1. In the linear sigma model (*cf.* previous homework), calculate the tree-level decay rate $\Gamma(\sigma \to \pi\pi)$.

2. Consider two Dirac fields $\Psi_1(x)$ and $\Psi_2(x)$ coupled to the a real scalar field $\Phi(x)$:

$$\begin{aligned} \mathcal{L} \; &= \overline{\Psi}_1(i\,\partial\!\!\!/ - m_1)\Psi_1 \;+\; \overline{\Psi}_2(i\,\partial\!\!\!/ - m_2)\Psi_2 \;+\; \tfrac{1}{2}(\partial_\mu\Phi)^2 \;-\; \tfrac{1}{2}M^2\Phi^2 \\ &- \; g_1\Phi\overline{\Psi}_1\Psi_1 \;-\; g_2\Phi\overline{\Psi}_2\Psi_2\,. \end{aligned} \tag{1}$$

   At the tree level, calculate the matrix element, the partial cross-section and the total cross-section for scattering of one type of a fermion off the other type, $f_1 + f_2 \to f_1 + f_2$. Take the initial fermions to be unpolarized and sum over the final fermion's polarizations.

   Hint: Prove and use

$$\begin{aligned} \tfrac{1}{2}\sum_{s,s'} \left|\bar{u}(p',s')u(p,s)\right|^2 \;&=\; \tfrac{1}{2}\,\mathrm{tr}\left((m+p\!\!\!/')(m+p\!\!\!/)\right) \\ &=\; 2(m^2 + EE' - \mathbf{p}\mathbf{p}'). \end{aligned} \tag{2}$$

3. Consider Rutherford scattering of an electron off a static point-like electric charge. See problem 4.4 of the *Peskin & Schroeder* textbook for details and follow steps (a), (b) and (c) therein.

   (d) In addition, calculate the scattering cross-section for a relativistic unpolarized electron.

   Hint: Prove and use

$$\begin{aligned} \tfrac{1}{2}\sum_{s,s'} \left|\bar{u}(p',s')\gamma^0 u(p,s)\right|^2 \;&=\; \tfrac{1}{2}\,\mathrm{tr}\left((m+p\!\!\!/')\gamma^0(m+p\!\!\!/)\gamma^0\right) \\ &=\; 2(m^2 + EE' + \mathbf{p}\mathbf{p}'). \end{aligned} \tag{3}$$