Consider muon decay, $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. Since neutrinos are hard to detect experimentally, the readily measurable quantities for this process are the total muon decay rate $\Gamma_\mu = 1/\tau_\mu$ and the energy distribution of electrons produced by decaying muons; the latter is known to have a maximum at the highest kinematically allowed value of E_e.

According to the Fermi theory of weak interactions, the matrix element for muon decay is

$$\left\langle e^-, \bar{\nu}_e, \nu_\mu \right| \mathcal{M} \left| \mu^- \right\rangle = \frac{G_F}{\sqrt{2}} \left[\bar{u}(\nu_\mu)(1-\gamma^5)\gamma^\alpha u(\mu^-) \right] \times \left[\bar{u}(e^-)(1-\gamma^5)\gamma_\alpha v(\bar{\nu}_e) \right]. \tag{1}$$

The modern Standard Model of particle interactions produces essentially the same answer at the tree level of the perturbation theory.

1. Show that

$$\tfrac{1}{2} \sum_{\substack{\text{all} \\ \text{spins}}} \left| \left\langle e^-, \bar{\nu}_e, \nu_\mu \right| \mathcal{M} \left| \mu^- \right\rangle \right|^2 = 64 G_F^2 (p_\mu \cdot p_{\bar{\nu}})\,(p_e \cdot p_\nu). \tag{2}$$

2. Now calculate the $d\Gamma/dE_e$ and the Γ_{tot} in the muon's rest frame. This is a straightforward but non-trivial exercise because of three particles in the final state, with momenta subject to constraints

$$\mathbf{p}_e + \mathbf{p}_\nu + \mathbf{p}_{\bar{\nu}} = \mathbf{0}, \qquad E_e + E_\nu + E_{\bar{\nu}} = M_\mu \approx 105.66\,\text{MeV}. \tag{3}$$

   Fortunately, the neutrinos are massless while the electron may be approximated as massless because in most decay events the electron's energy $E_e = O(M_\mu) \gg m_e$. You are advised to take this approximation $m_e \approx 0$ as it simplifies the calculation quite a bit.