

SUSY Summary

This summary covers all interesting supersymmetries which exist in spacetime dimensions $11 \geq d \geq 3$. In my notations, m, n, \dots are Lorentz indices (in d dimensions), a, b, \dots are Dirac or Majorana indices, α, β, \dots and $\dot{\alpha}, \dot{\beta}, \dots$ are left-handed and right-handed Weyl indices; \mathcal{N} is the number of supersymmetries (in units of the minimal SUSY in the same dimension d); $i, j = 1, \dots, \mathcal{N}$ are “flavor” indices of the R symmetry.

Maximal SUGRA

Maximal supersymmetry in any dimension d has 32 hermitian supercharges. The only consistent field theories with maximal SUSY are pure supergravities; their particle spectrum comprises one SUGRA multiplet and nothing else.

★ $d = 11, \mathcal{N} = 1$, the ‘mother of maximal supergravities’.

The supercharges $Q_a \in \mathbf{32}$ (real). No R symmetry.

Bosonic fields: the metric g_{mn} and a 3-form $C_{\ell mn}$.

Fermionic fields: one Majorana gravitino ψ_m^a .

★ $d = 10, \mathcal{N} = (2, 0)$, type IIB SUGRA.

The supercharges $Q_\alpha^i \in \mathbf{16}$ (real), $i = 1, 2$. R symmetry $R = SO(2)$.

Bosonic fields: the metric g_{mn} , two real scalars (the dilaton ϕ and the axion a , often combined into one complex scalar field $\tau = a + ie^\phi$), two 2-form fields B_{mn}^i and one self-dual 4-form field $D_{k\ell mn}$.

Fermionic fields: two Majorana–Weyl gravitini $\psi_m^{\alpha, i}$ (left) and two Majorana–Weyl spinors $\chi^{\dot{\alpha}, i}$ (right).

* All other maximal SUGRAs are dimensional reductions of the $d = 11$ theory.

● $d = 10, \mathcal{N} = (1, 1)$, type IIA SUGRA.

The supercharges $Q_\alpha \in \mathbf{16}$ (real) and $Q_{\dot{\alpha}} \in \mathbf{16}'$. No R symmetry.

Bosonic fields: the metric g_{mn} , a real scalar ϕ (the dilaton) a 1-form (vector) field A_m , a 2-form field B_{mn} and a 3-form field $C_{\ell mn}$.

Fermionic fields: one Majorana (left+right MW) gravitino ψ_m^a and one Majorana (left+right MW) spinor χ^a .

- $d = 9, \mathcal{N} = 2$ SUGRA.

The supercharges $Q_a^i \in \mathbf{16}$ (real), $i = 1, 2$. R symmetry $R = SO(2)$.

Bosonic fields: the metric g_{mn} , three real scalars, three vectors, two 2-form fields and one 3-form field.

Fermionic fields: two Majorana gravitini $\psi_m^{a,i}$ and four Majorana spinors.

- $d = 8, \mathcal{N} = 2$ SUGRA.

The supercharges $Q_\alpha^i \in \mathbf{8}$ and $\bar{Q}_{\dot{\alpha},i} \in \bar{\mathbf{8}}$ (complex plus h.c.), $i = 1, 2$.

R symmetry $R = U(2)$.

Bosonic fields: the metric g_{mn} , seven real scalars, six vectors, three 2-form fields and one 3-form field.

Fermionic fields: two Weyl (left) gravitini $\psi_m^{\alpha,i}$ and six Weyl (right) spinors, plus hermitian conjugates of opposite chirality.

- $d = 7, \mathcal{N} = 2$ SUGRA.

The supercharges $Q_a^i, \bar{Q}_{a,i} \in \mathbf{8}$ (pseudoreal), $i = 1, 2$. R symmetry $R = Sp(2)$.

Bosonic fields: the metric g_{mn} , 14 real scalars, ten vectors and five 2-form fields.

Fermionic fields: two Dirac gravitini $\psi_m^{a,i}$ and eight Dirac spinors, plus hermitian conjugates.

- $d = 6, \mathcal{N} = (2, 2)$ (AKA $\mathcal{N} = 4$) SUGRA.

