

1. In a Yukawa theory with $M_s > 2m_f$, the scalar particles are unstable against decay into fermion + antifermion pairs.

Write down the tree-level matrix element $\mathcal{M}(S \rightarrow f + \bar{f})$, sum $|\mathcal{M}|^2$ over final particles' spins, and calculate the total decay rate $S \rightarrow f + \bar{f}$.

2. Consider a Yukawa theory of two Dirac fields $\Psi_1(x)$ and $\Psi_2(x)$ coupled to the same real scalar field $\Phi(x)$:

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}_1(i \not{\partial} - m_1)\Psi_1 + \bar{\Psi}_2(i \not{\partial} - m_2)\Psi_2 + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M_s^2 \Phi^2 \\ & - g_1 \Phi \bar{\Psi}_1 \Psi_1 - g_2 \Phi \bar{\Psi}_2 \Psi_2. \end{aligned} \quad (1)$$

At the tree level, calculate the matrix element, the partial cross-section and the total cross-section for elastic scattering of one type of a fermion off the other type, $f_1 + f_2 \rightarrow f_1 + f_2$. Take the initial fermions to be unpolarized and sum over the final fermion's polarizations.

3. And now consider the pseudoscalar version of the Yukawa theory (1): Two Dirac fields $\Psi_1(x)$ and $\Psi_2(x)$ coupled to the same real pseudoscalar field $\phi(x)$:

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}_1(i \not{\partial} - m_1)\Psi_1 + \bar{\Psi}_2(i \not{\partial} - m_2)\Psi_2 + \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}M_p^2 \phi^2 \\ & - ig_1 \phi \bar{\Psi}_1 \gamma^5 \Psi_1 - ig_2 \phi \bar{\Psi}_2 \gamma^5 \Psi_2. \end{aligned} \quad (2)$$

- (a) Write down the Feynman rules of this theory.
- (b) Calculate the tree-level matrix element $\mathcal{M}(f_1 + f_2 \rightarrow f_1 + f_2)$ for elastic scattering of two distinct fermions off each other.
- (c) Calculate the partial cross section and the total cross-section for this scattering. Assume un-polarized beams of initial fermions and spin-blind detectors for the final particles.

4. Finally, let us finish the muon decay problem of homework #10: Given the matrix element

$$\langle e^-, \bar{\nu}_e, \nu_\mu | \mathcal{M} | \mu^- \rangle = \frac{G_F}{\sqrt{2}} [\bar{u}(\nu_\mu)(1 - \gamma^5)\gamma^\alpha u(\mu^-)] \times [\bar{u}(e^-)(1 - \gamma^5)\gamma_\alpha v(\bar{\nu}_e)], \quad (3)$$

show that

$$\frac{1}{2} \sum_{\text{all spins}} |\langle e^-, \bar{\nu}_e, \nu_\mu | \mathcal{M} | \mu^- \rangle|^2 = 64G_F^2(p_\mu \cdot p_{\bar{\nu}})(p_e \cdot p_\nu). \quad (4)$$