nu_0$, (e) $2.35 \nu_0$, (f) $1.75 \nu_0$, (g) $0.38 \nu_0$, (h) $0.43 \nu_0$.

6. After passing each other, what will be the frequency measured by A? (10 points)

(a) $1.20 \nu_0$, (b) $0.82 \nu_0$, (c) $1.25 \nu_0$, (d) $2.65 \nu_0$, (e) $2.35 \nu_0$, (f) $1.75 \nu_0$, (g) $0.38 \nu_0$, (h) $0.43 \nu_0$. 
7. When the muons are at rest, one measures a half-life of 1.52 \times 10^{-6} \text{s} for muon decay. A muon detector is placed on top of a mountain 2000 m high and the muon count is 1000 counts per second. One also measures that the muons are traveling at a speed of 0.99 \text{c}. If a similar muon detector is placed at the sea level, what do you expect the count rate will be? (12 points)

(a) 45 \text{c/s}, (b) 540 \text{c/s}, (c) 760 \text{c/s}, (d) 135 \text{c/s}, (e) 235 \text{c/s}, (f) 270 \text{c/s}, (g) 470 \text{c/s}, (h) 380 \text{c/s}

8. What is the kinetic energy of an electron having a momentum of 10 \text{GeV/c}? (12 points)

(a) 10 \text{GeV}, (b) 100 \text{GeV}, (c) 10.0005 \text{GeV}, (d) 9.9995 \text{GeV}, (e) 100.0005 \text{GeV}, (f) 99.9995 \text{GeV}, (g) 100.5 \text{GeV}, (h) 99.5 \text{GeV}.

9. Shown below is the radial probability distribution $P_{n,l}(r)$ of a particular electronic state of hydrogen atom. \(a_o = 0.51 \text{Å}\) is the Bohr radius. What is this state? Choose from the following list (12 points)

(a) 1s, (b) 2p, (c) 3d, (d) 4f, (e) 2s, (f) 3p, (g) 4d, (h) 3s

10. How about the state with the following radial probability distribution? (12 points)

(a) 1s, (b) 2p, (c) 3d, (d) 4f, (e) 2s, (f) 3p, (g) 4d, (h) 3s
11. The Balmer series of Hydrogen spectra can be expressed as

\[ \lambda = 364.56 \frac{k^2}{k^2 - 4} \text{nm} \]. By using expression, one finds an emission line at 656.21 nm corresponding to \( k = 3 \). Now you perform the experiment on a gas mixture of Hydrogen and Deuterium. In addition to this emission line of 656.21 nm, you find another line right next it to due to the Deuterium isotope. If we express this as 656.21 nm + \( \lambda \). What is the value of \( \lambda \)? (15 points)

(a) -0.53 nm, (b) -0.36 nm, (c) -0.18 nm, (d) -0.09 nm, (e) 0.09 nm, (f) 0.18 nm, (g) 0.36 nm, (h) 0.53 nm

12. A monoenergetic beam of electrons is incident on double slits separated by 50 nm. An interference pattern is form on a screen 51.5 cm from the slits. If the distance between successive minima of the diffraction pattern is 1 mm, what is the energy of the incident electron? (15 points)

(a) 160 eV, (b) 140 eV, (c) 120 eV, (d) 100 eV, (e) 80 eV, (f) 60 eV, (g) 40 eV, (h) 20 eV

13. Using the equipartition theorem, what will be the molar heat capacity of a linear array of harmonic oscillators in one dimension? (hint: to look at the energy expression for a simple harmonic oscillator) (15 points)

(a) 3.5 R, (b) 3 R, (c) 2.5 R, (d) 2 R, (e) 1.5 R, (f) 1 R, (g) 0.5 R, (h) 4 R

14. A metal has an electron density of \( n \). If we know the average scattering time is \( \tau \), we can calculate the electrical conductivity. Which of the following expression should you use? (12 points)

(a) \( \sigma = \frac{ne\tau}{m} \)  (b) \( \sigma = ne\tau \)  (c) \( \sigma = ne^2\tau \)  (d) \( \sigma = \frac{ne^2\tau}{m} \)  (e) \( \sigma = \frac{n^2 e\tau}{m} \)  (f) \( \sigma = \frac{n^2 em}{\tau} \)

(g) \( \sigma = \frac{ne^2 m}{\tau} \)  (h) \( \sigma = \frac{ne m}{\tau} \)

15. Consider a semiconductor of band gap 1.0 eV. Assuming that this semiconductor is extremely pure, what would be the ratio between the conductivity measured at 300K and that
measured at 600K, $\frac{\sigma_{300K}}{\sigma_{600K}}$? (choose the closest one if you can not find the exact match to your calculation) (15 points)

(a) $1.5 \times 10^{-9}$, (b) $2.0 \times 10^{-7}$, (c) $2.3 \times 10^{-5}$, (d) $6.7 \times 10^{-3}$, (e) $1.6 \times 10^{2}$, (f) $4.4 \times 10^{4}$, (g) $5 \times 10^{6}$, (h) $6.6 \times 10^{8}$.