The supercharges $Q_\alpha^i \in \mathbf{4}$ and $Q_{a\dot{\alpha}}^i \in \mathbf{4}'$ (both pseudoreal), $i = 1, 2$.

R symmetry $R = Sp(2) \times Sp(2)$.

Bosonic fields: the metric g_{mn} , 25 real scalars, 16 vectors and six 2-form fields.

Fermionic fields: two Dirac (left+right Weyl) gravitini $\psi_m^{a,i}$ and ten Dirac (left+right Weyl) spinors, plus h.c.

- $d = 5, \mathcal{N} = 4$ SUGRA.

The supercharges $Q_a^i, \bar{Q}_{a,i} \in \mathbf{4}$ (pseudoreal), $i = 1, \dots, 4$. R symmetry $R = Sp(4)$.

Bosonic fields: the metric g_{mn} , 42 real scalars and 27 vectors.

Fermionic fields: four Dirac gravitini $\psi_m^{a,i}$ and 24 Dirac spinors, plus h.c.

- $d = 4, \mathcal{N} = 8$ SUGRA.

The supercharges $Q_\alpha^i \in \mathbf{2}$ and $\bar{Q}_{\dot{\alpha},i} \in \bar{\mathbf{2}}$ (complex plus h.c.), $i = 1, \dots, 8$.

R symmetry $R = U(8)$.

Bosonic fields: the metric g_{mn} , 70 real scalars and 28 vectors.

Fermionic fields: eight Majorana gravitini $\psi_m^{a,i}$ and 56 Majorana spinors.

- $d = 3, \mathcal{N} = 16$ SUGRA.

The supercharges $Q_a^i \in \mathbf{2}$ (real), $i = 1, \dots, 16$. R symmetry $R = SO(16)$.

Bosonic fields: 128 real scalars (the metric field is non-dynamical in $d = 3$).

Fermionic fields: 128 Majorana spinors.

Maximal Rigid SUSY

Maximally supersymmetric field theories without gravity are SSYM theories with 16 supercharges. The spectra of such theories are comprised of several vector supermultiplets in the adjoint representation of some gauge group G . In $d = 10$, the SSYM theories are chiral (and hence suffer from the axial anomaly)[★]. The $d < 10$ SSYM theories are dimensional reductions from $d = 10$ and the chirality is lost.

- ★ $d = 10, \mathcal{N} = 1$ SSYM (the ‘mother of all SSYM’).

Each vector multiplet contains one vector field A_m and one Majorana–Weyl spinor field ψ^α .

- $d = 9, \mathcal{N} = 1$ SSYM.

Each vector multiplet contains one vector field, one real scalar and one Majorana spinor.

- $d = 8, \mathcal{N} = 1$ SSYM.

Each vector multiplet contains one vector field, two real scalars and one Majorana spinor. (Or equivalently, one vector, one complex scalar and one Weyl spinor).

$R = SO(2) = U(1)$.

★ The anomalies cancel for $G = SO(32)$ or $G = E_8 \times E_8$ SSYM theories coupled to SUGRA.

- $d = 7, \mathcal{N} = 1$ SSYM.

Each vector multiplet contains one vector field, three real scalars and one Dirac spinor.

$$R = Sp(1) = SU(2) = SO(3).$$

- $d = 6, \mathcal{N} = (1, 1)$ (AKA $\mathcal{N} = 2$) SSYM.

Each vector multiplet contains one vector field, four real scalars and one Dirac spinor

(*i.e.*, Weyl left+right spinors). $R = Sp(1) \times Sp(1) = SO(4)$.

- $d = 5, \mathcal{N} = 2$ SSYM.

Each vector multiplet contains one vector field, five real scalars and two Dirac spinors.

$$R = Sp(2) = SO(5).$$

- $d = 4, \mathcal{N} = 4$ SSYM.

Each vector multiplet contains one vector field, six real scalars and four Majorana spinors. $R = SU(4) = SO(6)$.

Despite quantum effects, the $\mathcal{N} = 4$ SSYM theories in $d = 4$ are superconformally invariant.

- $d = 3, \mathcal{N} = 8$ SSYM.

