1. First, a reading assignment: §16.5 of the *Peskin & Schroeder* textbook. Make sure you understand the calculation of the (infinite parts of) the counterterms δ_1, δ_2 and δ_3 *in detail*.

2. Next, another reading assignment: §20.1 of the *Peskin & Schroeder* textbook, which introduces the *Higgs mechanism*. The abelian version of the Higgs mechanism — the photon becomes massive when a charged scalar field acquires a non-zero vacuum expectation value — was discussed in class in the Fall semester, but please refresh your memory. I will discuss the non-abelian version during the last week of this semester, but I'ld like you to read about it first so it wouldn't come as a shock. Pay attention to the Georgi–Glashow model and make sure you understand why — and how — two of the three gauge bosons become massive while the third remains massless.

3. Finally, an exercise: Consider the pseudoscalar Yukawa theory. In a previous homework (set#18, problem 2), you (should) have calculated to one-loop order the infinite parts of all the counterterms in this theory. Let us now use the results of all this work to calculate the one-loop β–functions

$$\frac{\partial g(M)}{\partial \log M} = \beta_g(g,\lambda) \quad \text{and} \quad \frac{\partial \lambda(M)}{\partial \log M} = \beta_\lambda(g,\lambda). \tag{1}$$

   (a) Argue that for off-shell renormalization conditions at some renormalization scale $M \gg m_f, m_s$, each of the logarithmically divergent counterterms δ_Z^ϕ, δ_Z^ψ, δ_g and δ_λ is

$$\delta = C \log \frac{(\Lambda^{\text{UV}}_{\text{cutoff}})^2}{M^2} + \text{finite} \tag{2}$$

   or

$$\delta = C\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{M^2}\right) + \text{finite} \tag{3}$$

   in dimensional regularization, where the coefficient C of the UV divergence has the same value for all renormalization schemes (on-shell or off-shell) and the 'finite' terms are not only finite but also M independent.

(b) Show that the bare and the renormalized couplings of the Yukawa theory are related to each other as

$$\lambda + \delta_\lambda = \lambda_0 Z_\phi^2 \,, \qquad g + \delta_g = g_0 Z_\psi \sqrt{Z_\phi} \,, \tag{4}$$

and use these relations to derive

$$\begin{aligned} \beta_\lambda &= 2(\lambda + \delta_\lambda) \frac{\partial \log Z_\phi}{\partial \log M} - \frac{\partial \delta_\lambda}{\partial \log M} \,, \\ \beta_g &= (g + \delta_g) \left(\frac{\partial \log Z_\psi}{\partial \log M} + \frac{1}{2} \frac{\partial \log Z_\phi}{\partial \log M} \right) - \frac{\partial \delta_g}{\partial \log M} \,. \end{aligned} \tag{5}$$

At the one-loop level, these formulæ simplify to

$$\begin{aligned} \beta_\lambda &= 2\lambda \frac{\partial \delta_Z^\phi}{\partial \log M} - \frac{\partial \delta_\lambda}{\partial \log M} \,, \\ \beta_g &= g \left(\frac{\partial \delta_Z^\psi}{\partial \log M} + \frac{1}{2} \frac{\partial \delta_Z^\phi}{\partial \log M} \right) - \frac{\partial \delta_g}{\partial \log M} \,. \end{aligned} \tag{6}$$

(c) Use the results of homework set #18 to evaluate the right hand sides of eqs. (6) and write down the β–functions (1) as functions of the renormalized couplings g and λ.