
First, a reading assignment: *Carefully* study section §4.5 of the *Peskin & Schroeder* textbook (same as last homework), and this time pay attention to the phase-space factors of decay and scattering processes when the final state has more than two particles.

And as a practical application of this reading assignment, consider the muon decay, $\mu^- \to e^-\bar{\nu}_e\nu_\mu$. Since neutrinos are hard to detect experimentally, the readily measurable quantities for this process are the total muon decay rate $\Gamma_\mu = 1/\tau_\mu$ and the energy distribution of electrons produced by decaying muons; the latter is known to have a maximum at the highest kinematically allowed value of E_e.

According to the Fermi theory of weak interactions, the matrix element for muon decay is

$$\left\langle e^-,\bar{\nu}_e,\nu_\mu\right| \mathcal{M} \left|\mu^-\right\rangle = \frac{G_F}{\sqrt{2}} \left[\bar{u}(\nu_\mu)(1-\gamma^5)\gamma^\alpha u(\mu^-)\right] \times \left[\bar{u}(e^-)(1-\gamma^5)\gamma_\alpha v(\bar{\nu}_e)\right]. \tag{1}$$

The modern Standard Model of particle interactions produces essentially the same answer at the tree level of the perturbation theory.

Later in this class we shall learn how to sum $|\mathcal{M}|^2$ over the fermionic spins. In particular, in a future homework you will show that

$$\frac{1}{2}\sum_{\substack{\text{all}\\ \text{spins}}} \left|\left\langle e^-,\bar{\nu}_e,\nu_\mu\right| \mathcal{M} \left|\mu^-\right\rangle\right|^2 = 64G_F^2(p_\mu\cdot p_{\bar{\nu}})\,(p_e\cdot p_\nu). \tag{2}$$

But in this homework, you should simply take this formula and use it to calculate the $d\Gamma/dE_e$ and the Γ_{tot} in the muon's rest frame. This is a straightforward but non-trivial exercise because of three particles in the final state, with momenta subject to constraints

$$\mathbf{p}_e + \mathbf{p}_\nu + \mathbf{p}_{\bar{\nu}} = \mathbf{0}, \qquad E_e + E_\nu + E_{\bar{\nu}} = M_\mu \approx 105.66\,\text{MeV}. \tag{3}$$

Fortunately, the neutrinos are massless while the electron may be approximated as massless because in most decay events the electron's energy $E_e = O(M_\mu) \gg m_e$. You are advised to take this approximation $m_e \approx 0$ as it simplifies the calculation quite a bit.