Problem **1**:

When a unit of something is prepended with suffix "micro" it means one millionth of that unit.[⋆] A century is a unit of time equal to 100 years, hence a micro-century is one millions of a century,

$$1\,\mu\text{century} \;=\; \frac{1}{1000000}\times 100\,\text{years} \;=\; 0.0001\,\text{year}.$$

To compare this period in minutes, we note that $1\,\text{year} = 365.25\,\text{days}$, $1\,\text{day} = 24\,\text{hours}$, and $1\,\text{hour} = 60\,\text{minutes}$, thus

$$\begin{aligned} 1\,\mu\text{century} &= 0.0001\,\text{year}\times 365.25\,\frac{\text{days}}{\text{year}} \;=\; 0.036525\,\text{days} \\ &= 0.036525\,\text{days}\times 24\,\frac{\text{hours}}{\text{day}} \;=\; 0.8766\,\text{hours} \\ &= 0.8766\,\text{hours}\times 60\,\frac{\text{minutes}}{\text{hour}} \;=\; 52.596\,\text{minutes} \\ &\approx 52.6\,\text{minutes}. \end{aligned}$$

In other words, one microcentury is just a couple of minutes longer then your class.

Problem **2**:

Let's convert everything to metric units. A furlong is 220 yards, a yard is 3 feet, a foot is 0.3048 meter, thus

$$1\text{ furlong} \;=\; 220\times 3\times 0.3048\text{ m} \;=\; 201.168\text{ m}.$$

A fortnight is 14 day&nights of 24 hours each, and an hour is 3600 seconds, thus

$$1\text{ fortnight} \;=\; 14\times 24\times 3600\text{ s} \;=\; 1209600\text{ s}.$$

⋆ In other context, "micro-" simply means small. For example, a micro-brewery is a small brewery rather than one millions of a brewery. But that's because a brewery isn't a unit of some quantity.

Hence, a furlong per fortnight is a unit of speed equal to

$$1\,\frac{\text{furlong}}{\text{fortnight}} \;=\; \frac{1\ \text{furlong}}{1\ \text{fortnight}} \;=\; \frac{201.168\,\text{m}}{1209600\,\text{s}} \;=\; \frac{201.168}{1209600}\,\text{m/s} \;\approx\; 1.663\times10^{-4}\,\text{m/s}.$$

So converting the naturalist's report of snail's average speed to metric units we get $1.663\times10^{-4}\,\text{m/s} = 0.1663\text{mm/s}$. But of course we should not write down so many significant digits because the original data (1 furlong/fortnight) wasn't so precise; instead, we should round-off the metric value down to one or two significant digits, thus $v \approx 0.17\text{mm/s}$ or $\frac{1}{6}$ millimeter per second.

At this average speed, the snail would crawl in $t = 1\,\text{minute} \;=\; 60\,\text{s}$ through the distance

$$L \;=\; v\times t \;=\; 0.17\,\text{mm/s}\times 50\,\text{s} \;=\; 10.2\,\text{mm} \;\approx\; 10\,\text{mm} \;=\; 1\,\text{cm}.$$

In other words, in one minute the snail would crawl through approximately one centimeter.

<u>Problem</u> **3**:

The volume of the water in the lake is given by the product of lake's area A and its average depth d, $V = A\times d$, but we should evaluate this formula in consistent units. In other words, if we measure depth on meters and area in square meters then the volume comes in cubic meters, if we measure depth on feet and area in square feet then the volume comes in cubic feet, if we measure depth on centimeters and area in square centimeters then the volume comes in cubic centimeters, *etc.*, *etc.*.

For the problem at hand, it's easier to use meters, so we let $d = 1$ m while the area of 1 square kilometer should be written as one million square meters, $A = 1\,\text{km}^2 = \;1\,(10^3\,\text{m})^2 \;=\; 10^6\,\text{m}^2$. Therefore, the water volume is

$$V \;=\; A\times d \;=\; 10^6\,\text{m}^2\times 1\,\text{m} \;=\; 10^6\,\text{m}^3,$$

i.e., one million cubic meters.

Finally, one cubic meters is a thousand liters, hence

$$V \;=\; 10^6\,\mathrm{m}^3 \times 10^3\,\mathrm{L/m}^3 \;=\; 10^9\,\mathrm{L},$$

in other words, the lake contains one billion liters of water.

Problem **4**:

First, let's calculate Earth's net volume. Approximating its shape as a sphere of radius $R = 6,370$ kilometers or 6.27×10^6 meters, we evaluate

$$\begin{aligned} V \;=\; \frac{4\pi}{3}\,R^3 \;&=\; \frac{4\pi}{3} \times (6.37 \times 10^6\,\mathrm{m})^3 \;=\; \frac{4\pi}{3} \times 6.37^3 \times 10^{18} \times \mathrm{m}^3 \\ &\approx 1083 \times 10^{18} \times \mathrm{m}^3 \;=\; 1.083 \times 10^{21} \times \mathrm{m}^3. \end{aligned}$$

Consequently, the average density of the Earth is

$$\rho_E \;=\; \frac{M}{V} \;=\; \frac{6 \times 10^{24}\,\mathrm{kg}}{1.083 \times 10^{21}\,\mathrm{m}^3} \;\approx\; 5.5 \times 10^3\,\mathrm{kg/m}^3.$$

Or compared to the density of water $\rho_{\mathrm{water}} = 1\ \mathrm{g/cm}^3 \;=\; 1,000\ \mathrm{kg/m}^3$, the Earth is on average 5.5 denser than water.