Chapter 10: Systems of Particles

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ENGINEERING PHYSICS I

PHY 303K

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Linear Momentum

Concept of momentum

- The linear momentum of a particle or an object that can be modeled as a particle of mass \( m \) moving with a velocity \( \vec{v} \) is defined to be the product of the mass and velocity:

\[
\vec{p} = m \vec{v}
\]

- Vector quantity
- SI unit of momentum is \( \text{kg m/s} \)

- In components:

\[
\begin{align*}
\vec{p}_x &= m \vec{v}_x \\
\vec{p}_y &= m \vec{v}_y \\
\vec{p}_z &= m \vec{v}_z
\end{align*}
\]

Linear Momentum

Conservation of momentum

- Two particles interact with each other:

\[
\vec{F}_{12} = -\vec{F}_{21}
\]

\[
\begin{align*}
F_{x1} + F_{x2} &= 0 \\
m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt} &= 0 \\
\frac{d(m_1 \vec{v}_1)}{dt} + \frac{d(m_2 \vec{v}_2)}{dt} &= 0 \\
\frac{d(m_1 \vec{v}_1 + m_2 \vec{v}_2)}{dt} &= 0
\end{align*}
\]

\[
\vec{p}_1 + \vec{p}_2 = \text{const}
\]

- Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant

The Center of Mass

Special point

- The center of mass of the system moves as if all the mass of the system were concentrated at that point

- Coordinate of the center of mass:

\[
x_{cm} = \left( \sum \frac{m_i x_i}{M} \right) / M
\]

- Thinking of extended object as a system of a large number of particles:

\[
x_{cm} = \lim_{N \to \infty} \left( \frac{\sum x_i m_i}{M} \right) / M
\]

- The center of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry.
The Center of Mass

How to determine the center of mass?

- An experimental technique for determining the center of mass of a wrench:
  - The wrench is hung freely first from point A and then from point C.
  - The intersection of the two lines AB and CD locates the center of mass.

Motion of a System of Particles

Motion of the center of mass

- The total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass:
  \[
  \vec{p}_{CM} = \frac{d}{dt} \left( \sum m_i \vec{v}_i \right) = \sum m_i \vec{v}_i \Rightarrow M \vec{v}_{CM} = \sum \vec{p}_i
  \]

- The net external force on a system of particles equals the total mass of the system multiplied by the acceleration of the center of mass:
  \[
  \vec{F}_{ext} = \frac{d}{dt} \left( \sum m_i \vec{a}_i \right) = \sum m_i \vec{a}_i \Rightarrow M \vec{a}_{CM} = \sum \vec{F}_i
  \]

- The center of mass of a system of particles of combined mass \( M \) moves like an equivalent particle of mass \( M \) would move under the influence of the net external force on the system.

Motion of a System of Particles

Motion of the center of mass

- A projectile fired into the air suddenly explodes into several fragments (see figure). What can be said about the motion of the center of mass of the system made up of all the fragments after the explosion?

SUMMARY

Systems of Particles

- Linear momentum: \( \vec{p} = m \vec{v} \)

- The law of conservation of linear momentum: \( \vec{p}_1 + \vec{p}_2 = \text{constant} \)

- Position vector of the center of mass:
  \[
  \vec{r}_{CM} = \frac{1}{M} \int \vec{r} \, dm
  \]

- Newton’s 2nd law applied to a system of particles:
  \[
  \sum \vec{F}_i = M \vec{a}_{CM}
  \]