Chapter 17: Sound

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**Sound Waves**

The most common example of longitudinal waves

- Travel through any material medium
- Speed depends on the properties of the medium
- Elements of the medium vibrate to produce changes in density and pressure along the direction of motion of the wave
- Sinusoidally vibrating source → sinusoidal pressure variations
- Mathematical description of sinusoidal sound waves = sinusoidal string waves

Three categories of sound waves:

- **Audible** → lie within the range of sensitivity of the human ear
- **Infrasonic** → have frequencies below the audible range
- **Ultrasonic** → have frequencies above the audible range

**Speed of Sound Waves**

Motion of 1D pulse through a long tube with compressible gas

- First, the gas in the tube is undisturbed → uniform density
- A piston at the left end is suddenly pushed to the right → the gas just in front of it is compressed (pressure and density in this region is higher)
- Piston comes to rest → compressed region continues to move to the right with speed \( v \)
- The speed of sound \( v \) in a medium depends on the compressibility and density of the medium

**Periodic Sound Waves**

Sinusoidal sound waves

- Oscillating piston produces a 1D periodic sound wave in the tube
- High-pressure (compression) and low-pressure regions (rarefactions) propagate along the tube with a speed of sound
- Piston oscillates sinusoidally → small elements of the medium move with simple harmonic motion

\[
\Delta P = \Delta P_\text{rms} \sin(\omega x - \phi)
\]

\[
P(x,t) = P_\text{rms} \cos(\omega t - kx)
\]

The pressure wave is 90° out of phase with the displacement wave
Intensity of Periodic Sound Waves

Transport of energy with waves

- Consider an element of air in front of a piston oscillating with a frequency $f$:

\[ v(x,t) = \frac{\partial}{\partial t} \left( \frac{1}{2} \rho v^2 + \frac{1}{2} \rho c^2 \right) \]

\[ \Delta K = \frac{1}{2} \rho v^2 \Delta x = \frac{1}{2} \rho c^2 \Delta x \]

\[ K = \frac{1}{2} \rho c^2 \Delta x \]

- Total mechanical energy for one wavelength gives the rate of energy transfer by the wave:

\[ E = K \times \lambda \]

\[ I = \frac{K}{\Delta x} \]

\[ I = \frac{P}{A} = \frac{P_0}{4\pi r^2} \]

The intensity decreases in proportion to the square of the distance from the source.

Intensity of Periodic Sound Waves

Sound Level in Decibels

- Sound level is defined by

\[ \beta = 10 \log \left( \frac{I}{I_0} \right) \]

- The constant $I_0 = 1 \times 10^{-12}$ W/m² is the reference intensity (threshold of hearing)

- $\beta$ is measured in decibels (dB)

Prolonged exposure to high sound levels may seriously damage the ear.

Intensity of Periodic Sound Waves

Loudness and Frequency

- Physical measurement vs psychological "measurement" of the strength of sound

Threshold intensity for hearing varies with frequency

Superposition and Interference

Interference of sound waves

- The lower path length $r_1$ is fixed but the upper path length can be varied by sliding U-shaped tube

- The difference in the path lengths

\[ \Delta r = r_2 - r_1 \]

- Constructive interference $\rightarrow$ $\Delta r = n\lambda, n = 0, 1, 2, \ldots$

- Destructive interference $\rightarrow$ $\Delta r = (n + 1/2)\lambda, n = 0, 1, 2, \ldots$

Standing Waves in Air Columns

Open at both ends

- The open end of an air column is approximately a displacement antinode and a pressure node (no pressure variation at an opened end)

\[ f_n = \frac{n}{4L}, n = 1, 2, 3, \ldots \]

In a pipe open at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency.
Standing Waves in Air Columns

Closed at one end, open at another

- The closed end of an air column is a displacement node (no longitudinal motion because of the wall) and a pressure antinode

\[ nL = \frac{\lambda}{2}, n = 1, 3, 5, ... \]

In a pipe closed at one end, the natural frequencies of oscillation form a harmonic series that includes only odd integral multiples of the fundamental frequency

Beats

Interference in time

Beating is the periodic variation in amplitude at a given point due to the superposition of two waves having slightly different frequencies

\[ y = y_1 + y_2 = A_1 \sin(n_1 \omega t + \phi_1) + A_2 \sin(n_2 \omega t + \phi_2) \]

The frequency \( f' \) heard by the observer is increased (decreased) when the observer moves toward (away from) the source

The Doppler Effect

Stationary source + moving observer

- An observer moves with a speed \( v_O \) toward a stationary point source (\( v_S = 0 \))
- The source emits a spherical wave having speed \( c = \lambda f \)
- The speed of the wave relative to the observer is \( v_f = v_O + v_S = \frac{c}{\lambda} \)

The frequency \( f' \) heard by the observer is increased (decreased) when the observer moves toward (away from) the source

The Doppler Effect

Moving source + stationary observer

- A source moves with a speed \( v_S \) toward a stationary observer (\( v_O = 0 \))
- The wavelength of the source is \( \lambda \)
- The wavelength measured by the observer \( \lambda' \) is shorter by amount \( v_O T = \frac{\lambda - \lambda'}{T} \)

\[ f' = \frac{f}{\lambda'} = \frac{f}{\lambda - v_O (f - f')} \left( \frac{f + f'}{f - f'} \right) \]

The frequency \( f' \) heard by the observer is increased (decreased) when the source moves toward (away from) the observer

The Doppler Effect

Moving source + moving observer

- If source moves with a speed \( v_S \) and an observer moves with a speed \( v_O = 0 \)

\[ f' = \frac{f}{\lambda} = \frac{f}{\lambda - v_O (f - f')} \]

- A positive value for speeds is used for motion of the observer or the source toward the other, and a negative sign for motion of one away from the other

The word toward is associated with an increase in observed frequency. The words away from are associated with a decrease in observed frequency
The Doppler Effect

Shock Waves

• What if the speed $v_S$ of a source exceeds the wave speed $v$?

• The envelope of sound waves generated by the source is a cone whose apex half-angle is ("Mach" angle)

$$\sin \theta = \frac{v_f}{v} = \frac{v}{v_x}$$

SUMMARY

Sound

• The speed of sound in a liquid or gas:

$$v = \frac{B}{\sqrt{\rho}}$$

• Temperature dependence of the speed of sound in the air:

$$v = \left[331 + \frac{\theta}{273} \right] \frac{5}{9}$$

• Variation in the position of an element of the medium and variation in pressure from equilibrium for sinusoidal waves:

$$x(t) = x_{rms} \cos(\omega t - \phi)$$

$$\Delta P = \Delta P_{rms} \sin(\omega t - \phi)$$

• The intensity of a sound wave is power per unit area:

$$I = \frac{1}{2} \rho v C^2$$

• The sound level of a sound wave in dB:

$$P = 10 \log_{10} \frac{I}{I_0}$$

• The change in frequency heard by an observer whenever there is relative motion between the source of waves and the observer is called the Doppler effect:

$$f' = \frac{v + v_x}{v - v_y} f$$