16. Photoelectric effect
A stopping potential of 3.2 V is needed to stop all the photoelectron emission from a solid surface radiated by a UV light of wavelength 200 nm. What is the work function of this sample? (10 points)

(a) 6.2 eV, (b) 9.4 eV, (c) 3.0 eV (d) 12.4 eV (e) 8.6 eV (f) 1.8 eV, (g) 5.0 eV, (h) 10.8 eV
II. Open book section (This section has 200 points)
In this section, you will get partial credit. **Try to keep everything in the exam sheet without using extra paper.**

You can use your own note, different kind of textbooks and math table. However you are not allow to use anybook that collect problems with solutions. Xerox copy of homework solution is also not allowed

1. Consider a 4f state, what is the expectation value of $\langle L_z \rangle$? What is the value of $\langle L_z^2 \rangle$, what will be the expectation value of $\langle L_z^2 + L_y^2 \rangle$? (40 points)
2. He\textsubscript{4} atoms are bosons, they therefore obey Bose-Einstein statistics. The expression of Bose-Einstein distribution is

\[ F_{BE}(E) = \frac{1}{\exp\left(\frac{(E - \mu)}{kT}\right) - 1}. \]

When one cools liquid helium below a transition temperature of 2.17 K it becomes superfluid. This temperature is often called the Einstein condensation temperature, \( T_E \), below which most of the particles occupy the ground state, forming the so-called Bose-Einstein Condensates. If one considers that \( 10^{22} \) helium atoms are contained in a volume of 1 cm\(^3\), one can estimate the energy difference between the ground state and first excited state to be

\[ \Delta E = 3\times \frac{\hbar^2}{2M} \left( \frac{\pi}{L} \right)^2 = 2.48 \times 10^{-30} \text{erg}. \]

In the temperature unit, \( \frac{\Delta E}{k} = 1.8 \times 10^{-14} K. \)

The astonishing fact is that at a temperature below \( T_E \), say 1K, almost 100% of particles occupy the ground state, despite that the thermal energy is 14 orders of magnitude larger than \( E \).

(a) With this fact that almost all of \( 10^{22} \) particles are occupying the ground state orbital, estimate the value of chemical potential \( \mu \) at 1K. (The thermal energy, \( kT \), at 1K is \( 1.38 \times 10^{-16} \) erg.) (35 points)

Note that we choose the ground state energy as the reference energy zero. Hint: the chemical potential should be a negative value but extremely small. Also try Taylor expansion of the exponential factor.

(b) Now estimate the # of the particle in the first excited state (this should be a number much smaller than \( 10^{22} \). (25 points)
3. For an infinite potential well of width $L$, you have learned that the solution of the ground state and first excited state are

$$\phi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

for ground state, and

$$\phi_2(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$$

for the first excited states

the energy levels for the two are

$$E_1 = \frac{\pi^2 h^2}{2mL^2}, \quad E_2 = \frac{4\pi^2 h^2}{2mL^2}$$

The time dependent of the wavefunction for these two states become

$$\psi_1(x,t) = \phi_1(x)\exp(-i\omega_1 t), \quad \psi_2(x,t) = \phi_2(x)\exp(-i\omega_2 t)$$

where $\omega_1 = \frac{E_1}{\hbar}$ and $\omega_2 = \frac{E_2}{\hbar}$.

(a) Calculate the probability distribution functions for these two states respectively. Are these states stationary or not? (A stationary state means that the probability distribution function does not have time dependence) (20 points)

(b) Now consider a wavefunction which is a superposition of the ground state and the first excited state. Its wavefunction at $t = 0$ can be written as

$$\Psi(x,0) = \frac{1}{\sqrt{2}} [\psi_1(x,0) + \psi_2(x,0)]$$

What will be the probability distribution function? Is this a stationary state? (20 points)
4. Consider a 1-D potential well shown below

\[
V(x) = 0 \text{ when } 0 < x < a/2 \\
= V_o \text{ when } x > a/2 \\
= \infty \text{ when } x < 0
\]

We are seeking for bound state solution. Namely, \( E < V_o \)

(a) write down the appropriate solutions in the two regions, \( 0 < x < a/2 \), \( x > a/2 \) and the appropriate boundary conditions at \( x = 0 \) and \( x = a/2 \). (25 points)

(b) Draw qualitatively the wavefunction of the ground state directly onto the plot. (10 points)

(c) For a fixed barrier height of \( V_o \), there is a minimum potential well width \( a/2 \) for the existence of at least one bound state solution. What should this width be? (25 points)