1. In a Yukawa theory with $M_s > 2m_f$, the scalar particles are unstable against decay into fermion + antifermion pairs.

   Write down the tree-level matrix element $\mathcal{M}(S \to f + \bar{f})$, sum $|\mathcal{M}|^2$ over final particles' spins, and calculate the total decay rate $S \to f + \bar{f}$.

2. Consider a Yukawa theory of two Dirac fields $\Psi_1(x)$ and $\Psi_2(x)$ coupled to the same real scalar field $\Phi(x)$:

$$\begin{aligned} \mathcal{L} \; &= \overline{\Psi}_1(i\partial\!\!\!/ - m_1)\Psi_1 \; + \; \overline{\Psi}_2(i\partial\!\!\!/ - m_2)\Psi_2 \; + \; \tfrac{1}{2}(\partial_\mu\Phi)^2 \; - \; \tfrac{1}{2}M_s^2\Phi^2 \\ &- \; g_1\Phi\overline{\Psi}_1\Psi_1 \; - \; g_2\Phi\overline{\Psi}_2\Psi_2\,. \end{aligned} \tag{1}$$

   At the tree level, calculate the matrix element, the partial cross-section and the total cross-section for elastic scattering of one type of a fermion off the other type, $f_1 + f_2 \to f_1 + f_2$. Take the initial fermions to be unpolarized and sum over the final fermion's polarizations.

3. And now consider the pseudoscalar version of the Yukawa theory (1): Two Dirac fields $\Psi_1(x)$ and $\Psi_2(x)$ coupled to the same real pseudoscalar field $\phi(x)$:

$$\begin{aligned} \mathcal{L} \; &= \overline{\Psi}_1(i\partial\!\!\!/ - m_1)\Psi_1 \; + \; \overline{\Psi}_2(i\partial\!\!\!/ - m_2)\Psi_2 \; + \; \tfrac{1}{2}(\partial_\mu\phi)^2 \; - \; \tfrac{1}{2}M_p^2\phi^2 \\ &- \; ig_1\phi\overline{\Psi}_1\gamma^5\Psi_1 \; - \; ig_2\phi\overline{\Psi}_2\gamma^5\Psi_2\,. \end{aligned} \tag{2}$$

   (a) Write down the Feynman rules of this theory.

   (b) Calculate the tree-level matrix element $\mathcal{M}(f_1 + f_2 \to f_1 + f_2)$ for elastic scattering of two distinct fermions off each other.

   (c) Calculate the partial cross section and the total cross-section for this scattering. Assume un-polarized beams of initial fermions and spin-blind detectors for the final particles.

4. Finally, let us finish the muon decay problem of homework #10: Given the matrix element

$$\left\langle e^-, \bar{\nu}_e, \nu_\mu \right| \mathcal{M} \left| \mu^- \right\rangle = \frac{G_F}{\sqrt{2}} \left[\bar{u}(\nu_\mu)(1-\gamma^5)\gamma^\alpha u(\mu^-)\right] \times \left[\bar{u}(e^-)(1-\gamma^5)\gamma_\alpha v(\bar{\nu}_e)\right], \quad (3)$$

show that

$$\frac{1}{2}\sum_{\substack{\text{all}\\ \text{spins}}} \left|\left\langle e^-, \bar{\nu}_e, \nu_\mu \right| \mathcal{M} \left| \mu^- \right\rangle\right|^2 = 64 G_F^2 (p_\mu \cdot p_{\bar{\nu}})\,(p_e \cdot p_\nu). \quad (4)$$