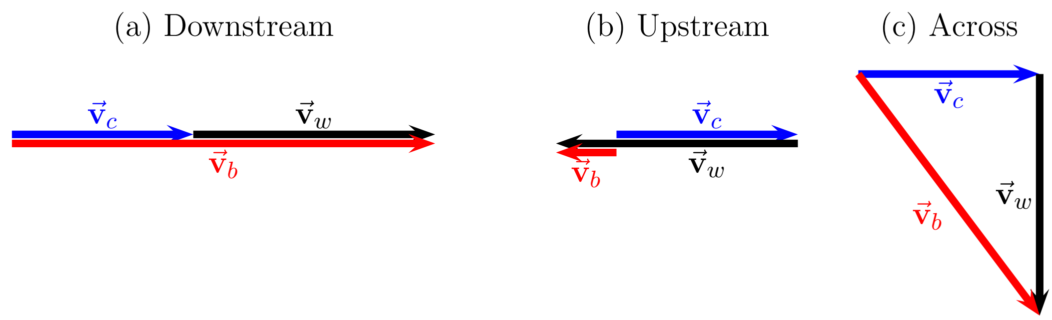
Problem **1**:

The velocity $\vec{\mathbf{v}}_b$ of the boat relative to the river's banks is the vector sum of its velocity $\vec{\mathbf{v}}_w$ relative to the water and the current's velocity $\vec{\mathbf{v}}_c$ (relative to the banks),

$$\vec{\mathbf{v}}_b = \vec{\mathbf{v}}_w + \vec{\mathbf{v}}_c .$$

Notice the velocities here add as vectors, so the magnitude of the sum depends on directions of the boat's motion relative to the current's.




(**a**) When the boat steams downstream, the vectors $\vec{\mathbf{v}}_w$ and $\vec{\mathbf{v}}_c$ are in the same direction, so their magnitudes add up:

$$|\vec{\mathbf{v}}_w + \vec{\mathbf{v}}_c| = |\vec{\mathbf{v}}_w| + |\vec{\mathbf{v}}_c| = 8 \text{ m/s} + 6 \text{ m/s} = 14 \text{ m/s}.$$

Thus, relative to the banks, the boat moves at 14 m/s.

(**b**) When the boat steams upstream, the vectors $\vec{\mathbf{v}}_w$ and $\vec{\mathbf{v}}_c$ have opposite directions, so their magnitudes subtract instead of adding up:

$$|\vec{\mathbf{v}}_w + \vec{\mathbf{v}}_c| = |\vec{\mathbf{v}}_w| - |\vec{\mathbf{v}}_c| = 8 \text{ m/s} - 6 \text{ m/s} = 2 \text{ m/s}.$$

Thus, relative to the banks, the boat moves at 2 m/s.

(**c**) When the boat steams across the river $\vec{\mathbf{v}}_w$ and $\vec{\mathbf{v}}_c$ are perpendicular to each other, so their sum's magnitude follows from Pythagoras's theorem:

$$|\vec{\mathbf{v}}_w + \vec{\mathbf{v}}_c|^2 = |\vec{\mathbf{v}}_w|^2 + |\vec{\mathbf{v}}_c|^2 = (8\text{ m/s})^2 + (6\text{ m/s})^2$$
$$= 64\ (m/s)^2 + 36\ (m/s)^2 = 100\ (m/s)^2$$

and therefore

$$|\vec{\mathbf{v}}_w + \vec{\mathbf{v}}_c| = \sqrt{100\ (\text{m/s})^2} = 10\text{ m/s}.$$

Thus, the speed of boat's motion relative to the river's banks is 10 m/s.

Problem **2**:

(a) As the Earth spins around its axis every 24 hours, the UT campus moves through a circle of radius $R = 5400$ km. The length of this circle is $2\pi \times R = 2\pi \times 5400\text{ km} = 33930$ km, so the speed of the circular motion is

$$v = \frac{2\pi R}{T} = \frac{33930\text{ km}}{24\text{ hours}} = 1414\text{ km/hr} = 1414 \times \frac{1000\text{ m}}{3600\text{ s}} = 393\text{ m/s}.$$

(b) Circular motion at constant speed has centripetal acceleration

$$a_c = \frac{v^2}{R} = \frac{(393\text{ m/s})^2}{5,400,000\text{ m}} = 0.028\text{ m/s}^2.$$

Problem **3**:

The force needed to accelerate the car follows from Newton's second law $F = ma$. But first, we need to calculate the car's acceleration. Assuming the car accelerates in a straight line at uniform rate, we have

$$a = \frac{\Delta v}{\Delta t} = \frac{60 \text{ miles/hr}}{6 \text{ s}} = 10 \text{ mile/hr/s}$$

or in metric units

$$a = 10 \text{ mile/hr/s} \times \frac{1609 \text{ m/mile}}{3600 \text{ s/hr}} = 4.47 \text{ m/s}^2.$$

Consequently, the force acting on the car is

$$F = m \times a = 1000 \text{ kg} \times 4.47 \text{ m/s}^2 = 4470 \text{ N}.$$

Problem **4**:

By Newton's third law, Bob pulls Alice with the same force as Alice pulls Bob. By the second law, we can find this force from Bob's mass and acceleration,

$$F = m_{\text{Bob}} \times a_{\text{Bob}} = 75 \text{ kg} \times 2 \text{ m/s}^2 = 150 \text{ N}.$$

The same force acting on Alice makes her accelerate at

$$a_{\text{Alice}} = \frac{F}{m_{\text{Alice}}} = \frac{150 \text{ N}}{50 \text{ kg}} = 3 \text{ m/s}^2.$$

Note that the 'action' and 'reaction' forces have equal magnitudes but opposite directions. This means that Bob and Alice pull each other in opposite directions, and consequently they accelerate in opposite directions as well. So if we take Bob's acceleration as positive, $a_{\text{Bob}} = +2 \text{ m/s}^2$, then Alice's acceleration is negative, $a_{\text{Alice}} = -3 \text{ m/s}^2$.

However, for the purpose of this homework it is enough to calculate the magnitude of Alice's acceleration, so saying $a_{\text{Alice}} = 3 \text{ m/s}^2$ is OK.