

1. Consider so-called Bhabha scattering $e^-e^+ \rightarrow e^-e^+$. In QED, there are two tree-level Feynman diagrams contributing to this process; their contributions must be added *before* squaring the amplitude and adding/averaging over spins,

$$|\mathcal{M}_1 + \mathcal{M}_2|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + 2\text{Re}(\mathcal{M}_1^* \mathcal{M}_2) \neq |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2. \quad (1)$$

Calculate the partial cross-section $d\sigma/d\Omega$ for the Bhabha scattering. For simplicity, assume $E \gg m_e$ and neglect the electron's mass throughout your calculation. You may find it convenient to use Mandelstam's Lorentz-invariant kinematic variables s , t and u , see eq. (5.69) of the Peskin&Schroeder textbook for details. Notice $s + t + u = 4m_e^2 \approx 0$ in the $m_e \approx 0$ approximation.

The answer to this problem is simple:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}} = \frac{\alpha^2}{2s} \left[\left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2 + \left(\frac{u}{s} + \frac{u}{t}\right)^2 \right] \quad (2)$$

but the intermediate steps are quite complicated, so beware.

2. The Z^0 (or simply Z) particle of the Standard Model is a massive ($M_Z \approx 91 \text{ GeV}$) neutral vector boson. The Lagrangian for the Z_μ field is

$$\mathcal{L}[Z] = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu + g'Z_\mu J_0^\mu + \dots \quad (3)$$

where $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, J_0^μ is the *neutral weak current* governing the Z^0 coupling to the leptons and quarks (*cf.* the charged weak currents J_\pm^μ governing the W^\pm couplings to the fermions) and the ' \dots ' stand for additional interaction terms involving the W^\pm , Higgs

and EM fields. The specific form of the neutral current is

$$J_0^\mu = \sum_{\text{fermions}} \bar{\Psi}(g_V + g_A \gamma^5) \gamma^\mu \Psi \quad (4)$$

where

$$\begin{aligned} g_V &= (\sin^2 \theta_W - \tfrac{1}{4}), & g_A &= +\tfrac{1}{4} & \text{for charged leptons } e^-, \mu^- \text{ and } \tau^-, \\ g_V &= (+\tfrac{1}{4}), & g_A &= -\tfrac{1}{4} & \text{for neutrinos } \nu_e, \nu_\mu \text{ and } \nu_\tau, \\ g_V &= (\tfrac{1}{3} \sin^2 \theta_W - \tfrac{1}{4}), & g_A &= +\tfrac{1}{4} & \text{for charge } -\tfrac{1}{3}e \text{ quarks } d, s \text{ and } b, \\ g_V &= (-\tfrac{2}{3} \sin^2 \theta_W + \tfrac{1}{4}), & g_A &= -\tfrac{1}{4} & \text{for charge } +\tfrac{2}{3}e \text{ quarks } u, c \text{ and } t, \\ g' &= \frac{e}{\sin \theta_W \cos \theta_W}, & \sin^2 \theta_W &\approx 0.23. \end{aligned} \quad (5)$$

The θ_W here is called the *weak mixing angle*; it is an experimentally determined parameter of the Standard Model.

Write down the Feynman rules for the Z field and its interactions with the leptons and the quarks and calculate the total decay rate of the Z particle and the branching ratios $B(Z \rightarrow e^+ e^-)$, $B(Z \rightarrow \mu^+ \mu^-)$ and $B(Z \rightarrow q \bar{q} \rightarrow \text{hadrons})$. Use tree-level approximation.

For your information, the top quark t is heavier than the Z particle while the other five quark flavors and all the leptons are so much lighter than the Z that you may neglect their masses altogether. You should also keep in mind that all quarks (but not the leptons) come in three colors. The color degree of freedom is fundamental to QCD but for the purposes of this calculation, it's just an index taking three values.

3. Finally consider the $e^+ e^- \rightarrow \mu^+ \mu^-$ pair production in the Standard Model. There are two tree-level Feynman diagrams contributing to this process: one involving a virtual photon and the other a virtual Z particle. In QED, there is only the first diagram — which was discussed in class in much detail — but at high energies available at the LEP $e^+ e^-$ collider both diagrams are equally important.

- (a) Write down the combined tree-level amplitude $\mathcal{M}(e^+ e^- \rightarrow \mu^+ \mu^-)$.

- (b) Assume both the electrons and the muons to be ultra-relativistic ($E_{\text{c.m.}} = O(M_Z) \gg m_\mu, m_e$) and evaluate the amplitude (a) for all possible particle helicities. (Use the center-of-mass frame.)

Hint:

$$(g_V + g_A \gamma^5) \gamma^\alpha = g_L \frac{1 - \gamma^5}{2} \gamma^\alpha + g_R \frac{1 + \gamma^5}{2} \gamma^\alpha$$

where $g_L = g_V - g_A$ and $g_R = g_V + g_A$.

- (c) Finally, calculate the total cross section $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ and the forward-backward asymmetry

$$A = \frac{\sigma(\theta < \pi/2) - \sigma(\theta > \pi/2)}{\sigma(\theta < \pi/2) + \sigma(\theta > \pi/2)} \quad (6)$$

as functions of the total energy $E_{\text{c.m.}}$. For simplicity, approximate $\sin^2 \theta_w \approx \frac{1}{4}$ and hence $g_V \approx 0$.

Note that in QED the tree-level pair production is symmetric with respect to $\theta \rightarrow \pi - \theta$; the asymmetry in the Standard Model arises from the interference between the virtual-photon and virtual- Z diagrams.