1. In class, I used functional formalism (path integrals) to prove the Ward–Takahashi identities of QED. Alternatively, one can follow Drs. Ward and Takahashi and derive the same identities from the QED Feynman rules. This derivation is explained in detail in section §7.4 of the *Peskin & Schroeder* textbook.

   (a) Read the textbook, then give a similar diagramatic proof of the Ward–Takahashi identities for the *scalar* QED.

   (b) Prove that the renormalized scalar QED has $Z_1^{1\gamma} = Z_1^{2\gamma} = Z_2$.

2. In any *even* spacetime dimension $d = 2n$, a *massless* Dirac spinor field $\Psi(x)$ has an axial symmetry

$$\Psi(x) \;\to\; e^{+i\beta\Gamma}\,\Psi(x), \qquad \overline{\Psi}(x) \;\to\; \overline{\Psi}(x)e^{+i\beta\Gamma}, \tag{1}$$

where $\Gamma = i^{n-1}\gamma^0\gamma^1\cdots\gamma^{(d-1)}$ generalizes the γ^5 to $d = 2n$. *Classically*, this symmetry leads to the conserved axial current

$$J_A^\mu \;=\; \overline{\Psi}\Gamma\gamma^\mu\Psi, \qquad \partial_\mu J_A^\mu(x) \;=\; 0. \tag{2}$$

In the quantum field theory however, the axial symmetry becomes anomalous when Ψ couples to a vector gauge field $A^\mu(x)$.

Show that in the $d = 2n$ dimensional QED,

$$\partial_\mu J_A^\mu(x) \;=\; -\frac{2}{n!}\left(\frac{e}{4\pi}\right)^n \epsilon^{\alpha_1\beta_1\alpha_2\beta_2\cdots\alpha_n\beta_n} F_{\alpha_1\beta_2}F_{\alpha_2\beta_2}\cdots F_{\alpha_n\beta_n}\,. \tag{3}$$