PHY–396 L.   Problem set #15.   Due February 10, 2005.

1. First, a reading assignment: §7.4 of the *Peskin & Schroeder* textbook about the dia-
   grammatic proof of Ward–Takahashi identities for QED.

2. Next, consider "scalar QED", the theory of a charged scalar field $\phi(x) \neq \phi^*(x)$ coupled
   to the electromagnetic field $A^\mu(x)$,

   $$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}^2 + D^\mu\phi^* D_\mu\phi - M^2\phi^*\phi - \tfrac{1}{4}\lambda(\phi^*\phi)^2. \tag{1}$$

   Write down Ward–Takahashi identities for this theory and prove them using the dia-
   grammatic method.

3. Finally, consider a theory comprised of the EM field $A^\mu(x)$, a charged scalar field $\phi(x)$,
   a charged Dirac field $\psi(x)$, and a neutral Dirac field $\chi(x)$, interacting with each other
   according to

   $$\begin{aligned}
   \mathcal{L} = &-\tfrac{1}{4}F_{\mu\nu}^2 + D^\mu\phi^* D_\mu\phi - M^2\phi^*\phi - \tfrac{1}{4}\lambda(\phi^*\phi)^2 \\
   &+ \overline{\psi}(i\,\slashed{D} - m)\psi + \overline{\chi}(i\,\slashed{\partial} - m')\chi - \gamma\phi\overline{\psi}\chi - \gamma\phi^*\overline{\chi}\psi.
   \end{aligned} \tag{2}$$

   Write down the Ward–Takahashi identities for this theory and sketch out a proof.