Each vector multiplet contains one vector field, seven real scalars and eight Majorana spinors, or equivalently 8 real scalars and 8 Majorana spinors.[†] $R = SO(8)$.

Tensor Theories with 16 Supercharges

In addition to all these SSYM theories, in $d = 6$ there also tensor theories with chiral $\mathcal{N} = (2, 0)$ SUSY. The basic multiplet of this SUSY is tensor rather than vector; it comprises five real scalar fields ϕ_I , two left Weyl spinors ψ_i^α and a self-dual 2-form field B_{mn} (the tensor). However, its dimensional reduction to $d < 6$ is equivalent to a vector multiplet.

Our present knowledge of interacting $\mathcal{N} = (2, 0)$ tensor theories is rather limited; mostly, it follows from dimensional “oxidation” of SSYM theories from $d = 5$ to $d = 6$.

[†] In $d = 3$ a vector field is dual to a scalar.

Theories with 8 Supercharges

Rigid theories with 8 supercharges allow for two kind of supermultiplets, namely vector and hyper. The vector multiplets form an adjoint repr. of some gauge group G ; the hypermultiplets may belong to any complete representations.

Again, all such supersymmetric gauge theories are dimensional reduction from the highest dimension which allows for 8 supercharges, namely $d = 6$. Consequently, in $d = 6$ we have chirality (and hence axial anomaly to contend with) but the $d < 6$ theories are non-chiral.

★ $d = 6, \mathcal{N} = (1, 0)$ (AKA $\mathcal{N} = 1$).

A vector multiplet comprises a vector field A_m and a right Weyl spinor field $\psi^{\dot{\alpha}}$. A hyper multiplet comprises a left Weyl spinor χ^{α} and four real scalars. $R = SU(2)$.

● $d = 5, \mathcal{N} = 1$.

A vector multiplet comprises a vector, a real scalar and a Dirac spinor. A hyper multiplet comprises a Dirac spinor and four real scalars. $R = SU(2)$.

● $d = 4, \mathcal{N} = 2$.

A vector multiplet comprises a vector, two real scalars and two Majorana spinors. A hyper multiplet comprises two Majorana spinor and four real scalars. $R = SU(2) \times U(1)$.

● $d = 3, \mathcal{N} = 4$.

A vector multiplet comprises a vector, three real scalar and four Majorana spinors, or equivalently four real scalars and four Majorana spinors. A hyper multiplet also comprises four real scalars and four Majorana spinors. However, the R symmetry $R = SU(2) \times SU(2)$ distinguishes between the vector and the hyper multiplets.

★★ In addition, the $d = 6, \mathcal{N} = (1, 0)$ SUSY has tensor supermultiplets, each comprising one real scalar ϕ , one left Majorana spinor χ^{α} and one self-dual 2-form field B_{mn} . The tensor multiplets are always neutral with respect to the gauge symmetry.

The dimensional reduction of a tensor multiplet to $d < 6$ is equivalent to a vector multiplet.

Theories with 4 Supercharges

Supersymmetric theories with only 4 supercharges (existing in $d \leq 4$) allow for Yukawa and scalar forces independent of the gauge couplings.

★ $d = 4, N = 1$ SUSY.

A chiral multiplet comprises a left Weyl spinor χ^α and a complex scalar field ϕ . Its hermitian conjugate — comprised of the right Weyl spinor $\bar{\chi}^{\dot{\alpha}}$ and the conjugate scalar ϕ^* — is a distinct anti-chiral multiplet.

A vector multiplet comprises a vector field A_m and a Weyl spinor ψ^α with its hermitian conjugate $\bar{\psi}^{\dot{\alpha}}$. $R = U(1)$

- $d = 3, N = 2$ SUSY is the dimensional reduction from $(d = 4, \mathcal{N} = 1)$. In $d = 3$, the vector, the chiral and the anti-chiral multiplets become equivalent *on shell*; each comprises two real scalar fields ϕ_i and one Majorana spinor χ^a . $R = SO(2) = U(1)$.

Theories with 2 Supercharges

★ $d = 3, N = 1$ SUSY.

On shell, there is only one kind of a supermultiplet; it contains a real scalar field and a Majorana spinor. There is no R symmetry.