Problem **17**:

The average acceleration is the ratio of the velocity change to the times in which this change happens,

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}.$$

The car decelerates from initial velocity $v_1 = 50$ mile/hr to final velocity $v_2 = 0$ in times $\delta t = t_2 - t_1 = 25$ seconds, thus

$$a_{\text{avg}} = \frac{0 - 50\,\text{mile/hr}}{25\,\text{s}} = -2\,\text{mile/hr/s} \approx 0.9\,\text{m/s}^2.$$

Note the negative sign of the average acceleration.

Problem **19**:

A freely falling body accelerates downward with a constant acceleration $a = g \approx 9.8\,\text{m/s}^2$. Assuming zero initial velocity $v_0 = 0$ of the rock at time $t = 0$, we have

$$v(t) = g \times t \qquad \text{and} \qquad x(t) = \tfrac{1}{2} g \times gt^2$$

where we measure x *down* from the initial position of the rock. Evaluating the above formulæfor times $t = 1$, 2, 3, 3.5, and 4.5 seconds, we find

t	v	x
1.0 s	9.8 m/s	4.9 m
2.0 s	19.6 m/s	19.6 m
3.0 s	28.4 m/s	44.1 m
3.5 s	34.3 m/s	60.0 m
4.5 s	44.1 m/s	99.2 m

Problem **21**:

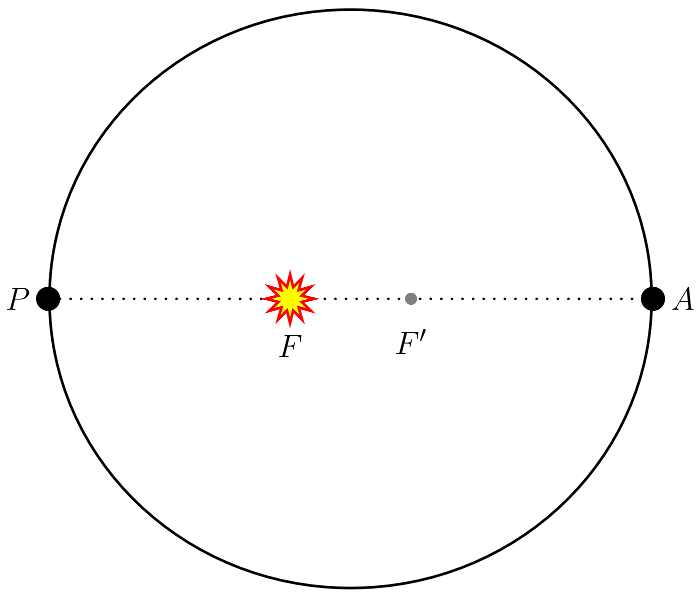
In the absence of air resistance, the horizontal motion of the ball is independent of its vertical motion and vice verse. Since the baseball was hit in the horizontal direction, its vertical motion is a free fall with zero initial velocity, while the horizontal motion has constant velocity $v_x = v_0 = 30$ m/s. Thus, counting x and y coordinates of the ball from the initial point at the top of the cliff, we have

$$x(t) \;=\; v_0 \times t \qquad \text{and} \qquad y(t) \;=\; -\tfrac{1}{2} g \times t^2.$$

Evaluating these formulæ for $t = 3$ seconds when the ball hits the ground, we find $x = 90$ m and $y = -44.1$ m, *i.e.*, the ball is 90 meters from the cliffs edge and 44.1 meters below the clifftop. In other words, the ball lands 90 m away from the cliff, and the cliff is 44.1 m high (assuming the ground below the cliff is level).

The non-textbook problem, part (a):

According to Kepler's first law, Mercury's orbit is an ellipse with the Sun in one of its focal points:



On this picture the focal points are labeled F and F' — the sun is at F while there isn't anything interesting at F', — P indicate the perihelion (the point of closest approach to the Sun) and A indicates the aphelion (the most distant point of the orbit). The perihelion and the aphelion lie at two ends of the major axis of the ellipse; one half of its length $a = \frac{1}{2}(PA)$ is called the major semi-axis. Non-technically, the semi-major axis of a planetary orbit is often referred to as the "average radius" of the orbit.

The eccentricity of an ellipse is the ratio of the distance between the focal points to the major axis's length,

$$\epsilon = \frac{(FF')}{(PA)}.$$

Note that the points P, F, F', and A all lie on the major axis, and the distances PA and AF' are equal by symmetry of the ellipse. Consequently,

$$(PA) = (PF) + (FF') + (F'A) = 2(PF) + (FF') = 2(PF) + \epsilon \times (PA)$$

and therefore

$$(PF) = \frac{1}{2}\Big((PA) - \epsilon \times (PA)\Big) = \frac{1-\epsilon}{2} \times (PA) = (1-\epsilon) \times a.$$

And for a planetary orbit, it means that the distance between the perihelion and the Sun — *i.e.*, the shortest distance between the planet and the sun — is $(1-\epsilon)\times a$. As for the longest distance — which happens at the aphelion — we have

$$(AF) = (PA) - (PF) = 2a - (1-\epsilon) \times a = (1+\epsilon) \times a.$$

For the specific case of Mercury, we have $a = 0.387$ au and $\epsilon = 0.2$, hence

$$\text{the shortest distance} = (1-\epsilon) \times a = 0.310 \text{ au} = 46.3 \times 10^6 \text{ km},$$
$$\text{the longest distance} = (1+\epsilon) \times a = 0.465 \text{ au} = 69.5 \times 10^6 \text{ km}.$$

The non-textbook problem, part (b):

By Kepler's third law, the square of a planetary orbital period (*i.e.*, the planetary year) is proportional to the cube of the semi-major axis of the orbit,

$$T^2 \;\propto\; a^3.$$

Comparing Mercury and Earth, we have

$$\left(\frac{T_M}{T_E}\right)^2 \;=\; \left(\frac{a_M}{a_E}\right)^3$$

and hence

$$T_M \;=\; T_E \;\times \left(\frac{a_M}{a_E}\right)^{3/2}.$$

Mercury has $a = 0.387$ au while Earth has $a = 1$ au (by definition of the astronomical unit), thus $a_M/a_E = 0.387$ and therefore

$$T_M \;=\; T_E \times 0.387^{3/2} \;=\; T_E \times 0.241\,.$$

In other words, one Mercurian year is 0.241 Earth years, or approximately 88 Earth days.